Weak oblique whistler / kinetic Alfvén wave turbulence

S. Galtier, R. Meyrand

Laboratoire de Physique des Plasmas, Ecole Polytechnique, F-91128 Palaiseau Cedex, France

Fluctuations in space plasmas exhibit a multitude of time and length scales such as the ion or electron cyclotron frequencies, inertial lengths and Larmor radii. In the solar wind *in situ* measurements reveal extended power law spectra for the turbulent velocity, magnetic and density fluctuations. For example, the large scale ($f < 1$Hz) magnetic spectrum is characterized by a narrow range of power law indices with a peak near $-5/3$ and is interpreted as an MHD turbulence cascade. But at sub-ion scales ($f > 1$Hz) the physical properties change drastically: after a stiff transition, a f^α spectrum is clearly observed such that,

$$-3.1 \leq \alpha \leq -2.5\,, \tag{1}$$

with a distribution peaked around -2.8 [1], whereas the typical solar wind plasma parameters are $\beta_i \sim \beta_e \sim 1$, $T_i/T_e \sim 1$, $d_i \sim 100\,$km (ion inertial length) and $f_{ci} \sim 0.1\,$Hz (ion cyclotron frequency). Both kinetic and fluid models are used to investigate plasma turbulence at sub-ion scales [2, 3]. For the solar wind, we often evoke kinetic Alfvén waves (KAW) and whistler waves as the main ingredients of the turbulence cascade. The main difference between KAW and whistlers is in the dynamics of ions which rapidly adjust to the fluctuating electric potential in the former case, whereas they are dynamically irrelevant in the latter.

By definition, classical fluid models (like Hall MHD or electron MHD) are not able to catch kinetic effects and their use at sub-ion scales is mainly relevant for the investigation of the turbulent dynamics. In the solar wind case it is believed that the origin of the power law spectra for $f > 1$Hz could be attributed mainly to turbulence which would imply that kinetic effects are irrelevant to understand the statistical properties of the magnetic fluctuations [4]. Hall MHD is a useful plasma model for length scales smaller than d_i ($d_i \equiv c/\omega_{pi}$ with c the speed of light and ω_{pi} the ion plasma frequency) and time-scales of the order, or shorter than, the ion cyclotron period ω_{ci}^{-1}. The limit of validity of Hall MHD may be discussed at the level of the dispersion relation. As shown by [5], the Hall MHD dispersion relation is a rigorous limit of Vlasov-Maxwell kinetic theory only in the limit of cold ions, *i.e.* $T_i = 0$. Under this limit, the ion Landau resonance becomes negligible which explains why physically the fluid model may be relevant. However, two important comments have to made here. First, this limitation was discussed quantitatively by [6] from the numerical resolution of the dispersion relations. It was found that even at $T_i = T_e$ (with $\beta = 1$) the parallel whistler, oblique whistler and kinetic Alfvén waves are well described by Hall MHD whereas for example the slow mode represents an unphysical/spurious wave that does not exist in a weakly collisional plasma. Second, the simple analytical demonstration made on the dispersion relation does not say anything about the validity of Hall MHD in the full turbulent regime for which the statistical contribution of kinetic effects is still not well documented. If this contribution is negligible [4] or if the three previous waves are not affected by the spurious waves, then Hall MHD may be a relevant model for plasmas even when $T_i \sim T_e$ (and $\beta \sim 1$). An example is provided by incompressible ($\beta \gg 1$) Hall MHD in the regime of wave turbulence: as shown by [7], it is possible to get spectral predictions only for the right circularly polarized wave (so without the feedback of the left circularly polarized wave, *i.e.* the slow wave) which mainly describes oblique whistler waves. Also, it is though that as long as a fluid model like Hall MHD is able to predict statistical properties compatible with the observations the pure turbulent cascade scenario has to be considered as a central mechanism to transfer energy scale by scale until the electron scales.

Three-dimensional (3D) Hall MHD turbulence is much more difficult to investigate numerically than pure MHD because the Hall effect brings a new kind of nonlinear term with a second-order derivative. Because of this difficulty, it is interesting to investigate first the incompressible limit for which the Hall effect is asymptotically large, *i.e.* the so-called electron MHD regime. In this limit, the ions can be considered as a motionless neutralizing background such that the electron flow determines entirely the electric current. Since the ions are static, electron compressibility corresponds to a violation of quasineutrality and reciprocally, therefore, electron MHD is only valid for large enough β_e. Direct numerical simulations of isotropic electron MHD show that the turbulent magnetic energy spectrum scales like $k^{-7/3}$ [8]. It is widely believed that Hall MHD should exhibit the same (magnetic) spectrum as electron MHD because the latter is simply the $kd_i \gg 1$ limit of the former. However, in the framework of 3D incompressible Hall MHD, a recent study has revealed the influence of the left polarity on the magnetic spectra with the possibility to get different power laws at different scales [9].

In addition to the large amount of research with fluid models, kinetic models are also widely used. A kinetic theory of plasma turbulence is, however, extremely difficult to reach because of the conceptual difficulty to manage *e.g.* with the multidimensional phase space and the multitude of phenomena that are included. For these reasons, simplifications are generally made in order to catch the most interesting part of the nonlinear dynamics. For example, most of the gyrokinetic theory/simulations [2] assumes that the distribution is close to a Maxwellian which is a rather strong assumption for space plasmas. Additionally, it is also assumed that the turbulent magnetic fluctuations are relatively small compared to the mean magnetic field, spatially anisotropic with respect to it and that their frequency is low compared to the ion cyclotron frequency. Under these hypotheses, it is possible to make numerical simulations and to follow *e.g.* the nonlinear dynamics driven by the KAW [10]. The Reynolds number – or in other words the size of the inertial range – is however still significantly limited compared to pure fluid simulations. Interestingly, the KAW cascade may also be described by a simplified system called reduced electron MHD (REMHD) valid for any temperature ratio T_i/T_e and β_i. The form of the REMHD equations is close to the (incompressible) electron MHD equations and in the strongly anisotropic limit ($k_\perp \gg k_\parallel$) they become even mathematically similar which means that the nonlinear dynamics and solutions of oblique whistler and KAW are the same (for any beta and any T_i/T_e) [11]. From this remark, we can conclude that the weak turbulence predictions [7, 12] are the same for both oblique whistlers and KAW.

The role of the magnetic helicity – the scalar product of the magnetic field with the magnetic vector potential – on the oblique whistler / KAW cascade has received only few attention until now. For example, 3D direct numerical simulations of electron MHD with a mean magnetic field revealed that the propagation of one wave packet moving in one direction leads to energy transfer towards larger scales [13]. This effect interpreted as an inverse cascade shows that one dispersive wave packet may produce another wave packet moving in the opposite direction whereas the magnetic helicity is well conserved. It is this conservation which is thought to be at the origin of the inverse cascade. Recently, the weak turbulence regime for KAW/oblique whistler has been reinvestigated in order to see the effect of the magnetic helicity. More precisely, we have derived the exact solutions at constant helicity flux [11]. Surprisingly, a family of exact solutions has been found which implies the entanglement of magnetic helicity and energy in the sense that the power law indices of the corresponding spectra are linked through the simple relation:

$$n + \tilde{n} = -6\,, \tag{2}$$

where n and $\tilde{n}$ are the power law indices of the magnetic energy $E(k_\perp)$ and helicity $H(k_\perp)$ spectra respectively (with $k_\perp$ the perpendicular wavevector; the cascade along the parallel $\mathbf{b_0}$ direction is neglected in the present discussion). We can even expect that $n \to -3$ for an asymptotically strong helicity flux which means that in principle the weak turbulence regime can explain the following values for the observed magnetic fluctuation spectra:

$$-3 < \alpha < -2\,. \tag{3}$$

In conclusion, if we assume that the kinetic effects have a negligible contribution on the statistics of the magnetic fluctuations, then the scaling laws may be seen as the signature of a pure turbulence cascade. The weak turbulence results [11] suggest that the wide range of values observed in the solar wind for the magnetic spectrum power law indices can find its origin in the magnetic helicity and its inverse cascade. Since signatures of a non-zero reduced magnetic helicity have been reported in the solar wind at sub-ion scales [14], it would be interesting to check if a negative magnetic helicity flux is also present. Our recent results are also interesting because they show – for the first time – that a theory is able to predict rigorously steep power laws for the magnetic fluctuation spectrum at sub-ion scale: indeed, previous theories based on the energy cascade of oblique whistler / KAW were mostly able to propose an index of $-7/3$ for strong turbulence or -2.5 for weak turbulence. Note that recently a spectrum close to $-8/3$ has been found numerically by using REMHD and anisotropic electron MHD [15, 3], and explained differently by invoking the dimensions of the dissipative structures (sheets and filaments respectively). Although the origin of this difference is unclear (since basically the equations simulated are the same) we think that the strength of the external magnetic field plays certainly an important role, *e.g.* in destabilizing the current sheets as it is also observed with PIC simulations [16].

References

[1] F. Sahraoui et al., Astrophys. J. **777**, 15 (2013)

[2] A.A. Schekochihin et al., Astrophys. J. Suppl. **182**, 310 (2009)

[3] R. Meyrand and S. Galtier, Phys. Rev. Lett. **111**, 264501 (2013)

[4] W.H. Matthaeus et al., ArXiv:1404.6569 (2014)

[5] A. Hirose, A. Ito, S.M. Mahajan and S. Ohsaki, Phys. Lett. A **330**, 474 (2004)

[6] G.G. Howes, Nonlin. Processes Geophys. **16**, 219 (2006)

[7] S. Galtier, J. Plasma Physics **72**, 721 (2006)

[8] D. Biskamp, E. Schwarz and J.F. Drake, Phys. Rev. Lett. **76**, 1264 (1996)

[9] R. Meyrand and S. Galtier, Phys. Rev. Lett. **109**, 194501 (2012)

[10] G.G. Howes et al., J. Geophys. Res. **113**, 5103 (2008)

[11] S. Galtier and R. Meyrand, arXiv:1406.4638v1 (2014)

[12] S. Galtier and A. Bhattacharjee, Phys. Plasmas **10**, 3065 (2003)

[13] J. Cho, Phys. Rev. Lett. **106**, 191104 (2011)

[14] G.G. Howes and E. Quataert, Astrophys. J. Lett. **709**, L49 (2010)

[15] S. Boldyrev and J.C. Perez, Astrophys. J. Lett. **758**, L44 (2012)

[16] H. Karimabadi et al., Phys. Plasmas **20**, 012303 (2013)