GOLDEN AND NOBLE MAGNETIC SURFACES AND MAGNETIC SHEAR IN TOKAMAKS

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ABSTRACT: A new symplectic map in magnetic coordinates is developed to study the resilience of golden and noble magnetic surfaces in tokamaks against resonant magnetic perturbations. The parameter *s* in this logarithmic map controls the magnetic shear. The locations of the equilibrium magnetic surfaces with golden and noble values of safety factor *q* [F.A. Volpe, J. Kessler, H. Ali, T.E. Evans and A. Punjabi, *Nucl. Fus*. **52**, 054017 (2012), H. Ali and A. Punjabi, to appear in *REDS: Inc. Plasma Sc. Plasma Tech*] are calculated for varying magnetic shear. Magnetic perturbations with resonant surfaces just above and below these golden and noble surfaces are considered. These perturbations are applied with increasing amplitude and the critical levels of the perturbations δ_{crit} when the surfaces break down are found. As the magnetic shear is increased, the resonant surfaces come closer. There is competition between the closer proximity of resonant surfaces and the magnetic shear. The results of this study will be presented.

Magnetic shear plays an important role in magnetic confinement of fusion plasmas. In this paper, we construct a symplectic map in magnetic coordinates that calculates magnetic field line trajectories in single-null divertor tokamaks with variable shear. We use this map to study the resilience of golden magnetic surface in tokamaks against resonant magnetic perturbations. The mapping technique [1] is applied to integrate the field lines of toroidally confined plasma. In this formalism the equations for magnetic field lines take the Hamiltonian form

$$\frac{d\psi}{d\phi} = -\frac{\partial\chi}{\partial\theta}, \qquad \frac{d\theta}{d\phi} = \frac{\partial\chi}{\partial\psi}, \tag{1}$$

where $\psi = r^2/2$ is a toroidal magnetic flux canonically conjugated to the poloidal angle θ and ϕ is the toroidal angle. The field lines Hamiltonian

$$\chi = \chi_0(\psi) + \chi_1(\psi,\theta,\phi) \tag{2}$$

can be represented as a sum of the unperturbed flux

$$\chi_0 = \int \frac{d\psi}{q(\psi)} \tag{3}$$

and the perturbed part of the flux

$$\chi_1(\psi,\theta,\phi)=\sum_{m,n}\chi_{mn}(\psi)\cos(m\theta-n\phi+\xi_{mn}),$$

Where $\chi_{mn}(\psi)$ is the amplitude of the perturbation with mode numbers (m,n) with the phase ξ_{mn}. The map for the field line is in magnetic coordinates, and is given by

$$\psi_{n+1}=\psi_n-k\frac{\partial\chi(\psi_{n+1},\theta_n,\phi_n)}{\partial\theta_n},\qquad \theta_{n+1}=\theta_n+k\frac{\partial\chi(\psi_{n+1},\theta_n,\phi_n)}{\partial\psi_{n+1}}, \tag{4}$$

where k is the map parameter. It represents the step-size of integration. Here, k is kept fixed at k=2π/36.

In this paper, the safety factor q as a function of magnetic coordinate ψ is given by

$$q(\psi)=1-s\ln(1-\psi) \tag{5}$$

The parameter s in this logarithmic map controls the magnetic shear and ψ is normalized by ψ_{SEP}, the toroidal flux inside the separatrix surface. The safety factor at the magnetic axis, $q(\psi=0)$, is equal to unity. Fig. 1 shows safety factor as function of ψ. The equilibrium generating function is then given by

$$\chi_{EQ}(\psi)=\frac{1}{s}e^{1/s}\left\{\mathrm{Ei}\left[\ln(1-\psi)-\frac{1}{s}\right]-\mathrm{Ei}\left[-\frac{1}{s}\right]\right\}. \tag{6}$$

The exponential integral Ei(x) in eqn.(6) is defined as $\mathrm{Ei}(x)=\mathcal{P}\int_{-x}^{\infty}\frac{e^{-t}}{t}dt=\mathcal{P}\int_{-\infty}^{x}\frac{e^{t}}{t}dt$. This definition can be used for real nonzero values of the argument x, but the integral has to be understood in terms of the Cauchy principal value due to the singularity of the integrand at zero. Fig. 1(b) shows the equilibrium generating function χ_{EQ} as a function of ψ. For magnetic perturbation, we choose two tearing modes with mode numbers (m,n)={(3,2),(2,1)}, and

$$\chi_1(\theta,\phi)=\delta\left[\cos(3\theta-2\phi)+\cos(2\theta-\phi)\right]. \tag{7}$$

Typically, for standard H-modes, q on the magnetic axis is the minimum q-value and is just below unity, while q close to the plasma boundary, q_{95}, is 3 or above. q_{95} is the safety factor of the surface that has 95% of the poloidal flux compared to the flux inside the separatrix. Magnetic shear and q_{95} play an important role in plasma stability and confinement. Experiments are beginning to access the external kink stability boundary at edge safety factor q_{95}=5 and in [2], authors reported results of experiments performed at q_{95}~9 in JT-60U, in order to attain high enough βp, which is significantly higher than that observed in the baseline scenario for diverted machines or in the ITER design. Fig. 2 shows safety factor at the 95% flux surface, q_{95}, as a function of the shear parameter s for the generating function given in eqn. (6) for our map.

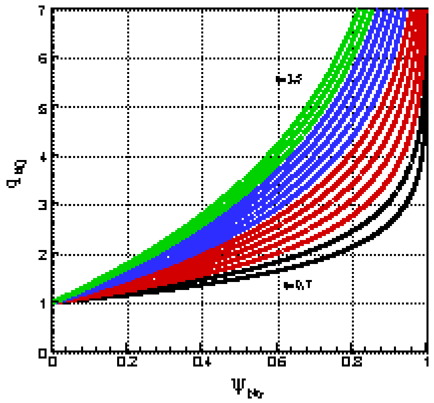

Fig. 1(a) Safety factor q as a function of ψ for s=0.7, 0.9, .., 3.5.

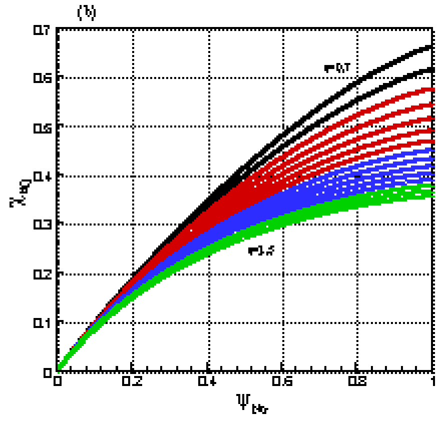

Fig. 1(b) Equilibrium generating function, χ_{EQ}, as a function of ψ s=0.7, 0.9, .., 3.5

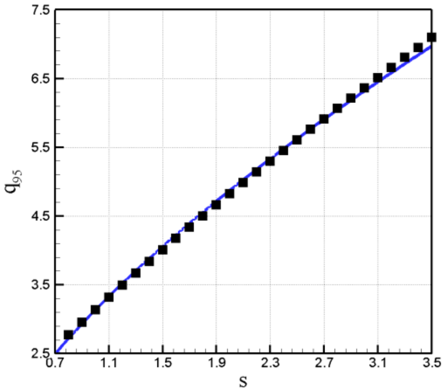

Fig. 2. The q_{95} as a function of toroidal flux for the shear parameter s. Curve through the data points is a power fit: q_{95} scales as $s^{2/3}$.

From Fig. 2, we see that for s=0.7, q_{95}=2.58 and for s=3.5, q_{95}=7.1. The locations of the resonant magnetic surfaces are calculated for varying magnetic shear. The locations of the equilibrium magnetic surface with golden values of safety factor q [3,4] are calculated for varying magnetic shear. We call these surfaces golden surfaces. As the magnetic shear parameter is increased from 0.7 to 3.5, the resonant surfaces come closer. There is competition between the closer proximity of resonant surfaces and the magnetic shear. See Fig. 3. The perturbation given by eqn.(7) is applied to these surfaces with increasing amplitude; and the critical levels of the perturbations δ_{crit} when the surfaces break down are calculated. We found that as the shear parameter increases, the critical perturbation, δ_{crit} scales approximately exponentially with the shear parameter, *s*. See Fig. 4.

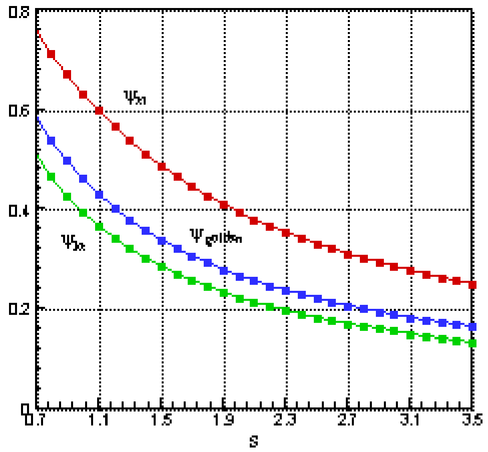

Fig. 3(a) Resonant and equilibrium golden magnetic surfaces as a function of the shear parameter *s*.

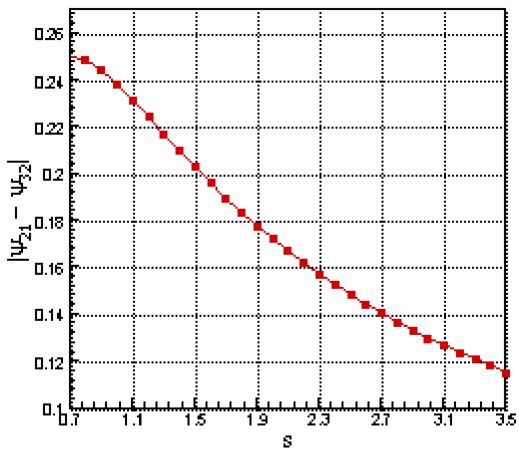

Fig. 3(b) Distance between resonant magnetic surfaces as a function of the shear parameter *s*.

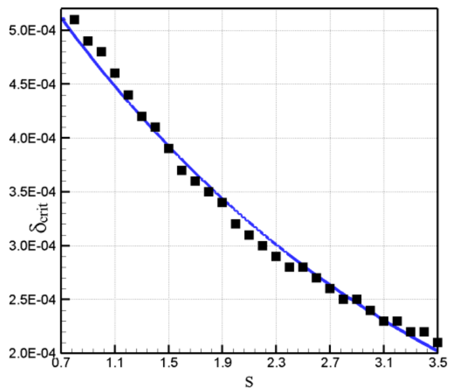

Fig. 4. The critical amplitude, δ_{crit} as a function of the shear parameter *s*. Curve through data point is an exponential fit: δ_{crit} scales as $e^{-s/3}$.

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