Alfvénic Instabilities Excited by Runaways

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Introduction

It is well known that runaway electron (RE) populations form in both space and laboratory plasmas, in the presence of a sufficiently strong electric field. An important example is the tokamak disruption [1] - a rapid thermal quench induces an electric field to maintain the plasma current against the increased resistivity of the cold plasma, $T \sim 2-10$eV. A highly relativistic RE beam can be generated, which may damage plasma facing components when it terminates, due to the localised energy deposition. The risk increases with plasma current, therefore the understanding and elimination of RE beam formation is important for future tokamak power plants. Several tokamaks have reported a threshold in toroidal magnetic field of ~ 2T for RE beam generation [2]. This threshold has been linked to a decrease in magnetic fluctuation levels, which appear to be effective in limiting the RE beam formation. TEXTOR has recently shown clearly the appearance of low toroidal mode number, $n \approx 1$, fluctuations in the frequency range $f \approx 60-260$kHz during disruptions of Ohmic discharges triggered by argon injection [2].

Alfvén wave destabilisation by particle resonance is commonly observed in both natural and laboratory plasmas. The free energy to excite low frequency waves originates from spatial inhomogeneity or an inverted energy distribution. The particle velocity must be a well defined fraction of the Alfvén velocity $v_A = B/\sqrt{\mu_0\rho_m}$ to allow resonance, where ρ_m is the mass density. This is increasingly demanding as the magnetic field B increases and we have considered here the potential for this effect to explain the experimentally observed magnetic field threshold to RE beam formation described above. The Toroidal Alfvén Eigenmode (TAE) is a typical Alfvénic instability in tokamaks [3]. It exists in the gap induced in the shear Alfvén continuum by toroidal shaping, thus is not subject to strong continuum damping. TAEs can have frequencies and mode numbers in the same range as the experimental observations reported in [2] and their excitation by a variety of energetic ion populations has been well studied.

Runaway ions can be generated by the same large electric field that accelerates the runaway electrons [5]. This requires that the frictional drag due to the drifting electrons does not cancel the electric force, which is possible in the presence of magnetic trapping or impurities of different charge to the ions. The latter is particularly relevant to disruption mitigation studies by massive gas injection (MGI). Neutrals are not typically expected to penetrate the RE beam, so only the effect of Coulomb collisions in determining the runaway ion distribution is included here. The ions experience a non-monotonic friction force - drag against ions dominates at low energy, decreasing to a minimum with increasing velocity, then increasing as drag against electrons takes over. When the electric field is large enough, ions from the tail of the thermal distribution will be accelerated. We have, therefore, considered here the TAE growth rate due to both runaway ions and electrons.

TAE growth rate

At large aspect ratio, the TAE is dominated by two toroidally coupled poloidal harmonics m and $m+1$, localised about the minor radius $r = r_0$ with safety factor $q_0 = (2m+1)/2n$ and frequency $\omega = v_A/2q_0R_0$, where R_0 is the major radius. The coupled harmonics allow resonant interaction for $|v_\parallel| \approx v_A/3$ and $|v_\parallel| \approx v_A$, so both runaway ions and electrons, which have oppositely directed velocities, may drive the mode. In the pre-disruption plasma, the thermal velocities satisfy $v_{Ti} \ll v_A \ll v_{Te}$. Two post-disruption regimes may be of particular interest - the increased density during MGI is expected to facilitate the interaction of runaway ions with the $v_A/3$ resonance, whilst the rapid temperature decrease in a spontaneous disruption may allow runaway electron resonance at v_A. The linear TAE growth rate due to a resonance with a low collisionality population, of distribution f and charge $q_i = Z_ie$, is given in [4]

$$\frac{\gamma_i}{\omega} = \frac{2\pi^2\mu_0 m_i^2 q_0^3 R_0}{B_0^2}\int_0^\infty dv_\perp v_\perp \sum_{v_r=v_A,v_A/3}\frac{v_r}{v_A}\left(v_r^2+\frac{v_\perp^2}{2}\right)^2\left(\omega\frac{\partial}{\partial\mathscr{E}}-\frac{n}{q_i}\frac{\partial}{\partial\psi}\right)f\Big|_{|v_\parallel|=v_r}, \quad (1)$$

where $\mathscr{E} = m_iv^2/2$, ψ is the poloidal flux and at large aspect ratio, $d\psi \simeq RB_\theta dr$.

The solution of the time-dependent ion kinetic equation accounting for acceleration by a parallel electric field was outlined in [5], in the limit of trace impurities. We have generalised this to allow for arbitrary impurity content, relevant to MGI scenarios, neglecting trapping effects at the low temperatures of interest. The solution follows from an expansion in $\delta = E_*T_i/E_DT_e \ll 1$, where E_D is the Dreicer field, and the effective electric field $E_* = E_\parallel + R_{ie\parallel}/Z_ien_i$ is the net electric field accounting for the effect of electron drag. The distribution is seen to develop a high energy tail, which peaks in the forward direction, as $\delta \ll 1$, around a pitch angle $\xi = 1$

$$f_{RI}(w,\xi,\tau) \propto \exp\left[-\frac{w^2}{2\delta}+\frac{w^4-(w^3-3\bar{n}\tau)^{4/3}H(w^3-3\bar{n}\tau)}{4\delta\bar{n}}+2w^2\sqrt{\frac{2(1+\xi)}{\delta Z_{\text{eff}}}}\right]. \quad (2)$$

Here H is the Heaviside step function, the effective charge $Z_{\text{eff}} = \sum_i n_iZ_i/n_e$, the independent variables $\tau = 3\delta^{3/2}(\pi/2)^{1/2}(n_e/n_iZ_i^2\tau_{ii})t$, $w = v(\delta m_i/T_i)^{1/2}$, and the ion self-collision time $\tau_{ii} = 3(2\pi)^{3/2}\varepsilon_0^2\sqrt{m_i}T_i^{3/2}/n_iZ_i^4e^4\ln\Lambda$, where $\ln\Lambda$ is the Coulomb logarithm. The parameter $\bar{n} = \left(n_iZ_i^2/n_e\right)\left[1+\sum_{z\neq i}\left(n_zZ_z^2m_i/n_iZ_i^2m_z\right)\right]$. In the trace impurity limit $\bar{n} = Z_i$ - for the cold plasmas of interest here $Z_zm_i/m_z < 1$, so $\bar{n}$ is always less than one if the main ions are hydrogenic.

The solution Eq. (2) is valid for small w or short times $\tau \ll 1$, but only holds for $w \leq 1$ when $\tau \geq 1$. By $\tau \sim 1$, the runaway population reaches only a few percent for $\delta \ll 1$, so we neglect the time dependence of the normalization constant and take $N(\tau) \simeq N(\tau = 0) = n_i/(\sqrt{\pi} v_{Ti})^3$. The evolution of the ion distribution is illustrated in Fig. 1. For typical experimental parameters $\tau \sim 1$ is a fraction of a microsecond, so the time to establish the runaway population is short. Note that we consider only the initial phase of the wave-particle interaction and the potential linear instability drive. If Alfvénic instabilities are excited as the runaway ion population builds up, the analysis leading to the form of f_{RI} above will break down.

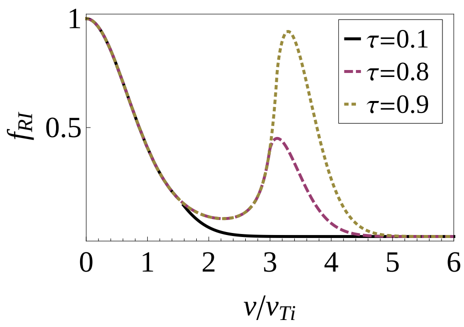

Figure 1: *Evolution of the runaway ion distribution Eq. (2) at $\xi = 1$, in the trace impurity limit, for $\delta = 0.1$ and $T = 10$ eV.*

The post-disruption parameters are not typically well known. During the disruption, impurities mix into the core and their charge states will vary rapidly with time and space. At the final, low temperature, it may be expected that the species' temperatures will equilibrate and the impurities will not be fully ionised. Taking typical values for massive argon injection into a deuterium plasma with an initial carbon impurity, $n_{Ar} = 0.1 n_D = 2 \times 10^{18}\text{m}^{-3}$, $n_C = 0.08 n_D$, $B = 2$T and $T_e = 10$eV, the condition $v_{\|} = v_A/3$ requires ions with velocity $v_{RI} \approx 0.65 v_{Te}$. At low $m \sim 1$, with $q_0 = 1.5$ and $R_0 = 1.75$m, ions accelerated to these velocities would drive a TAE with frequency 112kHz, which is comparable to the observed frequency range in [2]. At low temperatures the approximate analytical distribution Eq. (2) breaks down before the ions can be sufficiently accelerated and the inverted energy distribution appear at the resonance. Thus, a numerical solution of the ion kinetic equation is required to calculate the growth rate. However, an illustrative evaluation may be given at $T = 500$eV, which may be relevant during the cooling process. The energy gradient $\partial f_{RI}/\partial \mathscr{E} = (2\delta/m_i v_{Ti}^2 w)(\partial f_{RI}/\partial w)$, and taking $\xi \approx 1$ at the resonance, only the term in v_r^5 contributes in Eq. (1). At $\tau = 2.3$, for $Z_{Ar} = 2$, $Z_C = 6$, the runaway density is 2.3% of the initial bulk ion density and the normalized drive at the $v_A/3$ resonance is 3.8%. When the radial profile of runaway ions is peaked on axis, the term proportional to $\partial f/\partial \psi$ will give an additional positive contribution to the growth rate.

The form of the runaway electron distribution is well known. It does not typically have an inverted energy gradient and the anisotropy in pitch-angle known to drive instability will not be effective at the low mode frequency of interest here. However, steep density profiles can appear at the radially localised current sheets which form as the result of a thermal instability [6].

When the post-disruption plasma is sufficiently cold, the primary resonance condition $v_{\parallel} = v_A$ may move out to velocities greater than the electron thermal speed, allowing energy transfer to the mode from the well populated lower energy region of the runaway electron distribution. Using the electron distribution function resulting from secondary generation given in [7], the TAE growth rate becomes

$$\frac{\gamma_e}{\omega}[\%] = \frac{v_A}{|\omega_{ce}|}\frac{\pi n q_0^4 v_A}{\varepsilon_0 c_z c}\frac{m_e}{m_i}e^{-\frac{v_A}{c c_Z}}\left(1+\frac{2c}{\alpha v_A}+\frac{2c^2}{\alpha^2 v_A^2}\right)\frac{n_{re,17}}{n_{0,19}}\frac{1}{L_n^s}, \tag{3}$$

where $\alpha = (\hat{E}-1)/(1+Z_{\text{eff}})$, $\hat{E} = |E_{\parallel}|m_e c^2/E_D T_e$, $c_z = \sqrt{3(Z_{\text{eff}}+5)/\pi}\ln\Lambda$ and $1/L_n^s = \partial_r n_{re}/n_{re}$. This growth rate can become significant at radial locations with large safety factors q_0 and short spatial gradient scale lengths.

Discussion

The potential for runaway particles produced during a tokamak disruption to resonantly drive TAEs is seen to be favoured at low magnetic field. For ion drive higher density is also favoured; for electron drive, lower temperatures. The self-consistent loss of REs due to the magnetic fluctuations associated with the mode presents a possible interpretation of the experimentally observed magnetic field threshold to RE beam generation. However, the growth rate will be countered by various damping mechanisms, which will depend on the scenario. Whilst the low temperature bulk ions and electrons are unlikely to cause strong resonant damping, the RE energy gradient is expected to damp the mode. The continuum damping will evolve with the parameter profiles during the disruption and requires simulation of the detailed TAE structure. The damping calculations are beyond the scope of this work. Finally, we note that excitation and observation of known Alfvénic modes, which are supported by the background plasma, offers a non-intrusive diagnostic for both bulk plasma and fast particle post-disruption properties.

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