

Cylindrical Ion Acoustic Waves in Multi-component plasma

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Abstract. The basic purpose of this paper is the investigation of the propagation of ion acoustic waves in argon rf discharge under typical laboratory experiments conditions: RF frequency (13.56 Mhz), low pressure (5 mTorr) and low temperature (300K°) is made in the present work. To obtain more realistic rf discharge conditions, the reactor's geometry was taking into account. The study was done by using the standard reductive perturbation technique. It was found that the dynamics of ion acoustic solitary waves (IASWs) is governed by a three-dimensional cylindrical Kadomtsev-Petviashvili equation (CKPE).

Introduction

The study of plasma-surface interaction is of technological as well as scientific interest. As the tremendous amount of scientific reports demonstrates [1-4], there is a long lasting interest in the properties of thin films for a variety of reasons. Besides, there is a great deal of interest in understanding different types of collective processes in plasmas. Indeed, the solitary waves in plasma have been extensively investigated and it was found that those solitary waves could be described by the Korteweg–deVries (KdV) equation, Kadomtsev–Petviashvili (KP) equation or Davey-Stewartson equation [5-8]. We point out, that the first observation of K-dV solitons in two-component quasi-neutral plasma was made by Ikezi *et al.* in a double plasma device [9]. Besides, ion-acoustic solitary waves have been also observed by K. B. Gylfason in a sputtering discharge, and he found that a simple KdV model is not sufficient to explain the solitary waves observed [10]. In this respect, in the present paper, using the standard reductive perturbation method, we report on the possibility of creation and propagation of ion acoustic waves in multi-component plasma. Our results are in agreement with the prediction of K. B. Gylfason [10]. Indeed, the solitary waves we found are not a simple K-dV Soliton but a cylindrical K-dV Soliton.

Model equation

We consider multi-component plasma comprised of inertial ions and Boltzmann distributed electrons and an admixture of dynamics ions with negative and positive charge in a nonplanar cylindrical geometry. We investigate the ion acoustic waves in cylindrical geometry and also

take into account the transverse perturbations. The normalized ion dynamic equations are as follows:

$$\frac{\partial n_i}{\partial t} + \frac{1}{r} \left(\frac{\partial n_i u_i}{\partial r} \right) + \frac{1}{r} \left(\frac{\partial n_i u_i}{\partial \theta} \right) = 0, \quad (1)$$

$$\frac{\partial n_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} + \frac{v_i}{r} \frac{\partial u_i}{\partial \theta} - \frac{v_i^2}{r} = \frac{\partial \phi}{\partial r}, \quad (2)$$

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial r} + \frac{v_i}{r} \frac{\partial v_i}{\partial \theta} + \frac{u_i v_i}{r} = \frac{1}{r} \frac{\partial \phi}{\partial r}, \quad (3)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = n_i + \mu_e \exp(\sigma_i \phi) - \mu_{i-} \exp(-\phi). \quad (4)$$

Where r, θ are the radial and polar angle coordinates u_i and v_i are the ion fluid velocity in r and θ directions. n_i and ϕ represent the negative ion density and the electrostatic potential. The variables t, r, n_i, u_i, v_i and ϕ are normalized to the negative ion plasma frequency $\omega_{pi}^{-1} = \sqrt{4\pi n_{i0} e^2 m_i}$ Debye radius $\lambda_D = \sqrt{k_B T_i / 4\pi n_{i0} e^2}$ unperturbed equilibrium ion density n_i , effective acoustic velocity $C_i = \sqrt{k_B T_i / m_i}$, and $k_B T / e$, respectively. Here we have denoted: $\mu_e = n_{e0} / n_{i0}$, $\mu_{i-} = n_{i0-} / n_{e0}$, $\mu_{i+} = n_{i0+} / n_{e0}$, $\sigma_e = T_e / T_{i+}$, $\sigma_- = T_{i-} / T_{i+}$, $\mu = \mu_+ / \mu_-$.

To study the dynamics of small amplitude dust-acoustic solitary waves, we use the so-called reductive perturbation method [30]. We can then expand the variables n_i, u_i and ϕ about the unperturbed states in power series of ε (ε is a small parameter) that means, we let,

$$\begin{cases} n_i = 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \dots, \\ u_i = \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \varepsilon^3 u_{i3} + \dots, \\ \phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \dots \end{cases} \quad (5)$$

We can rewrite Eqs.(1-4) taking into account Eqs.(5-7) and the stretched coordinates $\xi = \varepsilon^{1/2} (r - v_0 t)$, $\tau = \varepsilon^{3/2} t$, $\eta = \varepsilon^{-1/2} \theta$, where v_0 is the wave velocity, to get the following system equations, to the lowest order in ε we have

$$n_j = -\frac{1}{v_0^2} \phi_1, \quad u_j = -\frac{1}{v_0^2} \phi_1, \quad v_0^2 = 1 / (\mu_e \sigma_e^{-1} + \mu_{i-} \sigma_{i-}^{-1}), \quad \frac{\partial v_j}{\partial \xi} = -\frac{1}{v_0^2 \tau} \frac{\partial \phi_1}{\partial \eta}.$$

To the next order in ε we get the following set of equations;

$$\frac{\partial n_1}{\partial \tau} - v_0 \frac{\partial n_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \frac{\partial (n_j u_j)}{\partial \xi} + \frac{1}{v_0 \tau} \frac{\partial v_1}{\partial \eta} + \frac{1}{v_0 \tau} \left(2v_1 + \frac{1}{\eta} v_1 \right) = 0, \quad (6)$$

$$\frac{\partial u_1}{\partial \tau} - v_0 \frac{\partial u_2}{\partial \xi} + u_1 \frac{\partial u_1}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} = 0,$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} - \frac{1}{2} (\mu_e \sigma_e^{-2} + \mu_{i-} \sigma_{i-}^{-2} - \mu_{i+} \phi_1^2 - (\mu_e \sigma_e^{-1} + \mu_{i-} \sigma_{i-}^{-1} + \mu_{i+}) \phi_2 - n_{d2}) = 0.$$

After some algebra we obtain the SKP equation,

$$\frac{\partial}{\partial \xi} \left[\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + \frac{1}{\tau} \phi_1 \right] + C \left[\frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \phi_1}{\partial \eta} \right] = 0. \quad (7)$$

$$A = -\frac{1}{2} \left[\frac{3}{v_0} + v_0^3 (\mu_e \sigma_e^{-2} + \mu_{i-} \sigma_{i-}^{-2} - \mu_{i+}) \right], B = \frac{v_0^3}{2}, C = \frac{1}{2v_0 \tau^2}.$$

It is important to point out, that if the wave propagate without the transverse perturbation, the last term in the left side of **Eq. (7)** disappear and the SKP equation (7) reduce to the ordinary cylindrical KdV equation. We can find an exact solitary wave solution for the CKP equation (7) by using a suitable variable transformation [10]. In **Eq. (7)**, the two terms with variable coefficient, can be canceled if we assume $\zeta = \xi - \frac{v_0}{2} \eta^2 \tau$, $\phi_1 = \Phi(\zeta, \tau)$.

Then the SKP **Eq.(7)** is reduced to the standard KdV equation,

$$\frac{\partial \Phi}{\partial \tau} + A \Phi \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0. \quad (8)$$

$$\Phi(\zeta) = \frac{3U_0}{A} \operatorname{sech}^2 \left[\sqrt{\frac{U_0}{4B}} (\zeta - U_0 \tau) \right], \quad U_0 \text{ is a constant represent wave velocity. Thus we get an}$$

exact solitary wave solution of the SKP **Eq.(7)**

$$\Phi(\zeta) = \frac{3U_0}{A} \operatorname{sech}^2 \left[\sqrt{\frac{U_0}{4B}} \left(\zeta - \left(U_0 + \frac{v_0}{2} \right) \tau \right) \right]. \quad (9)$$

From **Eq.(9)** we could deduce that the amplitude and wave velocity of our solitary wave described by CKP equation are exclusively determined by the parameters of the system and only depending on the initial conditions. Equation (9) indicates that the phase velocity of the solitary wave is angle-dependent in the phase. This means that the ion acoustic wave will slightly deform as time goes on. Elsewhere, we note that, the amplitude of ion acoustic wave increases by decreasing the coefficient of nonlinearity “A”, and the width increases as “B” (coefficient of dispersion) decreased.

Figure1 provides an information about the variation in amplitude and width of solitary waves with change values of electron concentration (via μ_e). Increase μ_e enhances the amplitude but reduces the width of Soliton. Negative ion concentration also changes the solitary waves structures. It is seen that the reduction of negative ions concentration (via μ_{i-}) reduces the amplitude as well as width of soliton (**Figure2**).

To resume, let us recall that one of the central points of the present work, concerns the investigation of coherent structure’s occurrence in multi-component plasma. The emphasis

was on the possibility of generation of ion acoustic waves (IAW) by means of reductive perturbation technic. We found that the presence of negative ions leads on the generation of ion acoustic waves. We underline that; the present work can be suitable in studying collective process in plasma reactor, which is deemed useful in understanding plasma-surface interactions, and thin films properties.

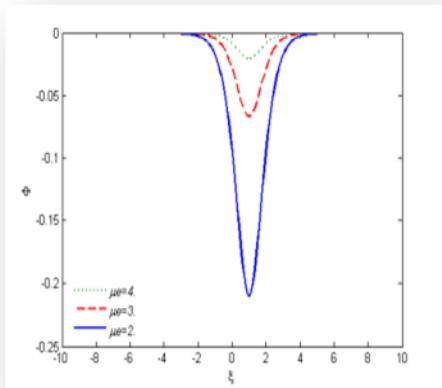


Figure 1. Electrostatic potential Φ vs ζ .

For $\sigma_e = 10, \sigma_- = 1$ and $\mu_- = 1$.

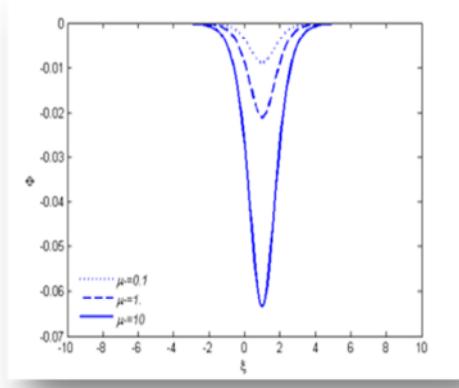


Figure 2. Electrostatic potential Φ vs ζ .

For $\sigma_e = 10, \sigma_- = 1$ and $\mu_e = 4$.

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