Fokker Planck kinetic modeling of suprathermal α-particles in a fusion plasma

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Context and purpose of the study

The design of Inertial Confinement Fusion (ICF) targets and the interpretation of ICF experiments rely on numerical simulations based on hydrodynamic Lagrangian codes where kinetic effects are only considered as corrections included in the transport coefficients. The fluid description is relevant if the mean free path of plasma particles, namely electrons and ions, is smaller than the characteristic length scale. Although this condition is reasonably fulfilled during the implosion stage, it does not apply to fast fusion products.

Nevertheless, in all major present-day fluid codes, multi-group flux limited diffusion schemes are usually employed to model suprathermal α-particles. This kind of methods, although computationally efficient, may not calculate energy and momentum deposition spatial profiles accurately. This may have an impact on the calculation of ignition thresholds and energy gains.

In this work we propose a full ion-kinetic description of suprathermal fusion products, treated self-consistently with the ion-kinetic modeling of the thermal imploding plasma. The difficulty lies in the coupling of ion populations characterized by two different energy scales: thermal D,T ions, which form the bulk of the imploding plasma and whose kinetic energy is in the keV range, are coupled to suprathermal α-particles, created at 3.52 MeV by fusion reactions.

To overcome the difficulty associated to the high energy contrast, we develop a *two-energy-scale approach* based on the analysis of the underlying physical model. We will show that it is possible to rearrange the terms of the Fokker-Planck equation governing the evolution of fast fusion products into a *system* coupling two components (namely a suprathermal and a thermal one) associated to the α distribution function.

Our two-scale approach enables us to design tractable numerical methods, which have been employed to build a new ion kinetic code FUSE (for FPION *[1] Upgrade with two Scales of Energy*). Based on a two-energy-scale formalism, FUSE is apt to investigate kinetic effects related to fusion reaction products on the ignition of the hot spot and on the subsequent propagation of the thermonuclear burn wave through the dense fuel with reasonable computational time.

Comparisons between the fluid code FCI1 and the kinetic code FUSE are presented during

the implosion and the propagation of the combustion flame through the dense fuel shell.

Physical model for the transport and collisional relaxation of α-particles

The distribution function $f_\alpha(\vec{r},\vec{v},t)$ of α-particles characterized by a charge $Z_\alpha e$ and a mass m_α is governed by the Vlasov-Fokker-Planck equation, modeling the transport, acceleration and collisions of α-particles :

$$\frac{\partial f_\alpha}{\partial t}+\vec{v}\cdot\frac{\partial f_\alpha}{\partial \vec{r}}+\frac{Z_\alpha e\vec{\mathscr{E}}}{m_\alpha}\cdot\frac{\partial f_\alpha}{\partial \vec{v}}=\sum_i \left.\frac{\partial f_\alpha}{\partial t}\right|_{\alpha i}+\left.\frac{\partial f_\alpha}{\partial t}\right|_{\alpha e}+\left.\frac{\partial f_\alpha}{\partial t}\right|_{\text{fuse}}. \tag{1}$$

The first term on the right hand side of this equation describes the collisional relaxation of α-particles with thermal ions (including thermalized α-particles) and can be written as :

$$\left.\frac{\partial f_\alpha}{\partial t}\right|_{\alpha i}=4\pi\Gamma_{\alpha i}\frac{\partial}{\partial \vec{v}}\cdot\left(\frac{m_\alpha}{m_i}f_\alpha\frac{\partial \mathscr{S}_i}{\partial \vec{v}}-\nabla_v^2\mathscr{T}_i\cdot\frac{\partial f_\alpha}{\partial \vec{v}}\right), \tag{2}$$

where $\mathscr{S}_i$ and $\mathscr{T}_i$ are the so-called Rosenbluth potentials [1] associated to the target ions i. They are defined by a set of Poisson equations in velocity space:

$$\Delta_v\mathscr{S}_i=f_i,\qquad \Delta_v\mathscr{T}_i=\mathscr{S}_i. \tag{3}$$

The coefficient $\Gamma_{\alpha i}=(4\pi Z_\alpha^2 Z_i^2 e^4/m_\alpha^2)\ln\Lambda_{\alpha i}$ is proportional to the Coulomb logarithm $\ln\Lambda_{ij}$ (for any species i,j including electrons) related to the Coulomb potential screening and taking quantum effects into account.

The second term on the right hand side of Eq. (1) describes the collisional relaxation of α-particles with electrons. Since the electron equilibration time τ_{ee} is much smaller than the mean ion-ion collision time τ_{ii}, the electron kinetic model reduces to a fluid equation. The Fokker Planck term modeling the effect of collisions between α-particles and electrons then simplifies to :

$$\left.\frac{\partial f_\alpha}{\partial t}\right|_{\alpha e}=\frac{1}{\tau_{e\alpha}}\frac{\partial}{\partial \vec{v}}\cdot\left[(\vec{v}-\vec{u}_e)f_\alpha(\vec{v})+\frac{T_e}{m_\alpha}\frac{\partial f_\alpha}{\partial v_\alpha}(\vec{v})\right], \tag{4}$$

where n_e, T_e, $\vec{u}_e$ are the electron density, temperature, mean velocity respectively, whereas $\tau_{e\alpha}$ is a characteristic $e-\alpha$ collision time.

The last term on the right hand side of Eq. (1) is the isotropic source term related to fusion reactions. It is calculated from the the distribution functions f_D and f_T, solutions of the Vlasov-Fokker-Planck equation written on the D,T ion species, respectively.

Two-component description of the α distribution function

3.52 MeV α-particles are firstly slowed down essentially by electrons. The first stage of the α slowing down is thus described by:

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{coll}\approx\frac{1}{\tau_{\alpha e}}\frac{1}{v^2}\frac{\partial}{\partial v}\cdot\left[v^3 f_\alpha(v)\right]. \tag{5}$$

The stationary solution of (5) behaves as $f_\alpha \sim 1/v^3$, where v is the suprathermal α-particle velocity. Consequently, as long as fast α-particles remain far from the thermal velocity region, their distribution function varies smoothly over the whole suprathermal velocity region. The associated velocity scale v_α^{ST} is in particular greater than the target thermal velocity v_i^{th}.

Then, when slowed down α-particles get closer to the thermal region but still remain suprathermal, thermal ions tend to dominate the end of the relaxation process, which is then governed by the equation:

$$\left.\frac{\partial f_\alpha}{\partial t}\right|_{\text{coll}} = \sum_i 4\pi\Gamma_{\alpha i}\frac{\partial}{\partial \vec{v}}\cdot\left(\frac{m_\alpha}{m_i}f_\alpha\frac{\partial \mathscr{S}_i}{\partial \vec{v}}\right), \tag{6}$$

Besides, the divergence with respect to velocity that appears on the right hand side of Eq. (6) can be expanded as follows:

$$\frac{\partial}{\partial \vec{v}}\cdot\left(\frac{\partial \mathscr{S}_i}{\partial \vec{v}}f_\alpha\right) \simeq \frac{\partial \mathscr{S}_i}{\partial \vec{v}}\cdot\frac{\partial f_\alpha}{\partial \vec{v}} + f_\alpha\Delta_v\mathscr{S}_i.$$

Using the approximation $f_i(\vec{v}) = n_i\delta^3(\vec{v})$, which is valid for suprathermal α-particles, the first Rosenbluth potential associated to the target ions i can be calculated explicitly: $\mathscr{S}_i(v) \sim -n_i/(4\pi v)$. Then, by calculating its derivative, the slowing down of α-particles can be modeled by:

$$\left.\frac{\partial f_\alpha}{\partial t}\right|_{\text{coll}} = \sum_i 4\pi\Gamma_{\alpha i}\frac{m_\alpha}{m_i}\left(\frac{\partial f_\alpha}{\partial \vec{v}}\cdot\frac{n_i}{4\pi v^2}\vec{e}_v + f_\alpha f_i\right). \tag{7}$$

The first term $\sim \partial f_\alpha/\partial\vec{v}$ corresponds to a conservative convection towards $v = 0$. It varies slowly and smoothly far from the thermal velocity region and can be characterized by a suprathermal velocity scale v_α^{ST}, which is greater than the typical thermal ion velocity $v_i^{th} = (T_i/m_i)^{1/2}$. The second term $\sim f_\alpha f_i$ appears highly localized in the thermal region of velocity space and behaves qualitatively as a δ-function for suprathermal α-particles. It thus seems natural to write the α distribution function as follows:

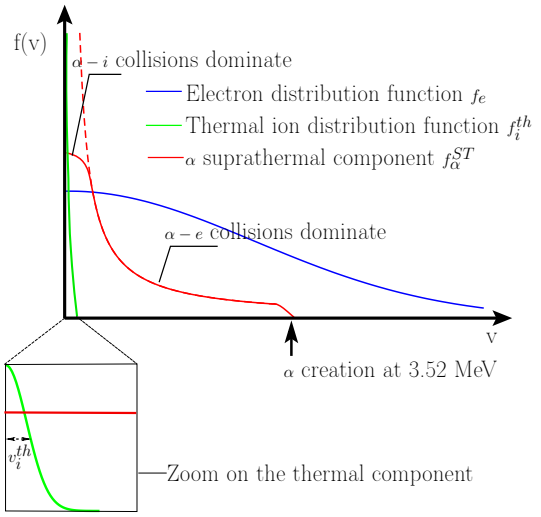

Figure 1: *The two-component α distribution function*

$$f_\alpha(\vec{v},t) = f_\alpha^{ST}(\vec{v},t) + f_\alpha^{T}(\vec{v},t), \tag{8}$$

where: f_α^{ST} denotes the suprathermal component. It is defined on a large velocity domain, spreading to the MeV range; f_α^T is the thermal component. It is localized in the region of velocity space corresponding to target thermal ion distribution functions and vanishes in the suprathermal velocity domain. The original Fokker-Planck operator given in Eq. (2) is then transformed

into a *system of two coupled equations* governing the two components f_α^{ST} and f_α^{T}, respectively:

$$\partial_t f_\alpha^{ST}\Big|_{\alpha i} = \Gamma_{\alpha i}\frac{n_i}{v^2}\partial_v f_\alpha^{ST} - n_i\Gamma_{\alpha i} f_\alpha^{ST}\frac{\delta(v)}{v^2},$$
$$\partial_t f_\alpha^{T}\Big|_{\alpha i} = 4\pi\Gamma_{\alpha i}\partial_{\vec{v}}\cdot\left(f_\alpha^{T}\partial_{\vec{v}}\mathscr{S}_i\right) + 4\pi\Gamma_{\alpha i} f_i f_\alpha^{ST}(v=0). \quad (9)$$

This re-arrangement of the terms of the Fokker Planck operator enables us to design tractable numerical methods treating the coupling between the two energy scales efficiently. Using our two-scale approach, it becomes feasible to simulate the implosion, ignition and combustion phases of real ICF target configurations at the ion-kinetic level [2].

Application on the ignition and thermonuclear burn of typical ICF capsules

We present the most striking differences between the classic fluid multi-group diffusion approach and our kinetic method, especially during the propagation of the burn front through the dense fuel shell. Comparing the birth of the burn front (Fig. 2-top panel) reveals that the α-particles tend to be more localized in the hot spot in the fluid (FCI1) calculation, where a multi-group diffusion model is applied. Indeed, the diffusion approximation does not hold for suprathermal α-particles in the hot spot, where the α mean free path is comparable with the hot spot radius. The diffusion model tends to artificially localize the energy deposition zone inside the hot spot. Conversely, the kinetic modeling tends to enhance the transport of α-particles *out of the central hot spot*, towards the inner surface of the dense fuel shell where they eventually transfer their energy to ions and electrons. This leads to a *pre-heating wave* (see Fig. 2-bottom panel) that tends to build up inside the dense fuel shell ahead of the main ion temperature front. This kinetic structure (not observed in the fluid model) is related to the Bragg peak of the D,T ions located in the dense cold fuel and may have significant consequences on the ignition conditions and the energy gain.

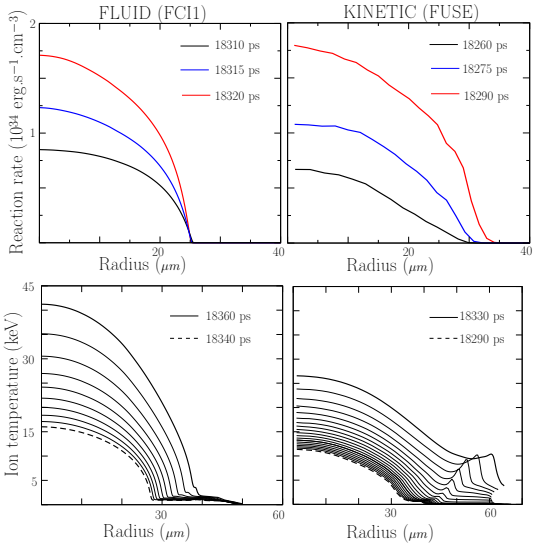

Figure 2: *Comparison of fluid and kinetic modeling during combustion.*

References

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