The Structure of Knotty Magnetized Configurations

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Abstract. The structure and evolution of a magnetized plasma fluid are affected by invariants which extend their topological information beyond the helicity. It is conjectured that various braided coronal magnetic configurations can be classified by the elementary building blocks of the 3-D structures – prime knots. The unique invariants of these knots may describe (a) the measured ion drop-outs in solar impulsive flare events and (b) the large deviations in the magnetic parcels of photospheric flux ropes carried into the interplanetary medium by the solar wind. Explicit examples of probable magnetic configurations at 1AU are presented.

I. Introduction

Magnetic fields are visualized as fictitious lines in three-dimensional space R^3, describing a curved vector field which is sustained due to the motion of electric charges (currents). In a common approximation these fields are embedded in an incompressible fluid and undergo transformations due to a continuous flow of the medium. The simplest naturally occurring magnetized configurations include the dipole and higher multi-pole structures resulting from a convection of electrically conducting fluid in the cores of celestial rotating bodies. Large scale galactic field is formed due to helical flow of plasma fluid on a much smaller scale (e.g. Brandenburg & Subramanian 2005). These motions form the basis of a dynamo operation: conversion of the kinetic into magnetic energy, as applied to the planetary, solar, or galactic environments. Additionally, various external perturbations or self-consistent oscillations may modify and destabilize the simple magnetic structures, resulting in significant deformations of the magnetic field with luminous emissions and ejection of plasma blobs. Terrestrial field exhibits the South American anomaly due to a non-symmetric core convection making the magnetic field imperfectly aligned with its geographic center and its poles. Large deformations of the geomagnetic tail due to the interaction with the solar wind plasma result in energetic electron fluxes which excite the terrestrial (planetary) auroras. Stressed, marginally stable coronal field loops may reach unsustainable equilibrium which undergoes rapid reconfiguration in the form of flares, resulting in the observed X-ray and gamma-ray images due to intense fluxes of energetic particles (electrons up to 1MeV and ions up to 100s of MeV). Sheared metastable configurations initiate coronal mass ejections (CMEs) - large interplanetary structures moving with a speed up to 3000 km/s and covering a fraction of AU; they can be traced by electromagnetic emissions, while they carry a shock which energizes particles over vast distances. Solar flares and CMEs are often correlated with filament ejections and brightening in Hα, EUV, or X-rays near the time of eruption onset, when coronal magnetic field exhibits kinking and writhing (Sterling et al. 2012). Magnetic clouds (MCs), a subset of the CMEs with low plasma beta and smoothly rotating magnetic field and bi-directional suprathermal electron fluxes indicate that their fields are twisted flux ropes (bundle of isolated strands of magnetic field with a central axis) connected at both ends to the Sun (Burlaga et al. 1982, Zurbuchen & Richardson 2006).

It has been known for many years that a vast majority of experimentally deduced or numerically simulated magnetic fields appear in helical shapes invoking twisting, bending and stretching of the field. Flux ropes exhibit besides the twisting of field lines adjacent to the axis also significant spatial deformation of the axis itself, forming a set of over/under crossings in a 2-D projection. These features of the magnetic field are important for their stability in the solar environment. Observations indicate that a necessary twist is present in the photospheric flux tubes which emerge from the convection zone (Pevtsov et al. 1995, Berger & Ruzmaikin 2000), which is essential in the evolution of a flux tube rising from the photosphere (Longcope & Klapper 1997), while numerical simulations (Longcope et al. 1996) confirm that an untwisted magnetic flux tube is easily fragmented. The partial conversion of twist into writhe may indicate the transformation of metastable configuration into a state of eruption (Török & Kliem 2005, Török et al. 2010). Hence, the complexity of the magnetic topology is of major importance with respect to the coronal stability and field eruption.

In topological terms most of these twisted configurations are equivalent through a one-to-one transformation (homeomorphism) to a two-dimensional (segment of a) circle. However, three-dimensionality allows formation of shapes which cannot be untangled to produce a simple loop, without making cuts in the magnetic lines. The observations and characterization of this important subset of magnetic structures is the topic of the present investigation.

II MHD Invariants

Magneto-hydrodynamics (MHD) describes the interaction of magnetic field with a plasma fluid, following both of them dynamically such that the flux is frozen into the fluid, i.e. magnetic flux embedded in the moving fluid stays constant in the zero-resistivity limit. Hence, the MHD velocity and magnetic field can be visualized as a collection of entangled, non-intersecting, slowly evolving fields. The solenoid magnetic field vector **B**(**x**)=rot **A** forms an important invariant of helicity: $H=\int \mathbf{A}.\mathbf{B}\, dV$, which can be viewed as an indicator of geometrical and structural complexity and important numerical and experimental investigative tool in magnetized plasma (Moffatt 1978; Berger & Field 1984; Ricca & Moffatt 1992). It has a topological interpretation as a linkage between two non-twisted isolated flux tubes and it measures the (sum of) twist and writhe of a flux rope (C˘alug˘areanu 1959). The twist indicates how much the field lines wind about the magnetic axis of the rope, whereas the writhe quantifies the helical deformation of the axis itself. In a closed system the helicity is modified only over the slow diffusive timescale and while magnetic energy is transformed into other energy forms, helicity is approximately conserved. Taylor (1986) hypothesis, states that in a turbulent plasma with small but non-vanishing resistivity, H is the only relevant constraint on the relaxation. However, various experiments and analyses indicated that wrapped or braided field configurations do not follow that conjecture, and the departure of the final state from the Taylor prediction indicate the presence of additional constraints (Berger 1990; Ruzmaikin and Akhmetiev 1994; Yeates 2010). These topological constraints emphasize that the magnetic structures and their relaxation processes may be affected by additional invariants beyond the helicity. Topological analysis based entirely on the 3D structure of the magnetic field forms the basis for these additional invariants.

III. Coronal and Solar Wind Topological Structures

The original suggestion of Parker (1979) and others states that the evolution of the coronal fields is controlled by the motion of highly conducting plasma in the photospheric foot-points, with (a) random rotations forming twists and (b) random walks forming braids in the overlying field lines (Wilmot-Smith et al. 2010). The emerging field can form twisted flux tube braids - number of intertwining strands attached between two photospheric foot-point plates, as well as link of two or more flux tubes. Some TRACE and Hinode images show twisting and braiding through X/EUV emissions. Solar wind which emanates from the solar atmosphere may carry the coronal field into the interplanetary space, allowing the remotely indiscernible details to be observed in situ. If the braiding structure, consisting of distinct magnetized plasma parcels preserves its topology, being stretched and deformed while convected passively from the Sun into 1 AU, a satellite should observe consecutive large diversion in the orientation of the magnetic field at 1AU (Borovsky 2008). When the braided or linked configuration becomes unstable causing flare deformation and ion energization, the magnetic structure convected into the solar wind may be observed as drop-out of energetic ion fluxes by a satellite which encounters different arcs of elongated braids emanating from a compact flare source (Mazur et al. 2000).

Mathematical braid is a collection of strands which weave over and under one another between fixed end-points, without looping back on themselves (Artin 1947). They visualize properly the coronal structures. Connecting the (photospheric) ends of two braids forms the same knot (link) if the braids differ by Markov (1935) moves; this procedure allows us to classify the braids with the help of the various topological knot invariants.

IV Knot Invariants

Knots - collection of entangled, closed loops of a non-self-intersecting curve, continuously deformed in R^3, following smooth changes in the surrounding viscous fluid, are depicted by 2D planar projections which preserve the over/under crossing of the 3D curve. General deformations which preserve the knot structure were described in the three link moves R_j, j=1, 3 (Reidemeister, 1928). Similarly to the integer prime numbers, knot which cannot be decomposed into two knots unless one of them is an unknot (circle) belongs to prime knots, the elementary building blocks of the 3D structures. It is conjectured that besides the trivial unknot, these prime knots are more likely to be observed in space. Their classification is obtained by assigning to each of the crossings a specific algebraic operation. Consistency of this operation while following the crossings over the whole knot results in a unique invariant: numerical value or algebraic function which will not change when the knot undergoes the R_j deformations (Alexander 1928; Kaufman 1987).

Alexander (1928) invariant is obtained when each under-crossing arcs a, b and over-crossing arc c are labeled via the rule: $a*b=c(t) = ta+(1-t)b$, for a variable t. The topological structure is then translated into algebra while labeling all the crossings forms a consistency matrix whose determinant becomes Polynomial $\Delta(t)$, invariant under R_j. The span of $\Delta(t)$ is also an invariant, suggesting that topological invariants with higher order polynomials are more impervious to modifications, resulting in slower dissipation for the more complex configurations. Figure 1 shows the randomly chosen (magnetic) prime knots with 6, 7, 8, 9, 10 crossings, after being dragged by the solar wind, together with their Alexander polynomials. The classification in the left column describes the crossing number enumerated in the knot atlas tables. Satellite traversing this structure would observe large deviations of the directional magnetic field which preserve the form of the coronal configuration.

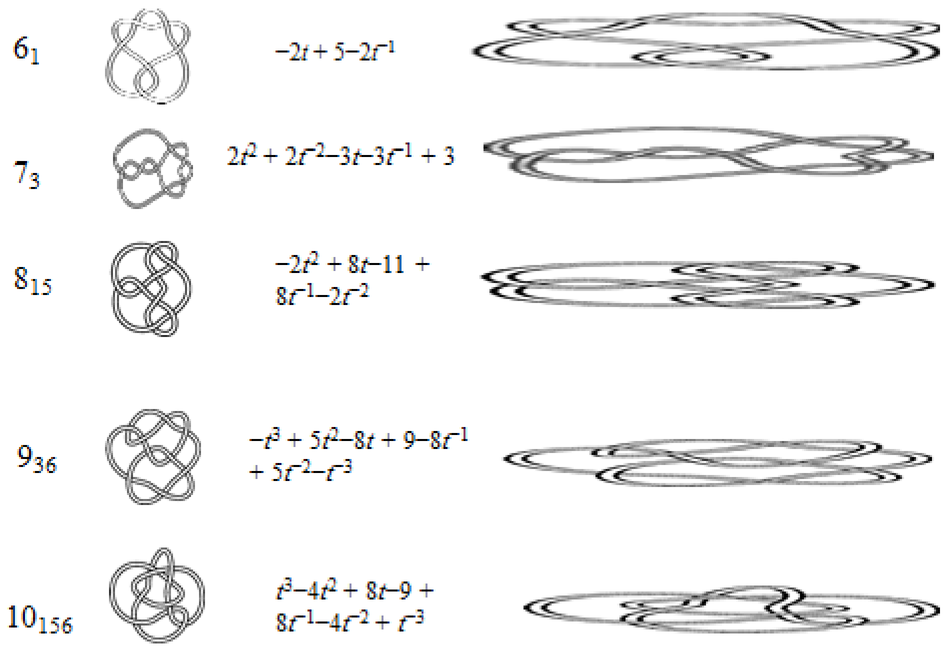

Figure 1. Random knots with 6-10 crossings, their invariants and presumed shapes at 1AU

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