RF induced density depletion due to the Ponderomotive force in presence of strong magnetic fields

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Abstract

Adopting the usual cold plasma dielectric tensor and an expression for the Ponderomotive force in presence of a strong magnetic field, it is demonstrated that a rapidly oscillating electric field gives rise to slow time scale drifts which cause density modifications near antennas. Neglecting the magnetic field when deriving the Ponderomotive force leads to a misinterpretation of the direction in which flows are induced. When a strong magnetic field is present, the poloidal gradients of the electric field (in particular near the antenna end points) are at the origin of radial displacements of the plasma while radial electric field gradients have the potential to trigger density inhomogeneity along the antenna. The RF induced plasma drifts are more prominent at higher power and for more evanescent modes.

Ponderomotive force in presence of a strong magnetic field

The usual expression for the Ponderomotive force (see e.g. [1]) is derived in absence of a strong magnetic field. For tokamak applications, it needs to be generalized. This can readily be done by adopting the same philosophy but including the forcing done by the confining field in the equation of motion from which the derivation starts [2]. For constant temperature, the relevant fluid type equation of motion for each type of particles is

$$\frac{d}{dt}\vec{v} = -v_t^2\frac{\nabla N}{N} + \vec{\varepsilon}e^{-i\omega t} + \Omega\vec{v}\times\vec{e}_{//}$$

in which $v_t^2 = kT/m$, $\vec{\varepsilon} = q\vec{E}/m$, $\Omega = qB_o/m$ and $\vec{e}_{//} = \vec{B}_o/B_o$; N is the density, m the mass, T the temperature, q the charge and $\vec{v}$ the velocity of the flow of one of the types of particles in the plasma, $\vec{E}$ is the electric field driven at frequency ω and $\vec{B}_o$ is the static magnetic field. Note that - aligned with what is done in deriving the usual cold plasma dielectric tensor and is fully justified for particles that are not too energetic - the *driven* magnetic field has been neglected since it is a small correction to the terms retained. For simplicity, it is assumed that the static magnetic field $\vec{B}_o$ is straight and homogeneous. Anticipating that the motion can be split into a fast dynamics part that responds on the time scales imposed both by the generator and the strong confining magnetic field, and a slow dynamics part which accounts for remaining net drifts when averaging over all fast time scales, we split both velocity $\vec{v} = \vec{v}_o + \vec{v}_1$ and position $\vec{x} = \vec{x}_o + \vec{x}_1$ into a zero order and a perturbed part. The electric field is inhomogeneous in general. Assuming the perturbation is small, $\vec{\varepsilon}$ can be approximated by the truncated Taylor series expansion $\vec{\varepsilon}(\vec{x}) \approx \vec{\varepsilon}(\vec{x}_o) + (\vec{x}_1 - \vec{x}_0).\nabla\vec{\varepsilon}(\vec{x}_o)$ so that the fast dynamics is governed by the equation

$$\frac{d}{dt}\vec{v}_1 = +\vec{\varepsilon}(\vec{x}_o)e^{-i\omega t} + \Omega\vec{v}_1\times\vec{e}_{//}$$

in which it has been assumed that the *fast time scale density* variations either are absent or can be neglected ($N \approx N_o$). The slow dynamics are captured by

$$\frac{d}{dt}\vec{v}_o = -v_t^2 \frac{\nabla N_o}{N_o} + \frac{1}{2} Re[< (\vec{x}_1^* - \vec{x}_0^*).\nabla\vec{\varepsilon}(\vec{x}_o)e^{-i\omega t} >] + \Omega\vec{v}_o \times \vec{e}_{//}$$

with $< ... >$ representing the averaging over *all* fast oscillations and '*' denoting the complex conjugate. The equation of motion for the fast dynamics can be solved and yields

$$x_{\perp 1} = x_{\perp 1,0} + \frac{1}{2}\Big[\frac{v_{+0}}{-i\Omega}e^{-i\Omega t} + \frac{v_{-0}}{i\Omega}e^{i\Omega t}\Big] - \frac{\omega\varepsilon_{\perp 1} + i\Omega\varepsilon_{\perp 2}}{\omega(\omega^2 - \Omega^2)}e^{-i\omega t},$$

$$x_{\perp 2} = x_{\perp 2,0} + \frac{1}{2i}\Big[\frac{v_{+0}}{-i\Omega}e^{-i\Omega t} - \frac{v_{-0}}{i\Omega}e^{i\Omega t}\Big] - \frac{\omega\varepsilon_{\perp 2} - i\Omega\varepsilon_{\perp 1}}{\omega(\omega^2 - \Omega^2)}e^{-i\omega t},$$

$$x_{//} = x_{//0} + v_{//0}t - \frac{\varepsilon_{//}}{\omega^2}e^{-i\omega t}.$$

In tokamaks, the magnetic field is of the order of a few Tesla, while the electric field excited by RF antennas is commonly of the order of a few $10^4 V/m$. Taking the thermal velocity as a measure for the typical velocities reached, the ion velocity is of order $10^4 m/s$ for typical edge temperatures of a few eV. Hence, the cyclotron gyration term and the electric field term are of equal importance in the edge ($|E/[vB_o]| \approx 1$) while in the hot plasma core where the electric field is propagative rather than evanescent, the more customary $|E/[vB_o]| << 1$ holds.

With the obtained expressions, the Ponderomotive term in presence of a strong magnetic field can now be evaluated. Only retaining net non-oscillatory terms one finds

$$\vec{a}_{Pond} = -\frac{1}{2}Re\Big[+\frac{1}{\omega(\omega^2-\Omega^2)}\Big([\omega\varepsilon_{\perp 1}^* - i\Omega\varepsilon_{\perp 2}^*]\frac{\partial}{\partial x_{\perp 1}}\vec{\varepsilon} + [\omega\varepsilon_{\perp 2}^* + i\Omega\varepsilon_{\perp 1}^*]\frac{\partial}{\partial x_{\perp 2}}\vec{\varepsilon}\Big)$$
$$+\frac{1}{\omega^2}\varepsilon_{//}^*\frac{\partial}{\partial x_{//}}\vec{\varepsilon} - \frac{iv_{//0}^*}{\omega}\frac{\partial}{\partial x_{//}}\vec{\varepsilon}\Big]. \quad (1)$$

Klima [4] followed a slightly different approach when deriving an expression for the Ponderomotive force in presence of a strong magnetic field. He limited himself to the purely driven motion at frequency ω when solving the equation of motion but retained the magnetic field corrections. Although the Lorentz force due to the static magnetic field is much stronger than the perturbed magnetic field so that the latter term plays a minor role in the equation of motion, including them allows to obtain a more symmetrical expression for the Ponderomotive acceleration:

$$\vec{a}_{Pond} = -Re\left[\frac{1}{4}\nabla\left(\frac{1}{\omega^2-\Omega^2}\Big[|\varepsilon|^2 + i\frac{\Omega}{\omega}\vec{e}_{//}.(\vec{\varepsilon}^* \times \vec{\varepsilon}) - (\frac{\Omega}{\omega})^2|\vec{e}_{//}.\vec{\varepsilon}|^2\Big]\right)\right].$$

Putting the obtained expression for the Ponderomotive acceleration into the equation of motion for the slow dynamics, the latter can now be solved. For typical parameters in the plasma edge, the 2 accelerations in $\tilde{\vec{a}} = -v_t^2[\nabla N_o]/N_o + \vec{a}_{Pond}$ are much smaller than the acceleration associated with the Lorentz force due to the confining field. By first dropping the acceleration $\tilde{\vec{a}}$ due to the slow dynamics and then solving the $full$ equation by variation of the constants, one gets $v_{o,\perp 1} = +\tilde{a}_{\perp 2}/\Omega$, $v_{o,\perp 2} = -\tilde{a}_{\perp 1}/\Omega$ which is of the usual form $\vec{v}_{drift,\perp} = \vec{F} \times \vec{B}_o/(qB_o^2)$ of drift velocities in presence of a strong magnetic field acting together with another force $\vec{F}$. For ion species whose fundamental cyclotron layer lies in the antenna region ($\omega \approx \Omega$), the Ponderomotive force effect is strongly enhanced compared to the case in absence of a magnetic field. The obtained expression thus also provides an explanation for the RF induced onset of a poloidal rotation/flow close to cyclotron resonances, be it that a reformulation is required to describe what happens when $\omega = \Omega$ is identically satisfied.

The key finding of this paper is that compared to the $\vec{B}_o$-less case, the *direction* of the net drift motion of the particles has changed. When $\vec{B}_o = \vec{0}$, the Ponderomotive effect radially chases particles away from the antenna region due to the radial field gradient, whereas $\vec{B}_o \neq \vec{0}$ requires the *poloidal* gradient of the electric field to cause a radial drift, the magnitude of which is pondered by the difference between the frequency and the cyclotron frequency. At the top of the antenna, this radial drift is opposite to that at the bottom. The *radial* component of the Ponderomotive force together with the toroidal field yield an upward or downward poloidal motion along the antenna. Since the evanescent electric field decays exponentially away from the antenna (in vacuum $k_\perp^2 = k_o^2 - k_{//}^2 \approx -k_{//}^2$ so the field decays as $exp[-k_{//}|x - x_{ant}|]$ for most $k_{//} >> k_o$ in the antenna spectrum), the various effects together give rise to a particle streaming along the antenna, a depopulation at one end of the antenna and a population at the other. Hence, a density that is poloidally homogeneous in absence of ICRH will be noticeably deformed by the presence of the RF waves as soon as the RF antennas are powered to a sufficiently high power level.

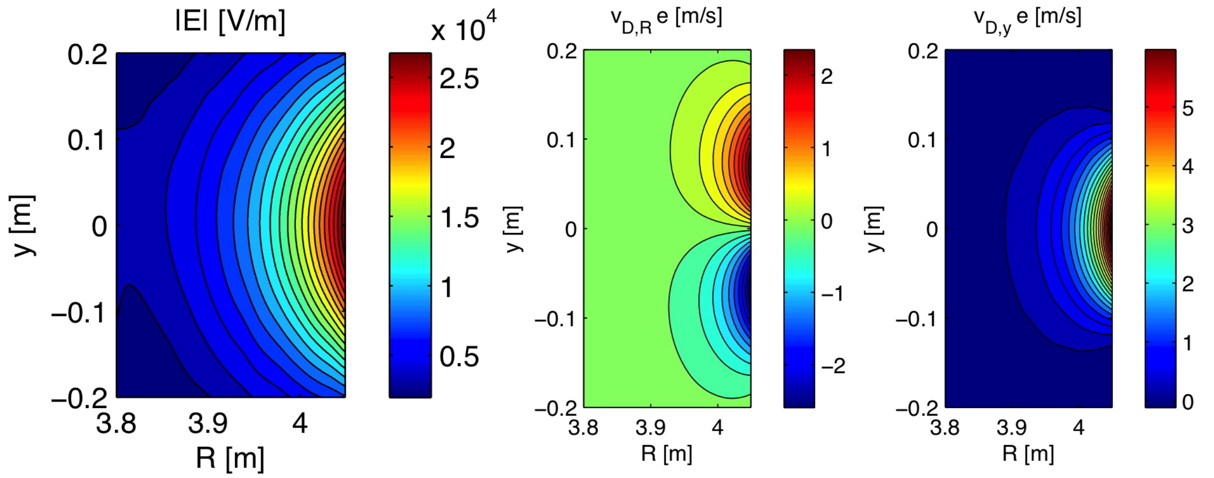

Figure 1: *Electric field amplitude (a) and corresponding radial (b) and poloidal (c) Ponderomotive force drift electron velocity components.*

Density modification

Once the solution of the equation of motion is known, the density modification for each type of species can be evaluated using the continuity equation, $\partial N_o/\partial t + \vec{v}_{drift}.\nabla N_o + N_o \nabla.\vec{v}_{drift} = 0$. Since the magnetic field cannot influence the parallel motion, the above equation has to be supplemented with an equation describing the parallel dynamics. Further upgrades would require accounting for the temperature variations, source and sink terms, the solving of which goes beyond the scope of the present paper which limits itself to showing the qualitative effect of the wave induced drifts.

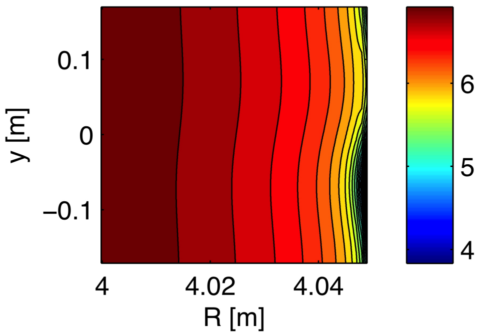

Figure 2: *ICRH induced e density deformation close to the antenna.*

Locally neglecting the small variations of Ω and curvature effects, the force in the drift term is the gradient of the sum of a potential term and the logarithmic derivative of the density and thus $\nabla.\vec{v}_{drift} = 0$. Figure 1 gives an example of the drift velocities resulting from the Ponderomotive force in presence of a strong magnetic field for typical JET-like $(H)-D$ parameters ($B_o = 3.45T$, $f = 51MHz$, $k_{//} = 6/m$, $X[H] = 5\%$). Starting from an exponentially decaying initial density profile with a decay length of $0.01m$, the cold plasma wave equations were solved to obtain the electric field pattern. In view of the fact that the density-related drift velocity changes with time, solving the continuity equation requires accounting for the density evolution. Finding the corresponding drift velocities and adopting a crude time stepping procedure, it can be shown that the density acquires poloidal asymmetries due to the presence of the ICRH electric field, as illustrated in Fig.2.

Towards a more accurate model

Wave induced poloidal as well as radial density asymmetries have experimentally been observed (see e.g. [5, 6, 7]). Although the here adopted model is much too crude to capture the full dynamics, it illustrates one mechanism via which waves influence the density close to the antenna. The very fact that flows are set up by electric fields hints, however, at the fact that the description of the dynamics requires more sophisticated modeling to capture the details of the interaction of waves and particles. At least, one should solve Maxwell's equations together with the continuity equation for the density and a parallel momentum equation. A basic ingredient in such a model - far outside the scope of the present paper - is the inclusion of slow *and* fast time scale dynamics, as well as their coupling. For example, charge imbalance due to differing dynamics of the various species - the ions react on a much slower time scale than the electrons - will give rise to electrostatic fields, in particular near metallic surfaces. The $\vec{E}\times\vec{B}$ flows created by such fields (see e.g. [8, 9]) cause supplementary drifts that equally have an impact on the density profile. Using the theoretical framework adopted in [7], work is underway to include the here described effect.

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