Adaptive particle-mesh simulation of laser driven plasma at ultra-high intensity

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The next generation of multi-petawatt laser facility foreseeable through the work of the Extreme Light Infrastructure (ELI) will lead Europe to the cutting edge of ultra-high laser plasma research. The forefront application of such highly nonlinear laser plasmas is design of novel sources of ultra-short pulses in the X-ray and γ-ray spectrum range. The extreme conditions reachable in laser plasmas serve as laboratory model for astrophysical objects such as neutron stars.

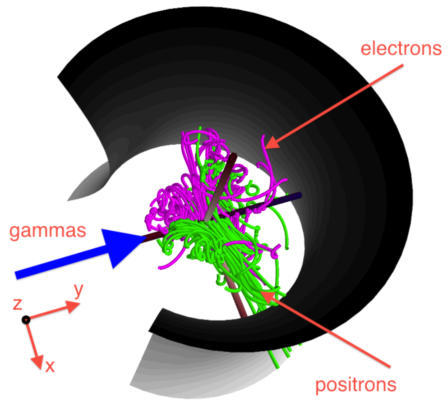

Figure 1: Electron-positron-photon electromagnetic cascade development in focused laser pulse at the intensity $I = 10^{25}\ W/cm^2$ [1].

The key phenomena in external electromagnetic fields of ultrahigh intensities $I > 10^{24}\ W/cm^2$ is the occurrence of QED cascades. These cascades (also called avalanches, or showers) are caused by successive events of hard photon emissions $e^\pm + n\hbar\omega \to \gamma + e^\pm$ due to nonlinear Compton scattering and electron-positron pair photo-production $\gamma + n\hbar\omega \to e^+ + e^-$ by hard photons. The emergence of this sort of cascades may become a dominating feature of laser-matter interactions at ultra-high intensities [1].

The theoretical framework used to describe the interaction of ultra-intense laser fields with the quantum vacuum or with matter makes use of extended Vlasov equations for electrons, positrons, and photons. They are given by

$$\begin{aligned}
&\left(\partial_t + \vec{v}\cdot\partial_{\vec{x}} + \vec{F}\cdot\partial_{\vec{p}}\right) f_\pm(\vec{x},\vec{p},t) \\
&= \int d^3k\, W_\gamma^{\vec{E},\vec{B}}(\vec{k},\vec{p}+\vec{k})\, f_\pm(\vec{x},\vec{p}+\vec{k},t) - f_\pm(\vec{x},\vec{p},t)\int d^3k\, W_\gamma^{\vec{E},\vec{B}}(\vec{k},\vec{p}) \\
&\quad + \int d^3k\, W_\pm^{\vec{E},\vec{B}}(\vec{k},\vec{p})\, f_\gamma(\vec{x},\vec{k},t)
\end{aligned} \tag{1}$$

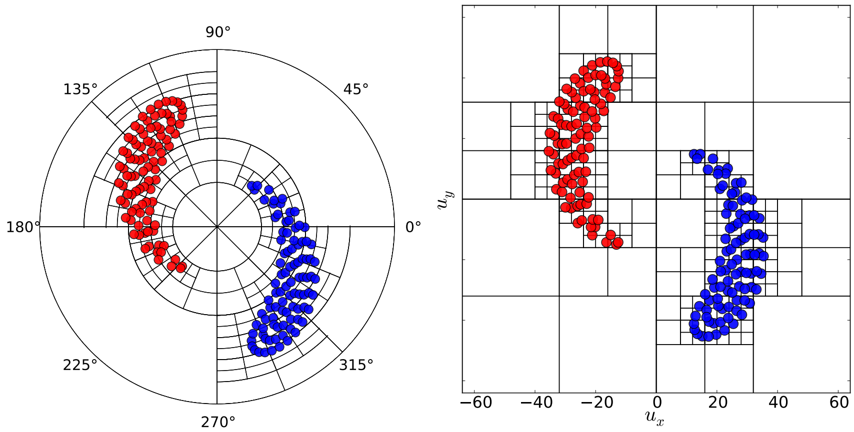

Figure 2: Two dimensional Cartesian and polar tree structure used to handle particles in momentum space.

and

$$\left(\partial_t + \frac{\partial \omega}{\partial \vec{k}} \cdot \partial_{\vec{x}}\right) f_\gamma(\vec{x},\vec{k},t) \tag{2}$$
$$= \int d^3p W_\gamma^{\vec{E},\vec{B}}(\vec{k},\vec{p}) \left[f_+(\vec{x},\vec{p},t) + f_-(\vec{x},\vec{p},t)\right]$$
$$-f_\gamma(\vec{x},\vec{k},t) \int d^3p W_\pm^{\vec{E},\vec{B}}(\vec{k},\vec{p})$$

The transition rates are computed in the limit of the constant crossed field approximation

$$\frac{dW_\gamma(\varepsilon_\gamma)}{d\varepsilon_\gamma} = -\frac{\alpha m^2 c^4}{\hbar \varepsilon_e^2} \left\{ \int_x^\infty \mathrm{Ai}(\xi)\, d\xi + \left(\frac{2}{x} + \chi_\gamma \sqrt{x}\right) \mathrm{Ai}'(x) \right\} \tag{3}$$

$$\frac{dW_\pm(\varepsilon_e)}{d\varepsilon_e} = \frac{\alpha m^2 c^4}{\hbar \varepsilon_\gamma^2} \left\{ \int_y^\infty \mathrm{Ai}(\xi)\, d\xi + \left(\frac{2}{y} - \chi_\gamma \sqrt{y}\right) \mathrm{Ai}'(y) \right\} \tag{4}$$

There are two approaches to calculate integrals in transport equations. One way is based on the Monte-Carlo method implemented in form of event generators. Recently, this approach was combined with particle-in-cell method [1, 2] for calculation of quantum effects of hard photon radiation and pair creation. However, the Monte-Carlo method heavily relies on the use of random sampling of transport integrals and may lead lead to noisy and possibly unstable solution of the transport equation. Also the event generator requires much finer time and space discretization than it is needed for accurate resolution of plasma effects. These limitations make difficult to study nonlinear cascading regime when radiation field is governed by accumulated electron-positron plasma. As an alternative approach we consider a class of deterministic adaptive particle-mesh methods. This method was realized first for the chase of small $\chi \ll 1$ when emission is dominated by soft photons $\omega \ll \varepsilon$. In this case the integrals in transport equation can be expanded with respect to wave number of photons $\vec{k}$. The detailed derivations are given in [4, 3]. Here we concern with the numerical solution of the Fokker-Planck type equation

$$\frac{df(\vec{r},\vec{p},t)}{dt} = \frac{\partial}{\partial \vec{p}}\left[\vec{I}(\vec{r},\vec{p},t)\right] + \frac{\partial^2}{\partial p_\alpha \partial p_\beta}\left[D_{\alpha\beta}(\vec{r},\vec{p},t) f(\vec{r},\vec{p},t)\right]$$

with the coefficients

$$\vec{I} = \int \vec{k} w_{rad}(\vec{r},t,\vec{p}\to\vec{k}) d^3k, \quad \vec{D}_{\alpha\beta}(\vec{r},\vec{p},t) = \frac{1}{2}\int k_\alpha k_\beta w_{rad}(\vec{r},t,\vec{p}\to\vec{k}) d^3k.$$

these integrals can be calculated analytically. This equation can be written in convenient form of nonlinear convection-diffusion form

$$\frac{\partial f}{\partial t} + \frac{\vec{u}}{\gamma}\frac{\partial f}{\partial \vec{u}} + \frac{\partial}{\partial \vec{u}}\left[f(\vec{F} - \vec{R} - \vec{I})\right] = \frac{\partial}{\partial u_\alpha}\left(D_{\alpha\beta}\frac{\partial f}{\partial u_\beta}\right) \tag{5}$$

where $R_\alpha = f \cdot \partial D_{\alpha\beta}/\partial u_\alpha$. The convection-diffusion form can be further converted into non-linear transport equation

$$\frac{\partial f}{\partial t} + \frac{\vec{u}}{\gamma}\frac{\partial f}{\partial \vec{r}} + \nabla \cdot \left(f[\vec{F} - R - I + \vec{U}]f\right) = 0, \tag{6}$$

where $\vec{U} = -D_{\alpha\beta} \cdot \nabla_{\vec{u}} f/f$. This equation is equivalent to "particles" equations

$$\frac{d\vec{r}}{dt} = \frac{\vec{u}}{\gamma}, \quad \frac{d\vec{u}}{dt} = \vec{F} - \vec{I} - \vec{R} + \vec{U} \tag{7}$$

To solve this system the Smoothed Particles Hydrodynamics method is adopted for momentum space. For this purposes we consider smooth kernel W and its derivative ∇W

$$W(\xi,\, h) = \frac{C}{h^D}\begin{cases} 1 - \frac{3}{2}\xi^2 + \frac{3}{4}\xi^3 \\ \frac{1}{4}(2-\xi)^3 \\ 0. \end{cases} \qquad \nabla W(\xi,\, h) = \frac{C}{h^D}\begin{cases} -3\xi\cdot\vec{\xi} + \frac{9}{4}\xi^2\cdot\vec{\xi}, & \xi < 1 \\ -\frac{3}{4}(2-\xi)^2\cdot\frac{\vec{\xi}}{\xi}, & 1 \le \xi \le 2 \\ 0, & \xi > 2. \end{cases} \tag{8}$$

where $\vec{\xi} = |\vec{u} - \vec{u}_j|/h$, D is the number of dimensions ($D = 2$ in our case) and C is the normalization factor that depends on the number of dimensions:

$$C_{1d} = \frac{2}{3}, \quad C_{2d} = \frac{10}{7\pi}, \quad C_{3d} = \frac{1}{\pi}.$$

Knowing W and ∇W we can approximate

$$\vec{U} = -D_{\alpha\beta}\cdot\frac{\sum_{j=1}^{N_p} c_j \nabla W(\vec{u} - \vec{u}_j)}{\sum_{j=1}^{N_p} c_j W(\vec{u} - \vec{u}_j)} \tag{9}$$

where c_j is the weight of particle. On each step the algorithm integrates equation (7) and reconstruct both distribution function and gradient of distribution function for calculation of diffusion

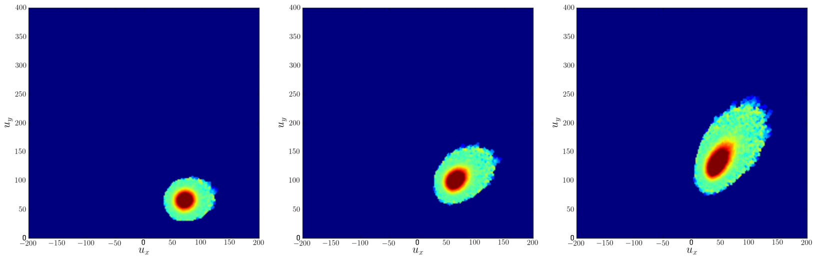

Figure 3: A bunch of electrons in rotating field simulated using the Fokker-Planck equation.

flow field $\vec{U}$. The calculation of $\vec{U}$ requires search for all neighbors particles within smoothing length $2h$. Our implementation of a searching algorithm makes use of two- and tree-dimensional spatial trees. Depending on problem one can choose the tree in Cartesian or polar geometry, see Figure 2. The tree is dynamically adapting if particles are moving. The tree structure is used also to control locally the smoothness of the distribution function. If necessary particles are added or deleted. To validate our code we perform preliminary simulation of an electron bunch in rotating electric field $\vec{E} = \{a_0 \cos(\omega t),\ a_0 \sin(\omega t), 0\}$, where $a_0 = eA/mc = 100$ and $\omega = 1$ eV. These parameters corresponds to intensity $I \sim 10^{22}\ W/cm^2$. The rotating field model problem allows to study the evolution of distribution function due to both classical and quantum radiation reaction. The impact of anisotropic phase space diffusion can be seen in Figure 3 where distribution function is shown at subsequent moments of time. Although results looks reasonable the Fokker-Planck approach must be compared with full solution of the transport equation. This will be the next step in our research.

References

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