Resistive wall effects on the plasma dynamics in tokamaks

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1. Introduction. In the theory of plasma stability in tokamaks, stellarators and pinches, the vacuum vessel wall is treated as either ideal or magnetically thin resistive (see, for example, [1, 2]). Accordingly, the models can be applied to the plasma motions with characteristic time scales either τ_A or τ_w, the Alfvén time and thin-wall resistive time. At typical conditions, they differ by 3–4 orders of magnitude, which leaves in between a wide area theoretically unexplored, though the events with growth rates and/or frequencies ω of the order of $1/\tau$ such that $\tau_A \ll \tau \ll \tau_w$ are often observed in experiments. These are faster than conventional slow resistive wall modes (RWMs) and are usually analyzed assuming the wall ideal [3]. However, the energy dissipation in the wall can still be an important factor in this range with τ comparable to the wall skin time τ_{sk} [4, 5]. For such events, the energy balance is different from those prescribed in the ideal-wall and thin-wall approaches to the plasma stability. Here we discuss the resistive wall effects on the plasma dynamics with $\omega\tau_{sk} \geq 1$ ("fast RWMs").

2. Theory overview. In the standard approaches to the stability analysis, the wall enters the problem through the boundary conditions at the inner side of the wall, S_w. Mathematically, a constraint is imposed on the Poynting vector $\mathbf{E}\times\mathbf{b}$ that determines the energy exchange between the plasma and external world. The perturbed electric $\mathbf{E}$ and magnetic $\mathbf{b}$ fields at S_w depend on the skin depth s. When $s/d_w \to 0$, where d_w is the wall thickness, the wall is assumed ideal with $\mathbf{E}\times\mathbf{b} = 0$ and $\mathbf{b}\cdot\mathbf{n}_w = 0$ at S_w ($\mathbf{n}_w$ is the outward unit normal to S_w). These restrictions are an inherent part of the classical energy principle of ideal MHD [6]. In the opposite case with $s/d_w \to \infty$, the wall can be considered as magnetically thin with $\mathbf{b}\cdot\mathbf{n}_w$ the same at both sides of the wall. This assumption gives rise to so-called thin-wall approximation [1, 2] extensively used in the RWM studies, as reviewed in [2, 4, 7].

In the intermediate case with s/d_w of order unity, the wall acts as resistive, providing an energy sink, but magnetically "thick", strongly inhibiting the outward flux of the energy because of the skin effect [5, 7]. This changes the energy balance in the system compared to that in the classical theory of MHD stability with an ideal wall [6] (for ideal MHD modes) or in the thin-wall models [1, 2] (for slow RWMs). As a consequence, a regime is possible

where the resistive dissipation due to the mode rotation with frequency ω damps a mode potentially unstable at $\omega = 0$. This is described by the dispersion relation [4, 5]

$$\gamma_R = \gamma_R^{ld}(1 - \omega^2 / \omega_{cr}^2) = \gamma_R^{ld}(1 - \nu^2 / \nu_{cr}^2),$$ (1)

where γ_R^{ld} and $\omega_{cr} = 2\pi\nu_{cr}$ are the constants determined by the plasma parameters, the time dependence was assumed as $\exp(\gamma t)$ with $\gamma = \gamma_R + in\omega$ so that γ_R and ω are, respectively, the real growth rate and angular frequency of the mode toroidal rotation (n is the toroidal wave number) and $\nu = \omega/(2\pi)$ is the frequency measured by the probes.

3. Estimates. The derivation procedure leading to Eq. (1) includes an expansion in s/d_w as a small parameter. However, predictions of Eq. (1) remain valid at a weaker constraint $s/d_w < 1$, which fact was proved numerically [8] without the expansion, though in the cylindrical model. Here the modes in this range (fast RWMs) are considered. The presented estimates serve to connect the theory leading to Eq. (1) with experimental observations.

For purely rotating perturbations, we have

$$\frac{d_w}{s} = \sqrt{\frac{n\omega\tau_{sk}}{2}} = \sqrt{\frac{\omega}{\omega_l}} = \sqrt{\frac{\nu}{\nu_l}},$$ (2)

where the constants ω_l and ν_l are defined by

$$n\omega_l\tau_{sk} = 2\pi n\nu_l\tau_{sk} = 2$$ (3)

and

$$\tau_{sk} \equiv \mu_0\sigma d_w^2 = \tau_w d_w / r_w$$ (4)

with σ and r_w the wall conductivity and minor radius. A noticeable skin effect (precisely, $s/d_w < 1$) is achieved at $\nu > \nu_l$ (or $\omega > \omega_l$) with a threshold determined by τ_{sk}. Note that τ_{sk} is smaller than τ_w by factor of d_w / r_w, which is typically of the order of 10^{-2}. Therefore, τ_{sk} lies well between τ_A and τ_w. In the existing tokamaks, τ_{sk} is expressed in units of 10^{-4} s. This is a good estimate for DIII-D with $\tau_w = 5$ ms [2] and $r_w/d_w \approx 50$ [9]. In the Alcator C-Mod tokamak, $\tau_{sk} = 4.2 \times 10^{-4}$ s [10]. In ITER, τ_{sk} will be 10 times larger.

With $\tau_{sk} = 10^{-4}$ s, Eq. (3) gives us $\nu_l \approx 3$ kHz as a lower level for oscillations that can be treated as fast RWMs affected by the skin effect in the resistive wall. The upper limit of applicability of this approach and Eq. (1) must be at d_w / s large enough to treat the wall as ideal. The transition point to this regime is determined by the energy balance. At modest $d_w / s = 5$, for example, the upper threshold for ν will be 75 kHz (at $d_w / s = 4$, near 50 kHz).

4. Applications. With natural $\tau_A \ll \tau_{sk} \ll \tau_w$ there is a wide range (for DIII-D, for example, a good part of the interval $3 < \nu < 75$ kHz) where the plasma-wall electromagnetic interaction can lead to the rotational stabilization described by (1). Obviously, many events registered by the Mirnov coils as rotating oscillations with frequencies of several kHz in tokamaks fall into this category. Such may be the tearing and neoclassical tearing modes, rotating precursors of various instabilities, edge originated magnetic perturbations (outer modes, etc.). Here we mention several representative examples that are well documented, can be easily quantified and compared with predictions of the model [4, 5] for the fast RWMs.

In recent studies of the plasma responses to an externally applied resonant magnetic perturbations (RMP) on the J-TEXT tokamak, the tearing mode locking occurred at the mode frequency $\nu < 5$ kHz [11]. This and $1.6 - 1.8$ kHz at the near-locked state in some cases is larger than $\nu_l \approx 0.7$ kHz obtained from (3) with $\tau_{sk} = 4.5 \times 10^{-4}$ s found from (4) with data given in [11]. Then the mode dynamics must be described by Eq. (1) with the observed locking frequency as the critical level ν_{cr}, which was obviously above ν_l. In the analysis [11], the wall effect was neglected, but our estimates prove that in these cases the plasma-wall interaction should be accounted for as proposed in [4, 5].

Similar logic, estimates and conclusions should be also applied to the edge harmonic oscillations (EHO) with frequency of 6–10 kHz in DIII-D [12, 13]. The oscillations (a series of $n = 1 - 4$ or $n = 1 - 6$ harmonics) have been observed using the magnetic probes on the DIII-D vessel wall, which itself is an excellent indication that the perturbation-induced currents in the wall must be large. With $\nu_l \approx 3$ kHz we have $d_w / s = \sqrt{3}$ from Eq. (2) for 9 kHz. This justifies the use of Eq. (1) and allows to interpret the long-living EHOs with finite amplitude as marginally stable ($\gamma_R = 0$) modes with $\omega = \omega_{cr}$. In other words, the EHOs can be explained and should be described as the modes whose dynamics is essentially affected by their interaction with the resistive wall.

Note that at $d_w / s = 1.6$, corresponding to $\nu \approx 8$ kHz at $\nu_l = 3$ kHz, the perturbation amplitude at the outer side of the wall will be 5 times smaller than that at S_w. The difference of $\mathbf{b} \cdot \mathbf{n}_w$ inside and outside the vacuum vessel can be easily measured by the magnetic probes. Once such measurements have been performed for the rotating perturbations on the ASDEX Upgrade [14]. At frequencies higher than 1 kHz, the measurements have shown a reduction of the time varying field at the outer vessel wall by more than a factor of 10. It was concluded that the vacuum vessel can be considered as a perfect conductor for $\nu > 1$ kHz.

Such modes are, indeed, far from the conventional slow RWMs because $\omega\tau_w = 188$ for $\nu = 1$ kHz and $\tau_w = 30$ ms, which is an estimate [15] for the ASDEX tokamak (we use definition (4) here). However, our analysis shows that the wall behaves as still resistive at such frequency and the observed rotating modes should be treated as fast RWMs (reduction of $\mathbf{b}\cdot\mathbf{n}_w$ by factor of 12 is obtained at $d_w/s = 2.5$).

5. Discussion. In the latter case, it was stated that the screening currents induced in the vessel wall and in the in-vessel components by the time varying perturbation fields influence the measured perturbation field and must be taken into account for a correct interpretation of the Mirnov measurements [14]. Logically, the field of these currents should be also included into the force balance for the plasma. To make it, one has to impose proper boundary conditions at the wall. The commonly used are the models with an ideal or thin resistive wall. Our estimates show that $1-10$ kHz oscillations, often observed in various regimes of tokamak operation, fall in between where the wall should be treated as magnetically thick, see [4, 5].

In [3], an interpretation was suggested that at $\omega\tau_w \gg 1$ (equivalent to $\omega\tau_{sk} \gg d_w/r_w$ in our notation) the magnetic perturbation does not have time to penetrate the wall, the wall appears as an ideal conductor, and the plasma modes are wall stabilized. Our analysis applied exactly to such modes shows that this explanation is incorrect because in a wide range at $\omega\tau_w \gg 1$ the mode dynamics must be essentially affected by the energy sink and braking torque due to the wall resistivity. The plasma with such steady-state activity should be considered as marginally stable, as described by Eq. (1) with $\gamma_R = 0$. It predicts that this dynamic equilibrium can be maintained as long as the mode rotation frequency is kept at the critical level. It also explains and quantifies a transition to instability at mode locking.

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