

Kinetic modelling of runaway electron avalanches in tokamak plasmas

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Introduction

Intense beams of highly energetic runaway electrons are often generated during tokamak disruptions when the toroidal electric field exceeds a density dependent critical field (E_c). Reaching energies of tens of MeVs, they may damage the first wall and therefore pose a serious threat for reactor-type experiments. In ITER, disruptions are expected to generate runaway electrons [1], mainly via knock-on collisions where enough momentum can be transferred from the relativistic to the slow electrons to transport the latter beyond a critical momentum p_c , setting off an avalanche of runaway electrons. The formation of runaway electrons is studied with the 3D Fokker Planck (FP) code LUKE [2], a well established code for calculating the electron distribution function. It reproduces the primary (Dreicer) growth rate of existing theory well [3], including the effect of effective ion charge. In order to describe the full generation of the runaway electron population, knock on collisions have been implemented in LUKE.

Runaway avalanche due to knock-on collisions

The description of runaway avalanches is beyond the FP approximation, which is only valid for weak deflections. Instead, the knock-on mechanism is described via a source term [4]:

$$S(p, \psi, \xi, \theta) = n_e n_r c \frac{d\sigma}{d\Omega} = n_e n_r \frac{1}{4\pi\tau \ln \Lambda} \frac{1}{p^2} \frac{d}{dp} \left(\frac{1}{1 - \sqrt{1 + p^2}} \right) \delta(\xi - \xi^*(p)) \quad (1)$$

where p is the momentum normalized to mc and ξ^* is the cosine of the pitch angle of the knocked-out electron. In (1), n_e is the bulk electron density, n_r is the runaway electron density, τ is the collision time for relativistic electrons, and $\ln \Lambda$ is the Coulomb logarithm. An analytic estimate of the avalanche growth rate is obtained by integrating the avalanche operator over the runaway region $p > p_c$ in momentum space. This approach may overestimate the actual avalanche rate, since it neglects the time it takes for an electron entering the runaway region to gain sufficient energy for contributing to secondary generation. The growth rate normalized to bulk density (n_e) is:

$$\frac{1}{n_r} \frac{\partial n_r}{\partial t} = \frac{1}{2\tau \ln \Lambda} (E/E_c - 1) \quad (2)$$

The runaway generation through knock on collisions in LUKE is benchmarked against the growth rate in Eq. 2 in the case of cylindrical geometry (Fig. 1(a)). The Rosenbluth model

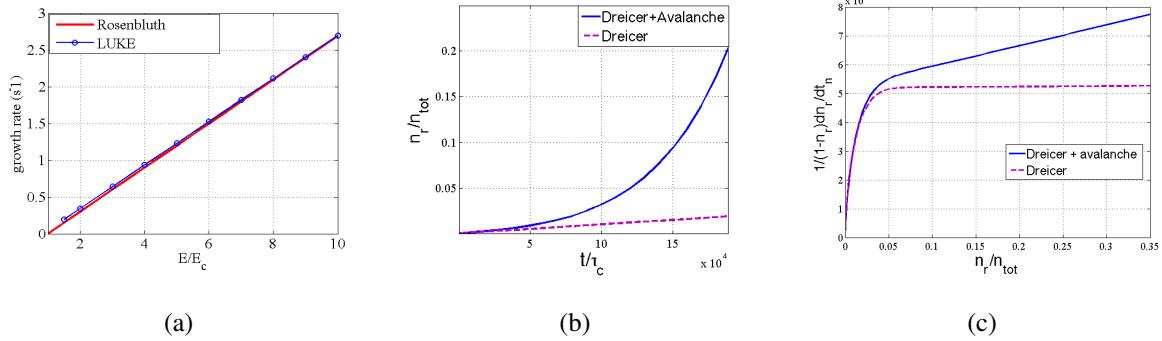


Figure 1: (a) Avalanche mechanism implemented and benchmarked in LUKE. (b) Runaway electron population grows exponentially when avalanche dominates. (c) Graphical representation of the combined Dreicer and avalanche growth rate.

(1) assumes that existing runaway electrons have an infinite momentum, but in practice a limit $p_{max} > p_c$ must be defined such that the growth rate is evaluated as the flux of electrons through p_{max} . We set $p_{max} = \max(p_c; 2\sqrt{2})$ to account for primary runaway electrons above 1 MeV. With $v/c \geq 0.94$, this criterion satisfies the assumptions in Ref. [4] and it is justified by the weak dependence upon the incident electron energy in the energy range 1-100 MeV [5].

To ensure a conservative form of the knock on mechanism, a sink term taking out the electrons transported from bulk to runaway momentum region is added along with the source term. The particle conserving knock on operator has the form $S = S_+ - \langle S_+ \rangle \frac{f_M}{\langle f_M \rangle}$, where f_M is the bulk distribution, assumed to be Maxwellian and $\langle \dots \rangle = \int \dots d^3p$. The growth rate consists of contributions from the two mechanisms Dreicer (D) and avalanche (A). The avalanche growth rate is proportional to the RE density (n_r) and can be quantified through an avalanche multiplication factor $\bar{\gamma}_A = \gamma_A/n_r$, so that $\frac{1}{(1-n_r)} \frac{\partial n_r}{\partial t} = \gamma_D + n_r \bar{\gamma}_A$, where n_r is normalised to the total electron density. Thus, the avalanche multiplication factor can be evaluated numerically from the slope of the curve in Fig. 1(c).

Influence of toroidicity

Since knock-on accelerated electrons emerge with high perpendicular momentum [4], it is necessary to properly account for guiding-center dynamics in non-uniform magnetic field geometry. The bounce averaged knock on operator derived and implemented in the LUKE code takes the form:

$$\begin{aligned} \{\bar{\mathbf{S}}\}(\psi, p, \xi_0) &= \frac{1}{\lambda \tilde{q}} \left[\frac{1}{2} \sum_{\sigma} \right]_T \int_{\theta_{min}}^{\theta_{max}} \frac{d\theta}{2\pi} \frac{1}{|\hat{\psi} \cdot \hat{r}|} \frac{r}{R_p} \frac{B}{B_p} \frac{\xi_0}{\xi} \bar{\mathbf{S}}(\psi, p, \xi) = \\ &= \frac{1}{2\pi^2} \frac{1}{\ln \Lambda^{\dagger} R_p} \bar{n}_e \bar{n}_r \frac{1}{\bar{p}^3 \gamma(\gamma-1)} \frac{B_0}{\lambda \tilde{q}} \frac{|\xi_0|}{(1-\xi_0^2)^2} \sum_{k=1,2} \left[\frac{1}{|\hat{\psi} \cdot \hat{r}|} \frac{r}{B_p} \frac{1}{|\Psi'|} \right]_{\theta_k^*} \end{aligned}$$

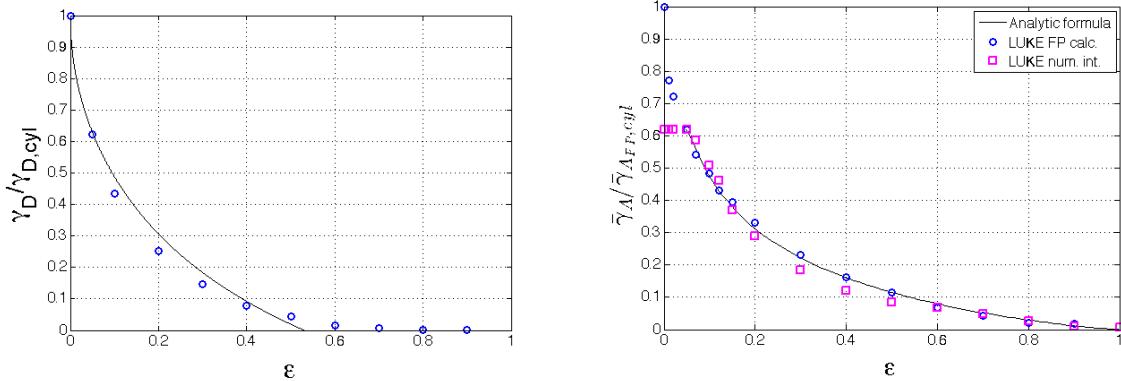


Figure 2: Radial dependence of Dreicer growth rate, normalized to the growth rate for cylindrical case ($\epsilon = 0$), compared to a fit $(1 - 1.2\sqrt{2\epsilon/(1+\epsilon)})$ (left) and multiplication factor for avalanches compared to analytic expression (right) ($E/E_c = 5$).

Conservation of momentum shows that a significant part of the knocked-on electrons can be magnetically trapped when born off the magnetic axis. Indeed, both Dreicer and avalanche runaway electron generation are strongly affected by a non uniform magnetic configuration, when the presence of magnetic trapping is accounted for (Fig. 2).

As an analytic estimate, we assume that all electrons with momentum $p > p_c$ will run away, except the electrons in the trapped region $p < p_T$. The latter criterion is obtained from the usual condition on magnetically trapped electrons $\xi_0(\psi) \leq \xi_{0T} = \sqrt{1 - 1/\Psi_{max}}$, where $\xi_{0T}(\psi)$ is the pitch angle, defined at the minimum $B_0(\psi)$ on a given flux surface, such that the parallel velocity of the particle vanishes at the maximum $B_{max}(\psi)$. An electron will run away if its momentum exceeds both the critical momentum and the trapping condition. Thus the lower integration limit p_{min} is given by $\max(p_c, p_T)$. From this condition an analytic expression of the growth rate including magnetic trapping is derived and compared to FP simulations in Fig. 2(b). For $\bar{E} \gg 1$, the growth rate is reduced by a factor $\frac{2}{\pi\sqrt{\epsilon\bar{E}}}(1-\epsilon)^{3/2}$. The overall runaway rate is further reduced by the effect of magnetic trapping on Dreicer generation.

Importance of avalanche effect

We quantify the importance of avalanche generation as a function of plasma temperature and electric field strength by letting a small fraction of the electrons run away in LUKE, and then evaluate the part that originate from knock on collisions. Fig. 3 shows the fraction of runaway electrons born due to knock on collisions, when 1% of the initial electron population has run away. The importance of secondary generation increases for lower temperature and electric field, as the slower primary generation rate allows for the avalanche mechanism to take off. Data from a typical DIII-D disruption [6], with the central electric field reconstructed with the

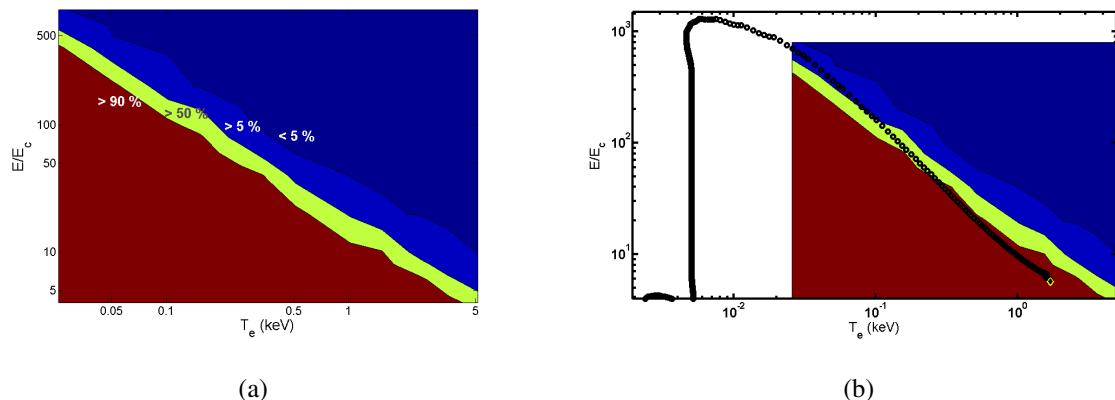


Figure 3: (a) The significance of the avalanche effect (n_A/n_r) (b) related to the electric field strength and plasma temperature in a DIII-D disruption. The start of the thermal quench is marked with a diamond.

GO code [7], related to the simulations suggests that knock on collisions play a role in typical disruptive scenarios. These results indicate that in scenarios with large temperature drops, avalanche generation may be a crucial ingredient in modelling of tokamak plasma disruptions.

Conclusion

A bounce-averaged knock-on collision operator from the Rosenbluth model is implemented in the kinetic equation solved by the Fokker-Planck code LUKE, which now describes the runaway electron generation through the combined effect of Dreicer and avalanche mechanisms. The growth rate is benchmarked in the cylindrical limit. An analytical expression for avalanche growth rate accounting for magnetic trapping due to a non uniform magnetic configuration has been derived. It is in good agreement with numerical results and shows that a significant proportion of secondary electrons could be knocked into the trapped region. In addition, LUKE simulations reveal that knock-on generation dominates Dreicer generation at lower values of the temperature and electric field, and likely play a significant role in tokamak disruptions.

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