Soft X-Ray cooling factor low dependency on transport: investigating the domain of validity

D. Vezinet[1], D. Mazon[1], R. Guirlet[1], M. Sertoli[2], J. Decker[1], Y. Peysson[1]

[1] *CEA IRFM, 13108 Saint Paul lez-Durance, France*

[2] *Max-Planck-Institut für Plasmaphysik, Euratom Association, Garching, Germany*

Both fast impurity density estimation from Soft X-Ray tomography [1] and accurate determination of impurity transport coefficients [2] can be made possible by low dependency of the cooling factor of some impurities on transport. This paper investigates the domain of validity of such robustness even when the fractional abundances $f_{S,z}$ are significantly affected by transport and differ from a local ionisation balance (LIB). The importance of the free-free contribution to the total SXR emissivity Σ_S^η of species S, of the plasma electron temperature T_e and of the detector spectral response η is underlined.

The local SXR emissivity Σ_S^η is related to the impurity density n_S via its cooling factor L_S^η. In this paper, spectrally differentiated quantities are written with lower case while upper case is used for spectrally integrated quantities.

$$\begin{cases} \Sigma_S^\eta = n_e n_S L_S^\eta \\ L_S^\eta = \sum_{z=0}^{Z_S} f_{S,z} K_{S,z}^\eta \end{cases} \quad \text{and} \quad \begin{cases} K_{S,z}^\eta = \int_0^\infty k_{S,z}(h\nu)\eta(h\nu)d(h\nu) \\ k_{S,z}(h\nu) = \left(k_{S,z}^{ff} + k_{S,z}^{fb} + k_{S,z}^{bb}\right)(h\nu) \end{cases} \qquad \textbf{(1)}$$

The ions cooling coefficients $k_{S,z}$ represent the free-free (ff), free-bound (fb) and bound-bound (bb) contributions. Robustness of the cooling factor versus transport (i.e. $L_S^\eta(\underline{\Gamma}_S) \approx L_S^\eta(\underline{0})$, where $\underline{\Gamma}_S$ is the flux of species S) can be achieved either by actual local ionisation balance ($\left\lfloor f_{S,z}(\underline{\Gamma}_S)\right\rfloor_{\forall z} \approx \left\lfloor f_{S,z}(\underline{0})\right\rfloor_{\forall z}$) or by very similar contributions of all ions (i.e.: $\left[K_{S,z}^\eta(T_e)\right]_{\forall z} \approx K_S^\eta(T_e)$). The latter possibility is studied in more detail in the following.

Bare nuclei, for example, emit no lines and their contribution is [3]:

$$k_{S,z}^{ff} \propto \frac{z^2}{\sqrt{k_B T_e}} e^{-\frac{h\nu}{k_B T_e}} G_{S,z}^{ff} \quad \text{and} \quad k_{S,z}^{fb} \propto \frac{z^4}{(k_B T_e)^{3/2}} e^{-\frac{h\nu}{k_B T_e}} \sum_{n \geq n_{\min}} \frac{G_{S,z,n}^{fb}}{n^3} e^{\frac{z^2 E_R}{n^2 k_B T_e}} \qquad \textbf{(2)}$$

Where k_B is Boltzmann constant, $G_{S,z}^{ff}$ (resp. $G_{S,z,n}^{fb}$) is the free-free (resp. fb) Gaunt factor (dimensionless and close to unity), $E_R \approx 13.6\,\text{eV}$ and $n_{\min}$ is the minimum principal quantum number n on which recombination can occur for a photon of energy $h\nu$ to be radiated.

From (2) it can be seen that two successive bare nuclei of charge z and z+1 have free-free contributions that differ only by $\Delta k_{S,z}^{ff} / k_{S,z}^{ff} = 2/z \le 10\,\%$ as soon as $z \ge 20$, meaning that if $k_{S,z}^{ff}$ is dominant, the resulting cooling factor will have low dependency on the ions fractional abundances. Such ionisation levels are obtained in today's Tokamaks for medium to high Z impurities and will be more frequent in ITER. We are now going to investigate the conditions in which the free-free contribution dominates for bare nuclei:

$$\frac{k_{S,z}^{ff}}{k_{S,z}^{fb}} \propto \frac{k_B T_e}{z^2} \times G_{S,z}^{ff} \times \left(\sum_{n \ge n_{\min}} \frac{G_{S,z,n}^{fb}}{n^3} e^{\frac{z^2 E_R}{n^2 k_B T_e}} \right)^{-1} \quad \textbf{(3)}$$

For fixed z and T_e, ratio (3) is thus a stair-shaped function (decreasing with $h\nu$ since n_{min} increases with $h\nu$), as represented on Figure 1 (b) for Ni^{28+} at T_e = 5 keV. The limits of the stairs correspond to recombinations on different quantum levels (see Figure 1 (a)).

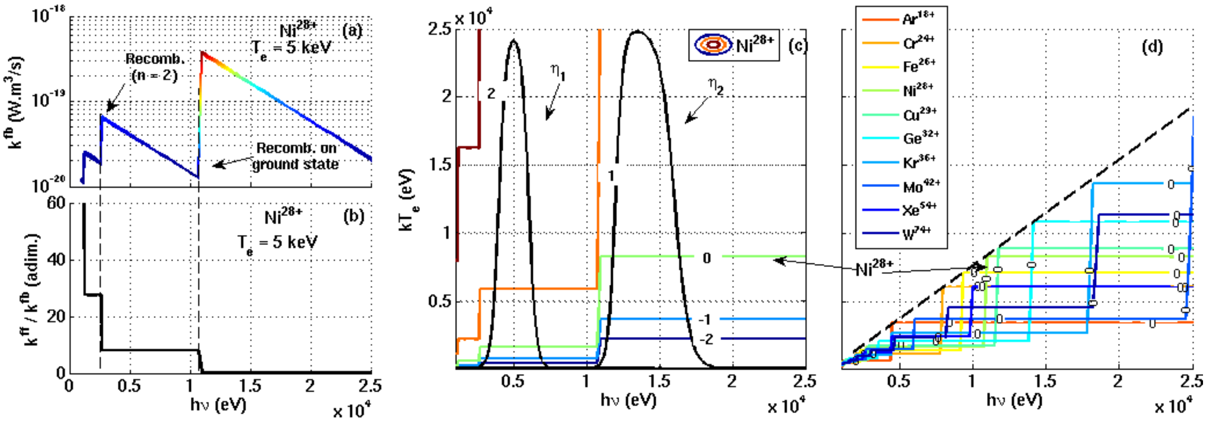

Figure 1: (a) k^{fb} for Ni^{28+}, showing the spectral intervals in which the ff/fb ratio (b) is constant. (c) 2D contour plot of $\log_{10}(k^{ff}/k^{fb})$ for Ni^{28+} in (hν,T_e) space and two examples of spectral responses (d) the limit ($\log_{10}$(ratio) = 0) of the validity domain for various bare nuclei

The $\log_{10}$(ff/fb) is plotted in the (hν,T_e) plane and its iso-contours are used to identify a domain in which the free-free contribution is dominant for Ni^{28+} (see Figure 1 (c)). For Ni^{28+}, selecting a low energy band using spectral response η_1 offers a much more favourable convoluted ff/fb ratio at low temperatures than η_2. The limit ($k^{ff} = k^{fb}$) has been plotted for various nuclei on Figure 1 (d), where the dashed straight line can be seen as global limit for bare nuclei. Quantitatively, the temperature limits are compatible with today's Tokamak operation on the low-energy part (1 keV – 10 keV), and remain reasonable for ITER-like plasma.

Two complications arise with more complex ions. First, incomplete screening effects and recombination of electrons on a limited number of available energy levels have to be taken into account. Second, line emission can have a significant contribution, meaning it must either be made negligible (with respect to Bremsstrahlung), or must be similar from one ion to the next.

In order to assess the quantitative relevance of this approach in experimental cases, we studied a Tore Supra scenario (cf. TS # 38464), during which various injections were performed, and which displays high electron temperature in the core plasma (≈ 5 keV), thus maximising the chances that the Bremsstrahlung be dominant. In [4] the authors identified a set of experimental transport coefficient profiles for this pulse and showed they were weakly dependent on the injected impurity nature. From this reference transport scenario (REF), a family of 5 diffusion (D) and 5 convection (V) profiles are generated (see Figure 2). The maximum difference between extreme profiles is more than one order of magnitude (a factor 25).

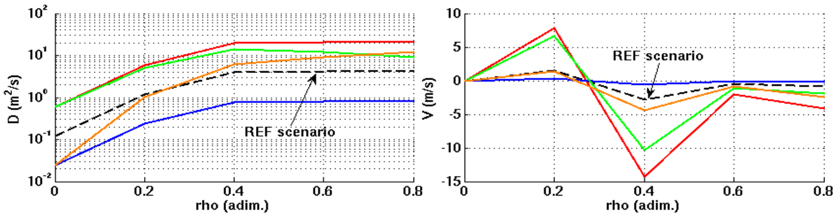

Figure 2: The 5 D and V profiles used to generate the 25 transport scenarii

All the possible (D,V) sets are then assembled, resulting in 25 transport scenarii used to study the robustness of the cooling factor with respect to modifications of the ionisation balance. The resulting 25 ionisation balances are represented on Figure 3 (c) which shows the expected significant variations between the 25 scenarios and with the LIB (Figure 3 (b)). The associated 25 cooling factor profiles (for η_1 and η_2) are also plotted on Figure 3 (d) and (e).

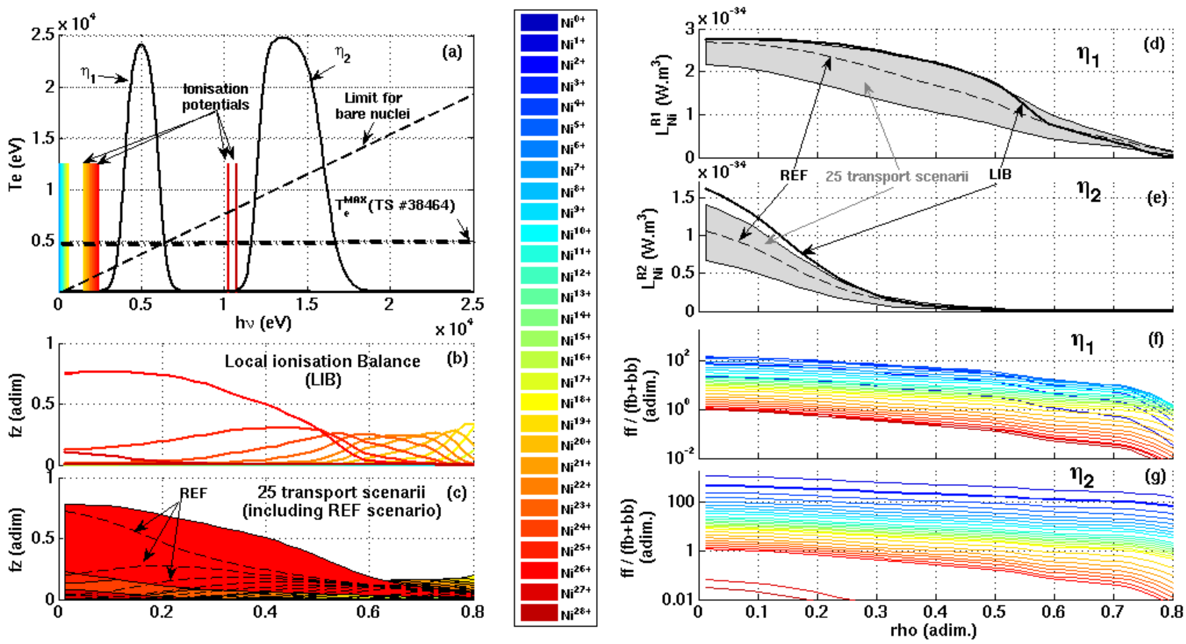

Figure 3: (a) similar to figure 1 (b) plus ionisation potentials of Ni ions (colors), a dotted-dashed line showing the max(T_e) in TS # 38464 and the two tested spectral responses (b) LIB of TS # 38464 (c) envelopes of the fractional abundances resulting from the 25 transport scenarii, cooling factor profile convoluted (d) by η_1 and (e) by η_2 (f) ratio of all the η_1-cooling coefficients (ff / (fb+bb)) (g) idem with η_2

The spectral responses were designed so as to minimise the influence of lines, hence the ratio ff/(fb+bb) behaves like ff/fb. It is clear that the ionisation balance is far more affected by transport than the cooling factor. In addition, as expected, the cooling factor is less sensitive to transport when it is convoluted by η_1 than by η_2. This results from the fact that η_1 corresponds to a spectral interval that is lower than the ionisation potentials of the last two Ni ions. Hence, the ff/fb ratio is considerably higher than with η_2 for Ni^{27+} and Ni^{28+} (see Figure 3 (f) and (g)), consistently with the stair-shaped iso-contours previously identified on Figure 1 (c). Quantitative estimations of the relative scattering of the cooling factor (total for the 25 scenarios and with respect to LIB, REF) are in Table 1.

(rho = 0.01)	η_1	η_2
Max. difference with LIB	20 %	56 %
Max difference from REF	18.5 %	33 %
Max scattering due to transport	25 %	100 %

Table 1: Relative scattering of the SXR cooling factor due to the 25 different transport scenarii, evaluated near the plasma centre with both spectral responses

Interestingly, ff/(fb+bb) ratio does indeed seem to be a monotonically increasing function of T_e (cf. Figure 3 (f) and (g)), suggesting that the results would be even better for hotter pulses.

Conclusion

As expected, these results underline the importance of three main parameters when considering the robustness of the cooling factor with respect to transport: the impurity atomic physics, the electron temperature (plasma parameter) and the detector spectral response (user-defined). An interpretation is provided for bare nuclei. It highlights the prominent role of Bremsstrahlung for medium to high z ions and applies successfully to Ni. The importance of the spectral response is underlined, thus calling for the use of SXR detector with tuneable spectral response [5].

Complementary work is currently in progress in order to define an optimised set of spectral responses adapted to several impurities and electron temperatures.

[1] D. Vezinet, D. Mazon, D. Clayton, R. Guirlet, M. O'Mullane, and D. Villegas, "Fast Nickel and Iron density estimation using Soft X-Ray measurements in Tore Supra: Preliminary study," *Fusion Sci. Technol.*, vol. 63, no. 1, pp. 9–19, 2013.

[2] M. Sertoli, C. Angioni, R. Dux, R. Neu, T. Pütterich, and V. Igochine, "Local effects of ECRH on argon transport in L-mode discharges at ASDEX Upgrade," *Plasma Phys. Control. Fusion*, vol. 53, p. 035024, 2011.

[3] H. R. Griem, *Principles of plasma spectroscopy*, 2nd ed. Cambridge: Cambridge University Press, 1997.

[4] T. Parisot, R. Guirlet, C. Bourdelle, X. Garbet, N. Dubuit, F. Imbeaux, and P. R. Thomas, "Experimental impurity transport and theoretical interpretation in a Tore Supra lower-hybrid heated plasma," *Plasma Phys. Control. Fusion*, vol. 50, p. 055010, 2008.

[5] D. Vezinet, D. Mazon, P. Malard, L. Gabellieri, R. Guirlet, D. Clayton, and A. Romano, "Spectrally adaptable Soft X-ray imaging on Tore Supra: preliminary results and simulations," presented at the 39th EPS Conference & 16th Int. Congress on Plasma Physics, Stockholm, 2012, vol. 36F - P1.050.