

Transport theory for energetic alpha particles in tokamaks with broken symmetry*

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Toroidal symmetry does not exist in real tokamaks because of the existence of the error fields, discrete toroidal magnetic field coils, and magnetohydrodynamic (MHD) activities. There are two mechanisms that break the toroidal symmetry in $|\mathbf{B}|$ spectrum. One is the direction addition of the perturbed magnetic field to the equilibrium magnetic field [1]. Another is the surface distortion mechanism by recognizing that it is the $|\mathbf{B}|$ spectrum on the distorted magnetic surface that is relevant to the transport processes [2,3]. Broken symmetry leads to enhanced particle, momentum, and energy transport losses. The asymptotic analysis of the linear drift kinetic equation has been used to calculate the toroidal plasma viscosity and the corresponding transport fluxes for thermal particles in tokamaks with broken symmetry. The theory consists of several asymptotic regimes depending on the collisionality, and its results are summarized in [4]. There is a steady state toroidal flow of the order of $v_{ti} \rho_{pi}/L$ in all regimes in the theory [3], which can be used to control MHD stabilities. Here, v_{ti} is the ion thermal speed, ρ_{pi} is the ion poloidal gyro-radius, and L is the radial gradient scale length. Approximate analytic expressions for the neoclassical toroidal plasma viscosity have been constructed [5] and are in good agreement with the numerical solution of the bounce averaged drift kinetic equation [6,7]. One of the key assumptions of the theory is that the perturbed magnetic fields do not create a new class of trapped particles. The transport fluxes in the low collisionality regime are caused by trapped particles, i.e., bananas, wobbling off the flux surface resulting from the perturbed magnetic field. The theory is also applicable for rippled tokamaks with negligible fraction of rippled trapped particles.

The energy transport loss rate increases with increasing particle energy in the theory [2-5]. Thus, energetic alpha particles are much more susceptible to broken symmetry in tokamaks. One of the distinct features in burning plasmas is self-heating from the fusion energy. In ignited burning plasmas, the fusion energy gain factor Q is infinity. To accomplish ignition, it requires that fusion born alpha particles transfer their energy to fueling plasmas

through the slowing down process. However, when the radial energy transport loss rate becomes comparable to the slow down rate, a significant portion of the fusion energy is lost before it has the opportunity to be deposited to the fuel plasmas. This limits the magnitude of the perturbed magnetic field strength. To have an accurate estimate of the tolerable magnitude of the perturbed magnetic field strength, a transport theory for the fusion born alpha particles for tokamaks with broken symmetry is needed.

Because energetic alpha particles are rather collisionless, it is natural to bounce average the linear drift kinetic equation to focus on the drift orbit dynamics of wobbling bananas. Thus, the transport theory for energetic alpha particles is similar to the theory for the neoclassical toroidal plasma viscosity for thermal particles. If the radial electric field is determined by the thermal fuel plasmas, it has little influence on the orbit dynamics of the energetic alpha particles. Thus, the dominant regimes are $1/v$ [3], superbanana plateau [8], and superbanana regimes [9]. The superbanana plateau and superbanana regimes are caused by the superbanana resonance where the bounce averaged toroidal drift speed vanishes. The resonance occurs at the tips of the superbananas where the drift speed vanishes. The physics associated with superbanana resonance has not been addressed in tokamaks with broken symmetry. It is neglected in the transport theory for rippled tokamaks [10]. The superbananas, discussed in [11], is from the resonance between the bounce motion and the drift motion. However, because particle drift speed is smaller than the bounce speed by a factor of ρ_α/L , transport fluxes associated with drift orbit dynamics are more significant, where ρ_α is gyro-radius of energetic alpha particles.

One of the differences between the transport theory discussed here and the theory for the neoclassical toroidal plasma viscosity is in the collision operator. For energetic alpha particles, the slowing down operator is also important besides the pitch angle scattering operator. Here, we discuss the transport consequences when the pitch angle scattering process dominates. The fluxes when the slowing down operator dominates will be presented elsewhere.

The Hamada coordinates are used in the theory in which the magnetic field is expressed as $\mathbf{B} = \psi' \nabla V \times \nabla \theta - \chi \nabla V \times \nabla \xi$, where V is the volume enclosed inside the flux surface, θ is the poloidal angle, ξ is the toroidal angle, $\psi' = \mathbf{B} \cdot \nabla \xi$, and $\chi = \mathbf{B} \cdot \nabla \theta$ [12]. The $|\mathbf{B}|$ spectrum can be expressed in a compact form, $B = B_0 (1 - \varepsilon \cos \theta) - B_0 \sum_n [A_n(\theta) \cos n \xi_0 + B_n(\theta) \sin n \xi_0]$, where B_0 is the magnetic field strength on the magnetic axis,

ε is the inverse aspect ratio, n is the toroidal mode number, $\zeta_0 = q\theta - \zeta$ is the field line label, and q is the safety factor.

The drift orbit dynamics dominates the transport processes when the collisionality parameter $\nu_* < 1$. Here, ν_* is defined as $\nu_* = v_j R q / (\varepsilon^{j+1/2} v_\alpha)$, where R is the major radius, v_α is the typical speed of the energetic alpha particles, $v_j = v_D$, the deflection frequency for $j = 1$, and $v_j = v_s$, the slowing down frequency for $j = 0$. The difference in the definitions for ν_* results from the fact that there is no enhancement due to the smallness of ε for the slowing down process, but there is for the pitch angle scattering.

The equation that governs drift orbit dynamics is the bounce averaged drift kinetic equation [13,14]

$$\langle \mathbf{v}_d \cdot \nabla \zeta_0 \rangle_b \frac{\partial f_{01}}{\partial \zeta_0} + \langle \mathbf{v}_d \cdot \nabla V \rangle_b \frac{\partial f_{01}}{\partial V} + \langle \mathbf{v}_d \cdot \nabla V \rangle_b \frac{\partial f_s}{\partial V} = \langle C(f_{01}) \rangle_b, \quad (1)$$

where f_{01} is the first order correction to $f_s(V)$, the slowing down distribution, $\langle \bullet \rangle_b = \sum_\sigma (\oint d\theta(\bullet) B / |v_\parallel|) / (\oint d\theta B / |v_\parallel|)$, σ is the sign of v_\parallel , and v_\parallel is the particle speed parallel to \mathbf{B} . Explicit expression for $f_s(V)$ is [15]

$$f_s(V) = \frac{N_\alpha}{4\pi v_0^3} \frac{3}{\ln[(v_0^3 + v_c^3)/v_c^3]} \frac{H(v_0 - v)}{(v^3 + v_c^3)/v_0^3}, \quad (2)$$

where H is the step function, and the density for the energetic alpha particle is $N_\alpha = (S\tau_{sa}/3) \ln[(v_0^3 + v_c^3)/v_c^3]$, where $S = N_D N_D \langle \sigma_F v \rangle$ denotes the isotropic source of alpha particles from D-T fusion reactions that have a cross section σ_F , v is the particle speed, v_0 is the birth energy of alpha particles, The $\partial f_{01} / \partial V$ term in Eq.(1) is needed to describe superbanana orbits when energetic alpha particles are in the superbanana regime. The collision operator consists of pitch angle scattering and slowing down operators. The deflection frequency for pitch angle scattering operator for energetic alpha particles is $v_D \approx v_D^{\alpha i} \approx v_{sa} v_b^3 / v^3$ is the deflection frequency. where $v_b^3 = 3\sqrt{\pi/2} T_e^{3/2} / (M_\alpha \sqrt{M_e}) (\sum_i N_i Z_i^2 \ln \Lambda_i) (N_e \ln \Lambda_e)^{-1}$, and $\tau_{sa} = v_{sa}^{-1}$ is the slowing down time for alpha particles. When $v^3 < v_b^3$, pitch angle scattering operator dominates.

The most important regime is the superbnana plateau regime for causing the largest transport losses. It appears when approximately $(\delta B/B_0)^{3/2} \varepsilon^{-1/2} c M_\alpha v_\alpha^2 \varepsilon' / (e_\alpha \chi') < v_D < c M_\alpha v_\alpha^2 \varepsilon' / (e_\alpha \chi')$. Because $\delta B/B_0 \ll 1$, this regime is nominally very broad in the collision

frequency domain. The energy transport coefficient χ_α for energetic alpha particles in the superbanana plateau regime scales as [14]

$$\chi_\alpha \sim \sqrt{\varepsilon} \frac{v_0^2}{\Omega_p} |n| \left(\frac{\delta B}{B_0} \right)^2, \quad (3)$$

and the corresponding the energy confinement time $\tau_{E\alpha}$ is $\tau_{E\alpha} \sim a^2/\chi_\alpha$, where Ω_p is the poloidal gyro-radius for alpha particles, and a is the minor radius. For averaged electron density $\langle N_e \rangle \sim 6.4 \times 10^{13} \text{ cm}^{-3}$ and temperature $\langle T_e \rangle \sim 18 \text{ keV}$, the slowing down time $\tau_{s\alpha} \sim 1.2 \text{ s}$. Thus, $\tau_{E\alpha}$ becomes comparable to $\tau_{s\alpha}$ for an $\varepsilon = 1/3$ tokamak with poloidal magnetic field strength $B_p = 0.57T$, and minor radius $a = 2m$, when $(\delta B/B_0)_t \sim 10^{-3}/\sqrt{|n|}$. The result implies that the larger the toroidal mode number n is, the smaller the tolerable magnitude of $(\delta B/B_0)_t$ is. For $|n| \geq 2$, the tolerable $(\delta B/B_0)_t$ is about few times 10^{-4} . This provides an estimate to the tolerable magnitude of $(\delta B/B_0)_t$ for the error fields or MHD activities that will not affect fusion energy gain factor Q significantly.

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