Numerical simulations of a strong laser pulse interaction with a relativistic electron beam for the near future ELI Beamlines experiment

Martin Mašek[1], Ondřej Klimo[1,2], and Martin Jirka[1,2]

[1] *ELI-Beamlines, Institute of Physics, ASCR, Na Slovance 2, 182 21 Prague, Czech Republic*

[2] *FNSPE, Czech Technical University, Břehová 7, 115 19 Prague, Czech Republic*

In the first half of the year 2015, the ELI Beamlines infrastructure near Prague will be available in its first stage for the installation of a 10 PW laser facility. When such laser beam is focused to a spot with diameter of several microns, its power density reaches the level of $I = 5 \times 10^{22}$ W/cm^2. Electric field of the laser pulse with such intensity and with the wavelength in the order of 1 μm reaches 10^{14} V/m. When an electron experiences such high field its motion is accelerated or decelerated and a relatively intense radiation is emitted. Usualy weak radiation reaction effect becomes important within such laser intensity and has to be taken into account when the interaction of an intense laser beam with a relativistic electron beam is studied. In laser acceleration, radiative reaction can limit the maximum energy reachable by the electrons. In the other situation of the laser-electron scattering in vacuum, the radiation reaction may affect the electron dynamic very much.

To study described situations we developed a fully relativistic, three-dimensional code based on single particle tracking which calculates the laser interaction with a relativistic electron beam in an arbitrary geometry.

Equation of motion

The radiated power by the electron in the non-relativistic case is given by the Larmor power formula (see e.g. [1])

$$P(t) = m_e \tau_0 (\dot{\boldsymbol{v}})^2, \tag{1}$$

where $\tau_0 = 2e^2/3m_e c^3$ is the characteristic time. For the relativistic case the Larmor power formula in the relativistic case using the 4-velocity u^α can be written as

$$P = -m_e \tau_0 \dot{u}_\alpha \dot{u}^\alpha \tag{2}$$

The radiation friction force is than [2]

$$G^\alpha = m_e \tau_0 \Big(\ddot{u}^\alpha + \frac{u^\alpha}{c^2} \dot{u}_\beta \dot{u}^\beta \Big) \tag{3}$$

The equation of motion with the Lorentz force expressed by the electromagnetic field tensor $F^{\alpha\beta}$ and the radiation reaction force term G^α is called the Lorentz-Abraham-Dirac (LAD) equation:

$$\frac{d(m_e u^\alpha)}{d\tau} = \frac{e}{c} F^{\alpha\beta} u_\beta + G^\alpha, \tag{4}$$

where τ is the proper time. An exact solution to the LAD equation has not been found so far, moreover certain number of problems, conceptual and practical, arises from the LAD equation. One of them appears due to the fact that the right hand side of the equation contains the second derivative of velocity, which implies an extra initial condition contrary to the classical mechanics. The other problems are related in the incostistency of the classical electrodynamics and lead to the known run-away solution with exponentialy increasing velocity [3]. Despite these difficulties an approximate solution can be found. For the purpose of this work, we use the approximate solution proposed by Landau and Lifshitz [3]. This solution is based on the assumption that the radiation friction force is much weaker than the electromagnetic force. We can than neglect its effect on particle behavior to the zeroth order. The spatial part of the LAD equation than can be written as follows ($\boldsymbol{\beta} = \boldsymbol{v}/c$, $\boldsymbol{u} = \gamma\boldsymbol{v}$)

$$\frac{d(m_e\boldsymbol{u})}{dt} = e(\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B})+\boldsymbol{g_1}+\boldsymbol{g_2}+\boldsymbol{g_3}, \tag{5}$$

$$\boldsymbol{g_1} = \frac{2e^3}{3mc^2}\gamma\left(\frac{\partial}{c\partial t}+\boldsymbol{\beta}\cdot\nabla\right)(\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B}), \tag{6}$$

$$\boldsymbol{g_2} = \frac{2e^4}{3m^2c^4}\left[(\boldsymbol{\beta}\cdot\boldsymbol{E})\boldsymbol{E}+(\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B})\times\boldsymbol{B}\right], \tag{7}$$

$$\boldsymbol{g_3} = -\frac{2e^4}{3m^2c^4}\gamma^2\left[(\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B})^2-(\boldsymbol{\beta}\cdot\boldsymbol{E})^2\right], \tag{8}$$

Method of solution

The LAD equation (5) - (8) is solved using predictor-corrector method. First, we do the timestep employing the fourth-order Runge-Kutta method. Before the particles are pushed to the new positions and their velocities are updated, the derivatives of forces appearing in (6) are calculated and the correction of the applied force can be thus done. The particles are pushed using corrected value of the force including the force of the radiation reaction. Moreover, the other correction to the force acting on the particles is done via calculation of the Lorentz invariant parameter [4]

$$\eta = \frac{e\hbar}{m_e^2c^4}|F_{\alpha\beta}u^\beta|, \tag{9}$$

which determines the importance of strong field quantum effects. For the counter-propagating electron ($E_k = 1$ GeV) and laser pulse ($a_0 = 51$) the typical value of this parameter is $\eta = 0.6$. On the contrary, it is completely negligible for the co-propagation. When the value of η is close to unity, the QED effects can no longer be neglected. The quantum emissivity is depleted for hard photons compared to the classical emissivity introduced in our model. This effect is taken into account by multiplying the radiation reaction force by the function $g(\eta)$ (see eq. (15) in ref. [5]).

Laser and electron beam

For description of electromagnetic field in the simulations, we use the Gaussian beam in the form:

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r})\exp\left[-4\ln 2\left(\frac{t-z/c}{\tau_L}\right)^2\right]\exp(\imath\omega t), \tag{10}$$

where τ_L is the full width at half-maximum (FWHM) pulse duration and $\boldsymbol{E}(\boldsymbol{r})$ is the solution of the Helmholtz equation in the paraxial approximation. Its x-component is [6]:

$$E_x(\boldsymbol{r}) = a_0\frac{w_0}{w(z)}\exp\left[-\frac{\rho^2}{w^2(z)}\right]\exp\left[-\imath kz-\imath k\frac{\rho^2}{2R(z)}+\imath\arctan\left(\frac{z}{z_0}\right)\right], \tag{11}$$

where $\rho^2 = x^2+y^2$, $z_0 = \pi w_0^2/\lambda$ is so called Rayleigh range, $w_0 = (\lambda z_0/\pi)^{1/2}$ denotes beam waist, the beam width can be calculated as $w(z) = w_0[1+(z/z_0)^2]^{1/2}$ and $R(z) =$

$z[1+(z_0/z)^2]^{1/2}$ expresses wavefront radius of curvature. The other electric and magnetic field components are calculated using $E_z = (\imath/k)\partial E_x/\partial x$ and $\boldsymbol{B} = -(\imath/\omega)\nabla\times\boldsymbol{E}$, where $k = 2\pi/\lambda$ is the wave number and $\omega = 2\pi c/\lambda$ is the angular frequency.

The electron beam is considered as produced by the strong laser wakefield acceleration with defined energy spread and defined divergence. It is generated within the desired length, radius and angle of incidence with respect to the laser pulse propagation.

Results of calculations

We investigate the case, when the laser pulse with the wavelength $\lambda = 800$ nm and the FWHM duration $\tau = 30$ fs focused to the spot with radius $w_0 = 2.4\ \mu$m is counter-propagating to the highly relativistic electron beam. The electron beam is assumed to be the product of laser wakefield acceleration and it is simulated with the radius $r_b = 0.12\ \mu$m (5 % of the laser focal radius) and with the length $l_b = 0.9\ \mu$m (10 % of the laser pulse length). The electron beam contains $N = 10,000$ particles, which is sufficient number for the statistics. We study laser-particle interaction in several cases of laser power and electron beam initial energy. In the present work we focus only to the case of head-on collision in the best focus of the Gaussian laser pulse.

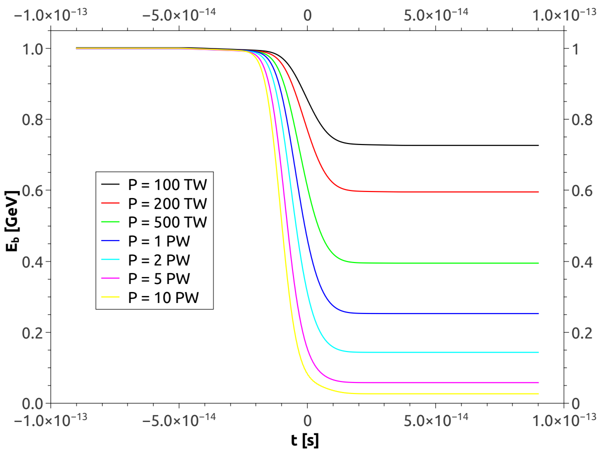

Figure 1: Temporal evolution of the mean beam energy.

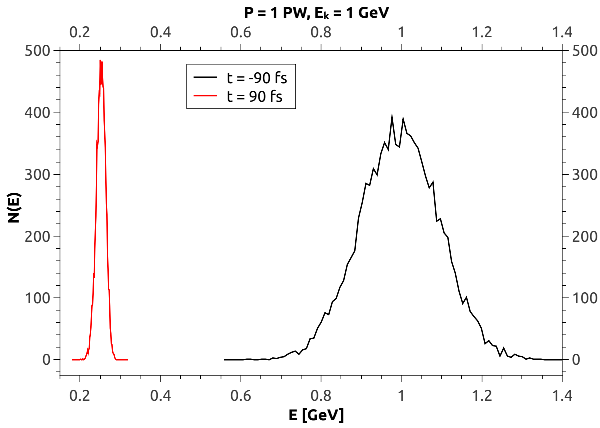

Figure 2: Comparison of the beam energy spectra in the beginning and at the end of the simulation.

The results of simulations in the range of laser power between $P = 100$ TW and $P = 10$ PW show the strong influence of the radiation reaction force to the particle dynamics. Fig. 1 illustrates the temporal dependence of average energy of the electron beam with the initial central energy $E_b = 1$ GeV and 10 % energy spread interacting with the laser pulse with power in the mentioned region. As expected the most powerful laser beam influences the electrons most significantly. The electrons radiate almost all of their kinetic energy and they are almost stopped by the laser beam. However, in the less intense case studied here ($P = 100$ TW) electrons radiate 27 % of their energy.

More interesting result can be find from the Fig. 2, where the electron energy spectra are depicted in the beginning of the simulation and after the interaction with the 1 PW laser pulse for the initial electron beam energy $E_b = 1$ GeV. As it is also visible from the Fig. 1, electrons are decelerated and their final energy decreases to the $E_b \approx 0.25$ GeV. Moreover, the interaction with the strong laser field leads to the narrower spectrum of the electron beam. In this particular case the spectral width drops from the initial $\sigma/E_b =$ 10 % to $\sigma/E_b = 4.6$ % of the mean value of the beam energy. On the other hand, the decrease of the spectral width is very significant in the absolute values. From initial $\sigma =$

0.1 GeV ($E_b = 1$ GeV) we got $\sigma = 0.01$ GeV ($E_b \approx 0.25$ GeV) at the end of the simulation.

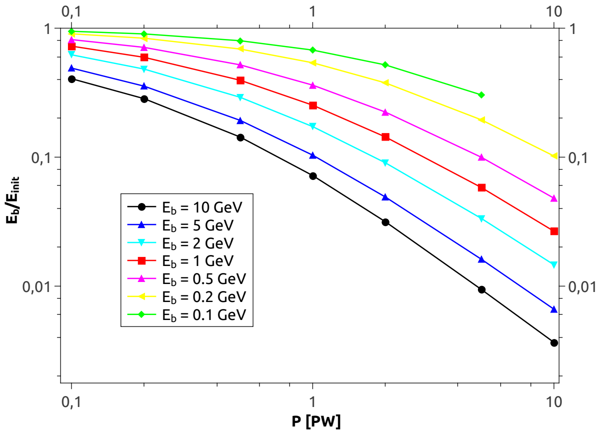

Figure 3: Electron beam mean energy relative to the initial energy as a function of laser power.

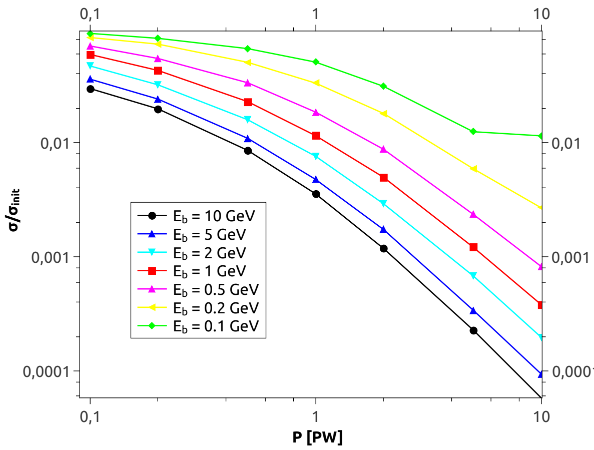

Figure 4: The beam energy spread relative to the initial beam energy spread (in each case $\sigma_{init} = 0.1E_b$) as a function of laser power.

Narrowing of the particle beam energy spectra is caused by the fact that the most energetic particles are the most affected by the laser field, while the less energetic particles radiate less. After the interaction they get close to each other in the phase space and the resulting energy spectrum is much narrower than the initial one.

We studied this effect also for the other values of the electron beam energy and laser pulse power. The results of such simulations are summarized in Figs. 3 and 4, where the mean energy relative to the initial energy and absolute energy spread of the beam relative to the initial value, respectively, are depicted as a function of laser power. It is clear from these figures that this process produces very fast electrons bunches with relatively low energy spread. E.g. for the 10 GeV beam with 10 % spread in energy we can produce 100 MeV electron bunch with energy spread approximately 3 MeV by using 5 PW laser. We see from our numerical results that in the most cases the energy spread decreases rapidly below 2 % (from the initial value 10 %) except the beams with the initial energies less than 200 MeV, where the decrease is slower and the resulting values for 10 PW laser pulse remain above 2 %.

Acknowledgements

This research has been supported by ELI-Beamlines (CZ.1.05/1.1.00/483/02.0061) and OPVK (CZ.1.07/2.3.00/20.0087) projects. The access to computing and storage facilities owned by parties and projects contributing to the National Grid Infrastructure MetaCentrum, provided under the programme "Projects of Large Infrastructure for Research, Development, and Innovations" (LM2010005) is highly acknowledged.

References

[1] J. D. Jackson, *Classical Electrodynamics*, 3rd Ed., John Willey & Sons (1999)
[2] P. A. M. Dirac, Proc. Roy. Soc. London Ser. A **167**, 148-169 (1938)
[3] L. D. Landau, A. F. Lifshitz, *The Classical Theory of Fields*, Pergamon, New York (1994)
[4] V. I. Ritus, Moscow Izdatel Nauka AN SSR Fiz. Inst. Tr. **51**, 5-151 (1979)
[5] J. G. Kirk, A. R. Bell and I. Arka, Plasma Phys. Control. Fusion **51**, 085008 (2009)
[6] B. E. A. Saleh, M. C. Teich, *Fundamentals of Photonics*, 2nd Ed., John Willey & Sons (2007)