Robustness of predator-prey models for confinement transitions in tokamak plasmas

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1. Introduction

Energy transport in toroidal magnetically confined fusion plasmas is determined, in most cases, by the effects of small-scale turbulence and larger scale coherent nonlinear structures, together with their mutual interactions[1,2]. These structures include zonal flows and geodesic acoustic modes, which are radially localised poloidal flows, and streamers, which are radially elongated and poloidally localised. Zero-dimensional models remain attractive as a simple and direct means to capture the physical origin of enhanced energy confinement and of transitions between confinement regimes[3] in tokamak plasmas. A key step is to establish agreement between the outputs of such models and the observed confinement phenomenology, which should ideally extend to the character of measured time traces of plasma properties near transitions. Recent experimental results from the DIII-D[4] and H2-LA tokamaks[5] are encouraging. The prime zero-dimensional model paradigm is predator-prey or Lotka-Volterra. Here we extend a successful three-variable (temperature gradient; microturbulence level; one class of coherent structure) model in this genre[6], constructed by Malkov and Diamond (hereafter MD), by adding a fourth variable to represent a second class of coherent structure. This requires a fourth coupled nonlinear ordinary differential equation. We investigate[7] (hereafter ZCD) the degree of invariance of the phenomenology generated by the two zero-dimensional predator-prey models, given the additional physics embodied in this change. We study and compare the long-time behaviour of the three-equation and four-equation systems, their evolution towards the final state, and their attractive fixed points and limit cycles. We find that, for example, an attractive fixed point of the three-equation system can become a limit cycle of the four-equation system[7]. Addressing these questions – which can be labelled “robustness” – is important for models which, as here, generate sharp transitions in the values of system variables which resemble some key features of confinement transitions in tokamak plasmas.

2. Model description

The ZCD model[7] encompasses drift wave turbulence level E, drift wave driving temperature gradient N, zonal flow velocity V_{ZF}, streamer flow velocity V_{SF}, and their nonlinear couplings, together with the heating rate q which is a control parameter of the system. This model thus extends, to the case when zonal flows are joined by streamers, the key physics encapsulated in the MD model[6]. The ZCD model equations are

$$\frac{dE}{d\tau} = \; N - a_1 E - a_2 d^2 N^4 - a_3 V_{ZF}^2 - a_3 V_{SF}^2 \;\; E$$

$$\frac{dV_{ZF}}{d\tau} = \left(\frac{b_{1Z} E}{1 + b_{2Z} d^2 N^4} - b_{3Z} \right) V_{ZF}$$

$$\frac{dV_{SF}}{d\tau} = \left(\frac{b_{1S} E}{1 + b_{2S} d^2 N^4} - b_{3S} \right) V_{SF}$$

$$\frac{dN}{d\tau} = - \;\; c_1 E + c_2 \;\; N + q \;\; \tau$$

Numerical solutions of this system have recently been generated and studied[7] for parameter regimes maximally close to those investigated previously in the three-variable model of MD[6].

3. Model results

An example of time traces for the dimensionless counterparts (E, N, U1, U2) of the variables (E, N, V_{ZF}, V_{SF}) is shown in Fig.1 for parameters relevant to MD. The initial evolution is akin to that in Fig.2 of MD, from overpowered H-mode via L-mode to T-mode. A new development is the subsequent transition, after a long time, from T-mode, which is no longer stable, to a new limit cycle. A corresponding plot of the phase evolution of the system is shown in Fig.2 below.

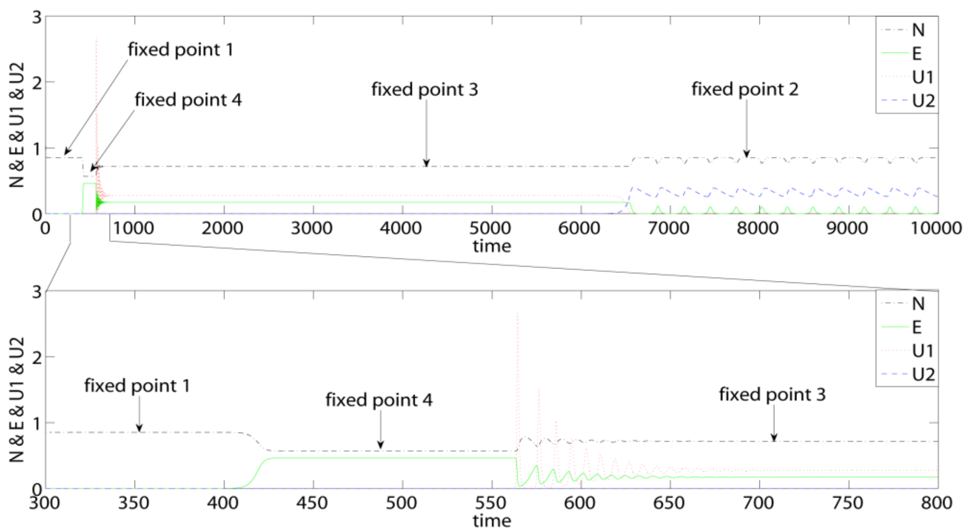

Figure.1. Time traces showing evolution from overpowered H-mode via L-mode to T-mode, in MD nomenclature; and a later transition to a new limit cycle. Numbers 1 to 4 relate to fixed points in Fig.2. Reproduced from Ref.[7].

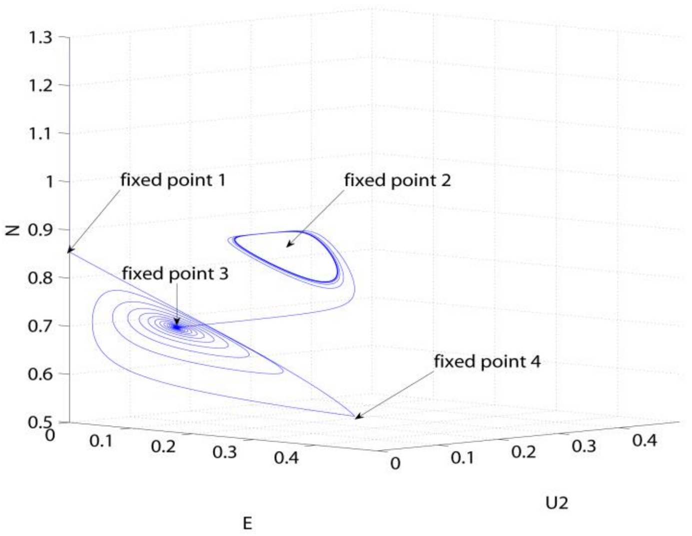

Figure 2. Evolution of the system, projected into (E, N, U2) phase space, corresponding to the time traces of Fig.1. Fixed points 1 to 4 are indicated also in Fig.1. Reproduced from Ref.[7].

Figure 3 (left panel) shows a projection of the limit cycle identified in Fig.3 of MD onto the (E, U) plane. The middle panel shows that the ZCD equivalent cycle in (E, U1) eventually becomes unstable: there is injection along the U2 direction followed by relaxation to a fixed point at the origin of the (E, U1, U2) phase space. The right panel, which plots 81 adjacent trajectories, shows that this outcome is robust.

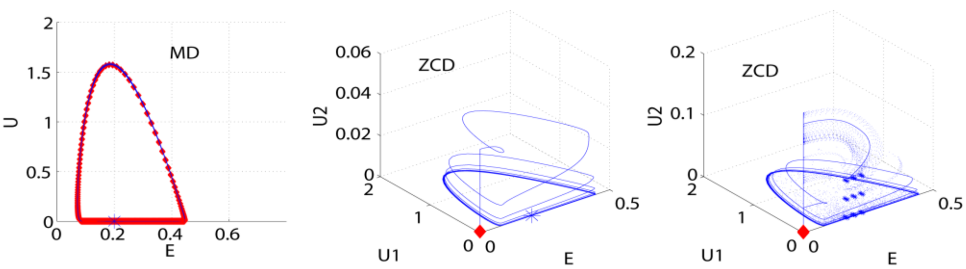

Figure 3. (Left) Limit cycle of MD system; (Middle) Instability of ZCD equivalent cycle; (Right) As middle panel, for 81 trajectories. Reproduced from Ref.[7].

In contrast to Fig.3, Fig.4 shows a case where the MD system dynamics is essentially invariant against the inclusion of a fourth variable in the ZCD approach. The model parameters for Fig.4 are close to, but slightly different from, those used to generate Fig.3. The two left panels depict the MD limit cycle attractor (Fig.3 of MD) for the case of a single coherent field U, and the two right panels show the corresponding system evolution in the (E,U1,U2) projection of phase space for the ZCD model. Over time, the U2 component diminishes to zero, and the ZCD limit cycle in (E,U1) is the same as the MD limit cycle in (E,U).

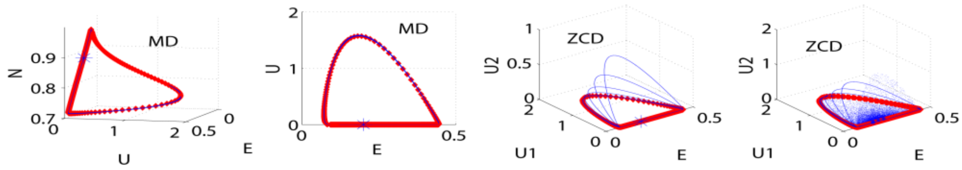

Figure 4. (Left) Limit cycle of MD system in (N, E, U) phase space; (Centre left) Projection of left panel on (E,U) plane; (Centre right) Evolution towards limit cycle of ZCD system in (E,U1,U2) phase space; (Right) As centre right, for 81 trajectories. Reproduced from Ref.[7].

4. Conclusions

Zero-dimensional models embody physically motivated narratives that aim to explain global fusion plasma confinement phenomenology. Ideally, the end states (attractors) of zero-dimensional models, together with the transitional behaviour *en route* from the initial configurations, should be robustly identifiable with fusion plasma confinement states and transitions. Zero-dimensional predator-prey models, constructed in terms of a small number of variables representing global quantities as in Eqs.(1) to (4), are intrinsically nonlinear. Nonlinearity implies the potential for a rich and varied set of attractors and transitional behaviour, together with strong dependence on the numerical values of model parameters. We have explored this[7] for the model of interest, in the case of parameter sets close to those studied previously in MD[6]. Our results appear to strengthen the links between zero-dimensional models and fusion plasma confinement phenomenology.

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[1] A Fujisawa, Plasma Phys. Control. Fusion **53**, 124015 (2011)
[2] G R Tynan, A Fujisawa, and G McKee, Plasma Phys. Control. Fusion **51**, 113001 (2009)
[3] F Wagner, Plasma Phys. Control. Fusion **49**, B1 (2007)
[4] L Schmitz *et al*., Phys. Rev. Lett. **108**, 155002 (2012)
[5] M Xu, G R Tynan, P H Diamond *et al*., Phys. Rev. Lett. **108**, 245001 (2012)
[6] M A Malkov and P H Diamond, Phys. Plasmas **16**, 012504 (2009)
[7] H Zhu, S C Chapman and R O Dendy, Phys. Plasmas **20**, 042302 (2013)