Time Delay Estimation based method for determining turbulent structure tilting

D. Guszejnov[1,2], A. Bencze[1], S. Zoletnik[1] and Andreas Krämer-Flecken[3]

[1] *MTA Wigner RCP, EURATOM Association, PO Box 49, H-1525 Budapest, Hungary*

[2] *Department of Nuclear Techniques, Budapest University of Technology and Economics, Association EURATOM, Műegyetem rkp. 9., H-1111 Budapest, Hungary*

[3] *Institute of Energy and Climate Research - Plasma Physics, Forschungszentrum Jülich, Association EURATOM-FZJ, D-52425 Jülich, Germany*

Introduction

Turbulence plays a key role in the transport of energy and particles in hot magnetized plasmas [1], but it is still not completely understood, despite intensive scientific investigation. Numerical simulations have shown that sheared flows play have a significant role in the controlling plasma turbulence [2], while one of the most significant experimental results of the last couple of years is the discovery of quasi-stationary [3] and oscillating flows (zonal flows) [4].

It is believed that the tilting of eddies could have a significant impact on the excitation of sheared flows [5]. Structures are inherently tilted in the radial-poloidal plane since their emergence (α_B – *ballooning angle*) and are further tilted by the sheared flows, resulting in a time dependent tilt angle (α). Theoretical studies of the ITG modes in toroidal geometry highlighted that this ballooning angle determines the linear growth rate of the instability as $\gamma \propto \cos\alpha_B$ [7], showing that the strongest modes are less tilted. Therefore the accurate measurement of the ballooning angle can give insight in the mode dynamics of the underlying instability.

Out goal is to present a time delay estimate (TDE) based method to determine the tilt angle of turbulent structures in experiments, where real time 2D measurements are not available. This model will be described in more detail in an upcoming publication [9].

Mathematical model

For our description of coherent density structures in the edge plasma, we assume a dominant scale on which coherent structures emerge, which take part in no significant nonlinear interaction during the timescale of the measurement. We further assume that the fluctuation of the plasma density is the superposition of small coherent structures. These have both Gaussian spatial distribution (in the direction of both their axes) and a

Gaussian time decay as experiments have shown, that edge and core coherent structures exhibit Gaussian-like shape [6] (unlike SOL structures which can be highly asymmetric). The model also assumes that the coherent structures move at a constant velocity and have the same size and orientation. These assumptions are generally true for neighboring observation channels of turbulence measurements – as the distance between them is usually 1-2 cms – except for the cases of strongly sheared flows. This means that the density fluctuation caused by structure i (n_i) can be expressed as

$$n_i(u,w,t) = G(u,u_i+v_u(t-t_i),\sigma_U)\times G(w,w_i+v_w(t-t_i),\sigma_W)\times G(t,t_i,\sigma_T) \qquad (1)$$

where u, w are coordinates in the coordinate system defined by the its axes (Fig. 1), v_u, v_w are the projected velocity components in these directions, while $G(x,x_i,\sigma_x)$ denotes a Gaussian function defined as

$$G(x,x_0,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(x-x_0)^2}{2\sigma^2}} \qquad (2)$$

Let us assume that there is a significantly large number of structures so that a statistical description is appropriate. For this description it is essential to know the distribution of the structure parameters . In our model we take the parameters of individual structures (u_0, w_0, t_0) to be independent, uniform random variables. Due to the fact that the coherent structures vanish at much smaller than the size of the poloidal plane and the time length of the measurement, temporal and spatial averages can be taken as infinite integrals (e.g. $\int_{-\Delta T/2}^{\Delta T/2} f(t)P(t)dt \approx \int_{-\infty}^{\infty} f(t)P(t)dt$).

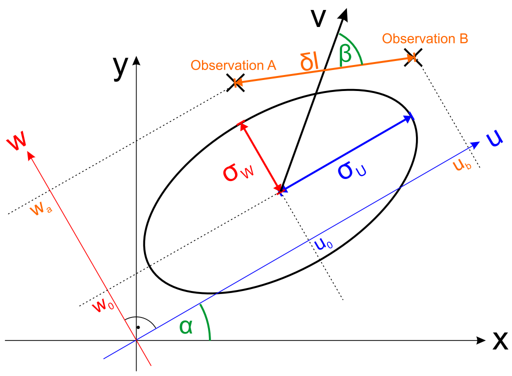

Figure 1: Coordinate system used for the modeling of coherent structures, including the observation points.

In signal processing the position of the cross correlation function (CCF) peak – from now on referred to as *time delay estimate* (TDE) – is essential in determining several key parameters of the turbulent structures (see Sec.). The TDE (denoted as $\hat{D}$) can be derived by solving

$$\left.\frac{dC_{a,b}}{d\tau}\right|_{\tau=\hat{D}} = \sum_{i,j}\left.\frac{dc_{a_i,b_j}}{d\tau}\right|_{\tau=\hat{D}} = 0, \qquad (3)$$

where $C_{a,b}(\tau)$ is the CCF between observation points a and b. Using our model the expected TDE $(\langle\hat{D}\rangle)$ becomes

$$\langle\hat{D}\rangle = \frac{\frac{v_u\delta u}{\sigma_U^2} + \frac{v_w\delta w}{\sigma_W^2}}{\kappa^2}, \tag{4}$$

where $v_u = v_z\sin\alpha + v_r\cos\alpha$, $v_w = v_z\cos\alpha - v_r\sin\alpha$, $\delta u = \delta z\sin\alpha + \delta r\cos\alpha$ and $\delta w = \delta z\cos\alpha - \delta r\sin\alpha$ (see Fig. 1). κ is the inverse of the characteristic decorrelation time [8] defined as

$$\kappa^2 \equiv \frac{1}{\sigma_T^2} + \frac{v_u^2}{\sigma_U^2} + \frac{v_w^2}{\sigma_W^2}. \tag{5}$$

Application to TEXTOR data

The results from Sec. allow a more detailed analysis of measured turbulence signals, for instance regarding the orientation of coherent structures. As a demonstration several parameters of turbulent structures in the TEXTOR tokamak ($R = 1.75\,\mathrm{m}$; $a = 0.47\,\mathrm{m}$; limited, circular plasma; $n_e = 10^{19}\,\mathrm{m}^{-3}$) were calculated. For that purpose measured data from the Lithium Beam Emission Spectroscopy (Li-BES) [10, 11] diagnostic was used. In the examined discharge (#113917, $I_p = 350\,\mathrm{kA}$, $B_t = -1.9\,\mathrm{T}$) the diagnostic was in '*fast deflection mode*', which means that during the discharge the beam was deflected by charged plates at high frequency before neutralization. This method allows the measurement of density fluctuations along not one but two beam lines hence it is called a '*quasi-2D*' measurement [12] (Fig. 2).

The TDEs for individual detector pairs were calculated from the experimental signals and a fitting algorithm was utilized to find the structure parameters for Eq. (4). It should be noted that the fitting procedure takes advantage of the fact that coherent structures in the plasma edge primarily propagate in the poloidal direction. In case of #113917 the zeroth order approximation of their poloidal velocity is $v_z \approx \Delta z/\hat{D} \approx 3.5\,\mathrm{km/s}$, while the apparent radial velocity is $v_r^{\mathrm{app}} = \Delta r/\hat{D} \approx 10\,\mathrm{km/s}$. This means that the high apparent radial velocity can only be explained by the presence of a tilt, which is responsible for the major part of v_r^{app}. This is fortunate, because in general the effects of radial propagation and structure tilt are hard to distinguish, but in this case the effects of v_r are negligible.

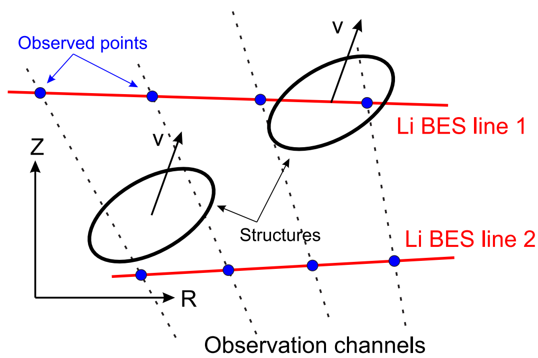

Figure 2: Measurement configuration for TEXTOR quasi 2D Li BES.

Figure 3 shows that the tilt angle of coherent structures is around 10-20 degrees around the midplane. It is important to note that at $R > 220\,\mathrm{cm}$ the velocity gradient steepens drastically, causing a significant deformation of the structures, thus violating the assumption of spatially constant structure parameters between observed points, thus fitted parameters in that range are likely erroneous.

The fitting results were compared against the results from the TEXTOR Correlation Reflectometry (CR) [13]. The CR results show, that poloidal velocity at $R = 216\,\mathrm{cm}$ is $-3.2\,\mathrm{km/s}$, while the tilt angle is 5.1 degrees. Although there is a discrepancy between this angle and Fig. 3, it is explained by the fact, that BES and CR measurements are carried out at different poloidal positions.

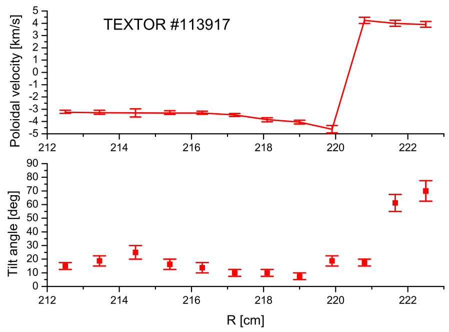

Figure 3: Fitted poloidal velocities and tilt angles for TEXTOR discharge #113917. The error bars are determined by $\chi^2_{\mathrm{red}} \leq 1$.

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