Trapping of electron Bernstein waves in drift-wave eddies and parametric decay instabilities at ECRH experiments in toroidal plasmas

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Introduction. High power ECRH is widely used nowadays and is considered for application in ITER. The standard theoretical analysis [1], which dealt with a monotonous plasma density profile, predicted that the parametric decay instabilities (PDIs) are deeply suppressed in ECRH heating experiments by convective losses of daughter waves from a decay layer along the magnetic field and in the radial direction. However, a number of observations obtained last decade evidences the presence of anomalous phenomena, eloquent example of which is anomalous ion heating often observed at ECRH experiments in toroidal devices [2–4]. In response to these experimental challenges, a theory [5–7] have been developed interpreting the anomalous ion heating in terms of the pump wave parametric decay into the electron Bernstein wave (EBW) and the low frequency wave, interacting with ion through the ion cyclotron resonance mechanism. The cornerstone of the theoretical model is a phenomenon of the 2D daughter wave trapping due to non-monotonous plasma density profile, produced by the pump out effect and the poloidal magnetic field inhomogenuity [5]. In the case of tokamak plasma possessing axial symmetry the 3D cavity for the daughter wave can be excited that leads to the pump 2nd harmonic extraordinary wave absolute PDI [6, 7]. As the EBW radial localization is solely possible in case of narrow enough density maximum, it's not universal. Here, we analyze a new universal scenario of the low-threshold PDI applicable to the stellarator experiment as well and based on the parametric excitation of EBW trapped in drift wave eddies, filaments or blobs possessing density maximum and aligned with magnetic field that is demonstrated both by ray tracing consideration and by analytical treatment.

Ray tracing of the EBW in the presence of a coherent turbulent structure. Due to complicated nature of the hot-plasma Bernstein modes, the only suitable approach to analysing their propagation is the ray tracing procedure. Assuming the EBW electromagnetic component negligible we use the non-relativistic equation dispersion function

$$D_\omega = q^2 + \frac{2\omega_{pe}^2}{\upsilon_{te}^2}\left[1 - \frac{\omega}{|q_\parallel|\upsilon_{te}}\sum_{m=-\infty}^{\infty} Z\left(\frac{\omega - m\omega_{ce}}{q_\parallel \upsilon_{te}}\right)\exp\left(-\frac{q_\perp^2\upsilon_{te}^2}{2\omega_{ce}^2}\right) I_m\left(\frac{q_\perp^2\upsilon_{te}^2}{2\omega_{ce}^2}\right)\right]. \quad (1)$$

Here $\xi Z(\xi) = \frac{\xi}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-s^2)}{\xi - s}ds = X(\xi) - iY(\xi)$ is the plasma dispersion function, $q_\parallel$, $q_\perp$ are the components of the wave vector parallel and perpendicular to the magnetic field $\vec{H}$, and I_m is the modified Bessel function of the first kind. In order to model the coherent turbulence structure, being aligned with the magnetic field line and possessing the finite size perpendicular to it, we introduce the density profile as a sum of the background term n_0 and a sharp perturbation, originated due to the drift-wave eddy, filament or blob:

$$\delta n = \delta_n n_0 \exp\left(-(\rho-\rho_0)^2/l^2 - \rho^2\left(\vartheta - \varphi/q(\rho)\right)^2/l^2\right), \quad (2)$$

Here ρ and ρ_0 are the magnetic surface radius and the radius at which the perturbation is maximal, ϑ and φ are the poloidal and toroidal angles, $q(\rho)$ is the safety factor, $\delta_n = \delta n / n_0 << 1$ and l is the width of the perturbation. For our analysis we choose the position of density perturbation (2) close to the magnetic axis $\rho_0 = 3\ cm$, $l = 0.3\ cm$ in the millimeter scale that corresponds to trapped electron mode typical of the ECRH experiments. We suppose also temporal variation of the density fluctuation negligible during period or inverse growth rate of the decay waves under consideration in this paper. In the presence of the density perturbation (2) in a close vicinity of the mid-plane the ray-tracing analysis for the TCV tokamak experimental conditions ($R_0 = 90.8\ cm,\ \ a = 24\ cm,\ T_e = 2keV,$

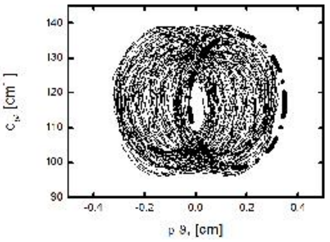

Fig. 1. Phase portrait of the ray trajectory in the poloidal direction.

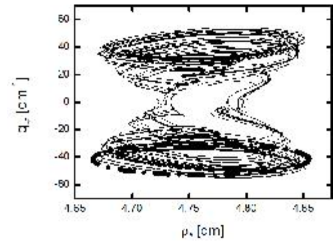

Fig. 2. Phase portrait of the ray trajectory along the flux coordinate.

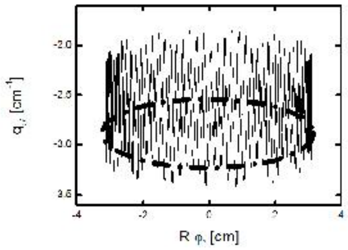

Figure 3. Phase portrait of the ray trajectory in the toroidal direction.

Dash - dotted lines – Eq. (3) for $m=1$, $n=0$, $k=0$; $n_0 = 3\cdot 10^{13} cm^{-3}$, $T_e = 2keV$, $H_0 = 13.7kGs$, $\omega_{EB}/2\pi = 82.3GHz$.

$n_0 = 3\cdot 10^{13} cm^{-3}$, $H_0 = 13.7kGs$) yields the phase dependences of the ray, shown in figures 1 – 3 and demonstrating its regular, finite but nevertheless complicated behaviour. The ray tracing procedure, the results of which are given in figures 4 – 6, shows also that the motion of the ray possesses quite different "time" scales $|\partial \ln q_\eta(t)/\partial t| >> |\partial \ln q_\rho(t)/\partial t| >> |\partial \ln q_\parallel(t)/\partial t|$, where $q_\eta = q_\vartheta H_\varphi / H - q_\varphi H_\vartheta / H$, $q_\parallel = \vec{q}\cdot\vec{H}/H$. It is worth noting that the EBW remains trapped, and the ordering of the ray trajectory oscillations persists if the relativistic corrections [8] to the electrostatic EBW dispersion relation $D'_\omega(\vec{q},\vec{r}) = 0$ are taken into account. The only difference important for the PDI analysis is that the frequency of the trapped EBW is downshifted due to the relativistic correction to the value $\omega_{EB}/2\pi = 81.65GHz$ for $T_e = 2keV$ and to $\omega_{EB}/2\pi = 80.8GHz$ for $T_e = 5keV$. Thus, even small amplitude density perturbation leads to the 3D EBW trapping in a volume less than the volume of the EBW toroidal cavity for an axisymmetric non-monotonic density profile [6, 7].

Semi-analytical description of the EBW trapped by the density perturbation. To describe analytically the 3D ray trapping, we use strong difference between "time" scales of the ray motions in different projections. In this way we can describe the ray propagation along η coordinate assuming its adiabatic dependence on the radial ρ coordinate and the coordinate along the magnetic field ξ. In its turn, the description of the ray motion along the radial ρ direction can be obtained assuming adiabatic dependence on the coordinate along the magnetic field ξ. To simplify the analysis we look for the solution of the Hamiltonian equations in a vicinity of the EBW turning point in the direction perpendicular to the magnetic field. Solving the system $D'_\omega(\vec{q},\vec{r},\omega)\big|_{\vec{q}_E,\vec{r}_E,\omega_E} \equiv D'_\omega\big|_0 = 0$, $\partial D'_\omega(\vec{q},\vec{r},\omega)/\partial q_\perp\big|_0 = 0$, $\partial D'_\omega(\vec{q},\vec{r},\omega)/\partial \eta\big|_0 = 0$, where the first equation is the EBW dispersion relation, the second one is a condition of the turning point of its dispersion curve and the latter is the extremum condition for the dispersion function over coordinate perpendicular to the magnetic field, gives $\omega = \omega_E$, $\vec{q}_E = (q_\rho = 0, q_\eta = q_E, q_\parallel = 0)$ and $\vec{r}_E = (\rho_E, \eta = 0, \xi = 0)$. Expanding the dispersion function into the Taylor series (for the details of the procedure we can refer readers to [5-7]) in a vicinity of $(\omega_E, \vec{q}_E, \vec{r}_E)$, using strong difference between "time" scales in different projections and imposing the conditions equivalent to the Bohr-Sommerfeld quantization finally gives parameters of the EBW WKB eigenmode:

$$\phi_\eta(\eta,m) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{Q_\eta(0)}{Q_\eta(\eta)}}\left[\exp\left(i\int^\eta Q_\eta(s)ds\right) + (-1)^m c.c.\right], Q_\eta = \left.\frac{\partial^2 D'_\omega}{2\partial q_\eta^2}\right|_\eta^{-1/2}\sqrt{-D'_\omega\big|_\eta - \left.\frac{\partial^2 D'_\omega}{2\partial \eta^2}\right|_\eta \eta^2},$$

$$\phi_x(x,n) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{Q_x(0)}{Q_x(x)}}\left[\exp\left(i\int^x Q_x(s)ds\right) + (-1)^n c.c.\right], Q_x = \left.\frac{\partial^2 \tilde{D}'_\omega}{2\partial q_\rho^2}\right|_x^{-1/2}\sqrt{-\tilde{D}'_\omega\big|_x - \left.\frac{\partial^2 \tilde{D}'_\omega}{2\partial \rho^2}\right|_x x^2}, \tag{3}$$

$$\phi_\xi(\xi,k) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{Q_\parallel(0)}{Q_\parallel(\xi)}}\left[\exp\left(i\int^\xi Q_\parallel(s)ds\right) + (-1)^k c.c.\right], Q_\parallel = \left.\frac{\partial^2 \bar{D}'_\omega}{2\partial q_\parallel^2}\right|_0^{-1/2}\sqrt{(\omega'_{EB} - \omega_E)\left.\frac{\partial \bar{D}'_\omega}{\partial \omega}\right|_0 + \bar{D}'_\omega\big|_0 - \left.\frac{\partial^2 \bar{D}'_\omega}{2\partial \xi^2}\right|_0 \xi^2}$$

where $D'_\omega|_\eta = D'_\omega(q_\eta = q_E, \eta = 0; q_\rho, q_\parallel, \rho, \xi)$, $\tilde{D}'_\omega == 0$ is a reduced dispersion function making connection between the arguments $q_\rho, q_\parallel, \rho, \xi$ it depends adiabatically on, $\tilde{D}'_\omega|_x = \tilde{D}'_\omega(q_\rho = q_{\rho 0}, \rho = \rho_E; q_\parallel, \xi)$, $\bar{D}_\omega = 0$ is a reduced dispersion function depending adiabatically on $q_\parallel, \xi$, $\bar{D}'_\omega\big|_0 = \bar{D}'_\omega(q_\rho = q_{\rho 0}, \rho = \rho_E)$. The EBW

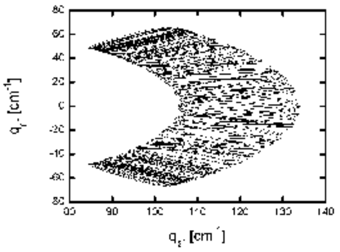

Fig. 4. Dependence of q_ρ on q_η for the same parameters as in figs 1-3.

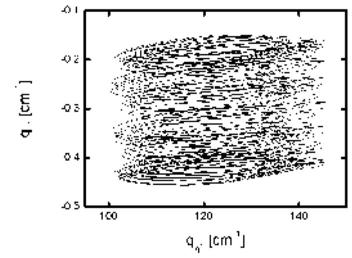

Fig. 5. Dependence of $q_\parallel$ on q_η for the same parameters as in figs. 1-3.

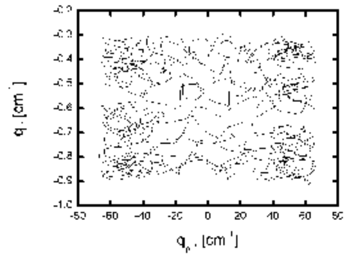

Figure 6. Dependence of $q_\parallel$ on q_ρ for the same parameters as in figures 1-3.

eigenfrequency is $\omega_{EB} = \omega'_{EB}(m,n,k) + i/4\pi \cdot \partial \bar{D}'_\omega / \partial\omega \big|_0^{-1} \sqrt{\partial^2 \tilde{D}'_\omega / \partial \rho^2 \big|_x \partial^2 \tilde{D}'_\omega / \partial q_\rho^2 \big|_x} \exp\left[-2\int_{x_3}^{x_2} Q_x(x)dx\right]$, where the last term describes damping of the trapped wave due to its radiative losses.

Parametric excitation of the trapped EBW by the 2nd harmonic extraordinary pump wave. To elucidate the physics of the 3D trapped EBW parametric excitation we analyze relevant to the experiment three-wave interaction model in which a wide X microwaves beam propagates from the launching antenna inwards plasma almost opposite to major radius in the tokamak mid-plane $E_{0y} = a_0/2\exp\left[-(\eta^2+\xi^2)/2w^2 + ik_{\rho 0}x + ik_{\parallel 0}\xi - i\omega_0 t\right] + \text{c.c.}$, $a_0^2 = 8P_0/cw^2$, P_0 and w are the pump wave power and the beam waist. The basic set of integral equations describing the decay of the X pump wave into the daughter EBW $\vec{E}_\omega = -\vec{\nabla}\phi_\omega \sim \exp(-i\omega t)$ and a low frequency potential daughter wave $\vec{E}_\Omega = -\vec{\nabla}\phi_\Omega \sim \exp(i\Omega t)$, $\Omega = \omega_0 - \omega << \omega_0$ is given by

$$\int_{-\infty}^{\infty} d\vec{q}d\vec{r}' D_\Omega(\vec{q},\vec{r}+\vec{r}')\exp\left[i\vec{q}(\vec{r}-\vec{r}')\right]\phi_\Omega(\vec{r}') = 4\pi\rho_\Omega(\vec{r}),\ \int_{-\infty}^{\infty} d\vec{q}d\vec{r}' D_\omega(\vec{q},\vec{r}+\vec{r}')\exp\left[i\vec{q}(\vec{r}-\vec{r}')\right]\phi_\omega(\vec{r}') = 4\pi\rho_\omega(\vec{r}), \quad (4)$$

where $\rho_\Omega \simeq -\chi_i(\Omega,q_I)/4\pi \cdot E_{0y}^*/3H \cdot \phi_\omega$, $\rho_\omega \simeq -\chi_i(\Omega,q_I)/4\pi \cdot E_{0y}/3H \cdot \phi_\Omega$ [9]. The expression for D_ω is given by (1). In a multi ion-component plasma D_Ω is [10,11]:

$$D_\Omega = q^2 + \frac{2\omega_{pe}^2}{\upsilon_{te}^2}\left[1 - \frac{\Omega}{|q_{I\parallel}|\upsilon_{te}} Z\left(\frac{\Omega}{q_{I\parallel}\upsilon_{te}}\right)\exp\left(-\frac{q_{I\perp}^2\upsilon_{te}^2}{2\omega_{ce}^2}\right) I_0\left(\frac{q_{I\perp}^2\upsilon_{te}^2}{2\omega_{ce}^2}\right)\right] + \sum_j \frac{2\omega_{pj}^2}{\upsilon_{tj}^2}\left[1 - X\left(\frac{\Omega}{q_{I\perp}\upsilon_{tj}}\right) - iY\left(\frac{\Omega}{q_{I\perp}\upsilon_{tj}}\right)\right] \quad (5)$$

where $\vec{q}_I = (q_{\rho E} - k_{\rho 0}, q_{\eta E}, -\omega_0 \sin\vartheta_0/c)$, $\vartheta_0 \approx 10^o \div 12^o$ is a tilt angle of the pump wave, and summation over the sorts of ions j is performed. In (5), we have kept in mind that for the low frequency oscillations, propagating obliquely $|q_\parallel| = \omega_0 \sin\vartheta_0/c \geq \max[\omega_{cj}/\upsilon_{tj}]$, the ion cyclotron resonance layers are strongly overlapped and thus the high frequency ion susceptibility appears to be "unmagnetized". We analyze the EBW cavity parametric excitation by using the perturbation theory approach. Following the algorithm proposed in [12], at the first step of the perturbation procedure we neglect both the trapped EBW dissipative and radiative losses and its non-linear pumping. In this approximation we represent the EBW potential as $\phi_\omega(\vec{r}) = b_\omega(\vec{r})\exp(iq_{\rho E}x + iq_{\eta E}\eta - i\omega t)$. Assuming the rhs of both equations constituting the system (4) small, we neglect them in zero order approximation and obtain unperturbed equation $\hat{D}'_\omega\{b_\omega\} = 0$ which, according to the previous section analysis, has at $\omega = \omega_{EB}$ the localized WKB solution $b_\omega^{mnk} = \phi_\eta(\eta,m)\phi_x(x,n)\phi_\xi(\xi,k)$. At the second step of the perturbation procedure we take into account the trapped EBW dissipative and radiative losses as well as the non-linear pumping. Substituting the unperturbed EBW solution into the low-frequency wave equation (4) and seeking the potential of these induced oscillations in the form $\phi_\Omega(\vec{r}) = b_\Omega(\vec{r})\exp(i(q_{\rho E} - k_{\rho 0})x + iq_{\eta E}\eta - ik_{\parallel 0}\xi + i\Omega t)$ yields their amplitude in the form $b_\Omega(\vec{r}) = -\chi_i(\Omega,q_I)/D[\Omega,q_I] \cdot a_0^* b_\omega^{mnk}(\vec{r})/6H\exp\left[-(y^2+z^2)/2w^2\right]$. $D[\Omega,q_I] = D'[\Omega,q_I] - iD''[\Omega,q_I]$, $D'[\Omega,q_I]$ is the residual part of the dispersion relation of the induced low frequency oscillations, $D''[\Omega,q_I]$ describes their strong absorption by the electrons or ions. Substituting $b_\Omega(\vec{r})$ into rhs of the high frequency equation in (4), multiplying it by $(b_\omega^{mnk})^*$ and integrating over the plasma volume, we arrive at the PDI growth (damping) rate

$$\gamma_{m,n,k} = \nu_0 \frac{V_{PDI}}{V_{cavity}} - \nu_E \tag{6}$$

where $\nu_0 = \frac{|a_0|^2}{36H^2} \frac{\chi_i^2(\Omega,q_I)D''(\Omega,q_I)}{|D(\Omega,q_I)|^2} \left|\frac{\partial D'_\omega}{\partial \omega}\right|_0^{-1}$ is the growth rate of the PDI in homogeneous plasma theory, ν_E characterizes the EBW damping and its radiative losses, $<...>$ stands for averaging over the eigenmode localization region, V_{cavity} is the volume of the eigenmode localization, V_{PDI} is proportional to the volume element within which the three-wave interaction of the pump and daughter waves occurs. The absolute PDI threshold can be found from the condition $\gamma_{m,n,k} = 0$ as follows $\nu_0\left(P_0^{th}\right) = \nu_E V_{cavity} / V_{PDI}$. For the deuterium discharge with small hydrogen impurity fraction ($n_H / n_D = 5\%,\ T_{D,H} = 0.3keV$) in TCV tokamak, we finally get an estimate of the PDI threshold $P_0^{th} = 7.4kW$. The PDI growth rate $2\gamma_{1,0,0} \simeq 1.2 \cdot 10^8 s^{-1}$ obtained for the pump wave power $P_0 = 1.5MW$ is much higher than the frequency range typical for the long-scale drift turbulence. In this case the low frequency daughter wave appears to be a slow ion sound wave heavily damped due to the ion Landau damping. This wave directly transfers the pump power to the ion component. In the course of the auxiliary ECRH the electron temperature raises abruptly up to $T_e = 5keV$ [13] at the approximately same or little less density. Under these conditions, the 2nd daughter wave is the heavily damped oscillation induced at frequency $\Omega = 1.6\ GHz$ by the EC and EB waves non-linear coupling. This oscillation interacts with accelerated ions. Thus the pump energy transfer to the ion channel occurs. In this case the PDI threshold is higher $P_0^{th} = 200kW$. However, it is still exceeded at the ECRH power values actually used in the TCV experiments. The growth rate for the pump wave power $P_0 = 1.5MW$ is about $2\gamma_{1,0,0} \simeq 0.9 \cdot 10^5 s^{-1}$ exceeding the frequency range typical for the long-scale drift turbulence component $\sim 10 \div 100kHz$. Note that a powerful ECRH system providing a total power of 4.5 MW is installed on TCV. A well focused system of the wave launchers allows for localized plasma heating and rising the electron temperature up to $4 \div 6\ keV$. Under these circumstances, more than one pump wave beam can experience a passage through the same turbulent structure, increasing thus the PDI growth rate up to $2\gamma_{1,0,0} \simeq 0.45 \times M \cdot 10^5 s^{-1}$ (M is an integer less than 7).

Conclusions. In this paper the low-threshold absolute PDIs of the 2nd harmonic X wave are analyzed. The key element of these models is the coherent structures always arising in the plasma turbulence and leading to both the non-monotonous plasma density profile and the axial symmetry breaking. It should be stressed the possible role of the instability in anomalous absorption of the microwave power by the ion component and, in particular, in fast ion production often observed in second harmonic ECRH in toroidal plasmas.

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[1] M. Porkolab et al., Nucl. Fusion 28, 239 (1988).
[2] A.N. Karpushov, et al., in Proc. 33rd EPS Conf. on Plasma Physics **30I**, P-1.152 (2006).
[3] D. Rapisarda, B. Zurro, V. Tribaldos, et al., PPCF **49**, 309 (2007).
[4] A.Bruschi, et al., in Proc.39th EPS Conf.&16th Int.Cong.on Plasma Physics **36F**, P-1.041 (2012).
[5] A.Yu. Popov, E.Z. Gusakov, A.N. Saveliev, JETP Letters **96**, 164–170 (2012).
[6] E. Gusakov, A. Popov, EPL 99, 15001 (2012).
[7] E.Z. Gusakov, A.Yu. Popov, A.N. Saveliev, EPJ Web of Conferences **32**, 01002 (2012).
[8] A.N. Saveliev, Plasma Phys. Control. Fusion 49, 1061 (2007).
[9] V.P. Silin, *Parametric Action of High Power Radiation on a Plasma* (Nauka, Moscow, 1973)
[10] D.G. Swanson, *Plasma Waves* (Academic Press, Boston, 1989).
[11] A.D. Piliya, A.N. Saveliev, *Plasma Phys. Control. Fusion* **36**, 2059 (1994).
[12] E.Z. Gusakov, V.I. Fedorov, *Plasma Phys. Reports* **5**, 827 (1979).
[13] S. Coda, I. Klimanov, S. Alberti, et al, *Plasma Phys. Control. Fusion* **48**, B359 (2006).