Role of Polarisation Current on Neoclassical Tearing Mode Threshold

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Introduction

Neoclassical tearing modes (NTMs) are plasma instabilities characterised by the formation and evolution of so-called magnetic islands. They can limit tokamak performance by enhancing radial transport of particles and heat, which reduces core plasma pressure and temperature. If the mode is allowed to grow, it could substantially reduce the fusion power in future tokamaks, such as ITER. It is therefore necessary to develop systems to control NTMs in order to prevent the growth of the modes.

Experimental observations show that the growth of small islands (typically $O(1\text{cm})$ in width) is suppressed; they heal themselves and shrink away. One possible source of this threshold mechanism is the neoclassical polarisation current, which is generated when an island propagates through the plasma. Because of the difference in ion and electron inertia ($m_i \gg m_e$), an electrostatic potential is generated to satisfy quasineutrality, resulting in the $\mathbf{E}\times\mathbf{B}$ drift of the particles. However, because of the difference in the ion and electron Larmor radii ($\rho_{Li} \gg \rho_{Le}$), the two particle species experience different *orbit-averaged* $\mathbf{E}\times\mathbf{B}$ drift, resulting in a net current. This is the classical picture of the polarisation current, relevant when the island width is comparable in size to the ion Larmor radius. In toroidal geometry, a fraction of the particles are trapped. These trapped particles are in closed orbits, whose width is characterised by the banana width, ρ_b. Again, because $\rho_{bi} \gg \rho_{be}$, this also results in a net current: the neoclassical polarisation current. Since $\rho_b \gg \rho_L$, the neoclassical contribution dominates the classical counterpart, except perhaps in the vicinity of the island separatrix.

Previous works have considered the contribution of the polarisation current to the island evolution in the limit of large island width (compared to the trapped ion banana width) [1, 2]. However, the physics of polarisation current is not well-understood when the two length scales are comparable, in the full toroidal geometry with trapped particle population. Furthermore, the contribution to the polarisation current from the narrow layer surrounding the island separatrix opposes that away from the island, nearly cancelling each other out [3]. In order to develop an effective NTM control system for ITER, it is essential to determine the overall sign of the polarisation current, and understand the physics that governs it. In this paper, we report on the progress of the theoretical development to determine the full contribution of the polarisation current in the tokamak toroidal geometry. We describe a new numerical algorithm and code to solve the drift kinetic equation for the ion response to the island perturbation, expanding the perturbed distribution function in the small ratio of the island width to the tokamak minor radius.

Ion Response

The drift kinetic equation describes the ion response to the magnetic island perturbation:

$$v_\parallel \nabla_\parallel f + \mathbf{v}_E.\nabla f + \mathbf{v}_b.\nabla f - \frac{\omega_c}{Bv}\left[v_\parallel \nabla_\parallel \Phi + \mathbf{v}_b.\nabla\Phi\right]\frac{\partial f}{\partial v} = C(f), \tag{1}$$

where $\mathbf{v}_E$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity and $\mathbf{v}_b = -\mathbf{v}_\parallel \times \nabla(v_\parallel/\omega_c)$ is the magnetic drift velocity. Φ is the electrostatic potential and $C(f)$ is the model collision operator. In order to solve this equation for the perturbed ion distribution function, we expand the solution in terms of the small ratio of island width to the tokamak minor radius:

$$f = \sum_j \Delta^j f_j, \qquad \Delta = \frac{w}{r}. \tag{2}$$

Then, working in the coordinate (ψ, θ, ξ), where ψ is the poloidal magnetic flux, θ is the poloidal angle (coordinate along the equilibrium magnetic field lines) and ξ is the helical angle which labels the equilibrium field lines, the leading order ($O(\Delta^0)$) contributions to the drift kinetic equation are:

$$\frac{v_\parallel}{Rq}\frac{\partial f_0}{\partial\theta}\bigg|_\psi - \frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial f_0}{\partial\psi} = 0, \tag{3}$$

where f_0 is the leading order perturbed ion distribution function. This can be written more compactly by introducing the particle orbit coordinate in terms of the toroidal canonical momentum, p_ϕ:

$$p_\phi = (\psi - \psi_s) - \hat{\rho}, \qquad \hat{\rho} = \frac{I(\psi)v_\parallel}{\omega_c}, \tag{4}$$

where ψ_s is the poloidal flux at the rational surface where the island is located, $I = RB_\phi$, ω_c is the ion gyrofrequency, $v_\parallel = \sigma v(1-\lambda B)^{1/2}$ is the parallel velocity and $\lambda = v_\perp^2/v^2 B$ is the pitch angle. In the new particle orbit coordinate system (p_ϕ, θ, ξ), the leading order contribution to the drift kinetic equation reduces to:

$$\frac{v_\parallel}{Rq}\frac{\partial f_0}{\partial\theta}\bigg|_{p_\phi} = 0. \tag{5}$$

Therefore, by writing the solution as a function of p_ϕ, we can eliminate the θ-dependence of f_0: $f_0 = \bar{f}_0(p_\phi, \xi, v, \lambda)$. Not only is this consistent with the assumption that the system is treated as axisymmetric, but it allows us to reduce the dimensions of the problem to 4-D.

The exact solution to f_0 is determined from the $O(\Delta^1)$ contribution to the drift kinetic equation:

$$\frac{v_\|}{Rq}\frac{\partial f_1}{\partial\theta}\bigg|_{p_\phi} + \frac{mv_\|}{Rq}\left(1-\frac{q}{q_s}\right)\frac{\partial f_0}{\partial\xi} - \frac{Iv_\|}{Rq}\frac{\partial}{\partial\theta}\left(\frac{v_\|}{\omega_c}\right)\frac{\partial}{\partial\theta}\left(\frac{Iv_\|}{\omega_c}\right)\frac{\partial f_0}{\partial p_\phi}$$
$$+\mathbf{v}_E.\nabla f_0 + \mathbf{v}_b.\nabla\xi\frac{\partial f_0}{\partial\xi} - \frac{\omega_c}{Bv}\left[v_\|\nabla_\|\Phi + \mathbf{v}_b,\nabla\Phi\right]\frac{\partial f_0}{\partial v} = C(f_0). \quad (6)$$

In the previous analytic works, the drift kinetic equation (1) was further expanded in terms of the small ratio of the ion banana width to the island width, $\delta = \rho_b/w$; the $O(\delta^1\Delta^0)$ equation determined g^{00} through a constraint equation, and $O(\delta^1\Delta^1)$ equation determined g^{10} (see below for the set of analytic solutions). When the island width is comparable to the ion banana width, however, the ratio δ is no longer small, and the entire equation (6) must be solver for f_0 for an arbitrary ion banana width. For passing particles, the term in f_1 can be eliminated by multiplying Eq.(6) by $Rq/v_\|$, integrating over a period in θ at fixed p_ϕ (i.e. along the particle orbits) and applying the condition: $f_1(\theta=-\pi) = f_1(\theta=+\pi)$. For trapped particles, the conservation of particles at the bounce points imply: $f_1(\sigma=-1,\theta=\pm\theta_b) = f_1(\sigma=+1,\theta=\pm\theta_b)$, where σ is the sign of $v_\|$. Hence, for the trapped particles, the term in f_1 can be eliminated by multiplying Eq.(6) by $Rq/|v_\||$, summing over σ and integrating over θ between the bounce points. For both passing and trapped particles, the result is a 4-D integro-differential equation for f_0:

$$C_1\frac{\partial f_0}{\partial p_\phi} + C_2\frac{\partial f_0}{\partial\xi} + C_3\frac{\partial f_0}{\partial v} - C_{ii}(f_0) = 0, \quad (7)$$

where $C_1 \sim C_3$ are θ-averaged coefficients (functions of p_ϕ, ξ, v and λ), $C_{ii}(f_0)$ is the model collision operator for ion-ion collisions:

$$C_{ii}(f) = 2\nu_{ii}(v)\left[\frac{(1-\lambda B)^{1/2}}{B}\frac{\partial}{\partial\lambda}\left(\lambda(1-\lambda B)^{1/2}\frac{\partial f}{\partial\lambda}\right) + \frac{v_\|\bar{u}_\|}{v_{\mathrm{th}}^2}\right], \quad (8)$$
$$\bar{u}_\|(f) = \frac{1}{n\{\nu_{ii}(v)\}_v}\int \mathrm{d}^3\mathbf{v}\, v_\|\nu_{ii}(v)f(v), \quad (9)$$

and $\bar{u}_\|$ is the momentum conservation term (note that the differentials in λ are at constant ψ). Before solving this equation for f_0, it is useful to recall the set of analytic solutions [1]:

$$f_0 = \left(1 - \frac{e\Phi}{T}\right) F_{\mathrm{M}} + g^{00} + g^{10}, \tag{10}$$

$$g^{00} = \frac{F_{\mathrm{M}}}{n}\frac{\mathrm{d}n}{\mathrm{d}\psi}\frac{(\omega - \omega_*^T)}{\omega_*}\left[\psi - h(\Omega)\right], \quad g^{10} = -\frac{I v_\parallel}{\omega_c}\left[\frac{\partial g^{00}}{\partial \psi} + \frac{F_{\mathrm{M}}}{n}\frac{\mathrm{d}n}{\mathrm{d}\psi}\frac{\omega_*^T}{\omega_*}\right] + \bar{h}. \tag{11}$$

Here, g^{00} describes the leading order ion response to the perturbed magnetic geometry, and g^{10} corresponds to the first order banana width expansion of $F_{\mathrm{M}} + g^{00}$, as well as an additional response, $\bar{h}$, which is determined from collisional effects. Here, the coordinate Ω is constant on the perturbed flux surfaces of the magnetic island. Since this solution is valid in the limit of small banana width, or equivalently, away from the magnetic island ($w \ll |r - r_s|$), it can be used as a boundary condition for our new numerical calculation. Hence, by separating f_0 into the analytic solution: $F_a (= F_{\mathrm{M}} + g^{00} + g^{10})$ and the as yet unknown numerical solution, g, the constraint equation (7) then becomes an inhomogeneous equation of the form:

$$C_1\frac{\partial g}{\partial p_\phi} + C_2\frac{\partial g}{\partial \xi} + C_3\frac{\partial g}{\partial v} - C_{ii}(g) = D(F_a). \tag{12}$$

Then, the boundary condition on g is that its gradient, $\partial g/\partial p_\phi$, tends to zero away from the island ($|r - r_s| \to \infty$), but shifted by a constant amount, which would account for $O(\rho_b^2)$ correction.

Computer Code Development

A new parallelised computer code has been developed to solve Eq.(12) as a matrix equation: $\mathbf{M}.\mathbf{g} = \mathbf{D}$, where $\mathbf{M}$ describes the 4-D differential operator, $\mathbf{g}$ is the solution vector and $\mathbf{D}$ is the right hand side vector in terms of F_a. An initial version of the code has been developed for comparison with previous analytic results. This neglects the term in total velocity differential (the term in C_3), and the geometry is reduced to simpler toroidal geometry with circular poloidal cross-section and large tokamak aspect ratio. In addition, the challenging task of implementing quasineutrality is omitted for the time being (this requires iterating over the calculation of the electrostatic potential - another computational challenge). The code is current undergoing benchmarking, with results anticipated soon.

References

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