Analysis of MHD signals for measuring systematic errors in the coil response

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**See the Appendix of F. Romanelli et al, Proc. of the 24th IAEA FEC, San Diego, 2012.*

Abstract

Magneto hydrodynamic (MHD) modes have a strong influence on the plasma behaviour. The measurement of their characteristics, i.e. amplitude, poloidal (m) and toroidal (n) numbers is essential for the early detection of potentially dangerous modes, and for the better characterization of plasma dynamics. Even though the phase characteristic of the coils can be measured before installation, an in-situ characterization is needed to include the effect of the various structural elements surrounding them and the long cables connecting them to the ADC. Six non uniformly toroidally distributed coils are used on JET to determine the n mode number [1]. Ideally, in presence of a single mode, the phase difference between the coils should be proportional to $e^{in\phi_k}$ where ϕ_k are the coil toroidal positions, while their amplitude should be the same. In practice, a systematic error is still present. A correction factor can be derived measuring the difference in phase and amplitude among the coil signals after having taken into account the factor $e^{in\phi_k}$. This correction factor mainly depends, as expected, on the mode frequency.

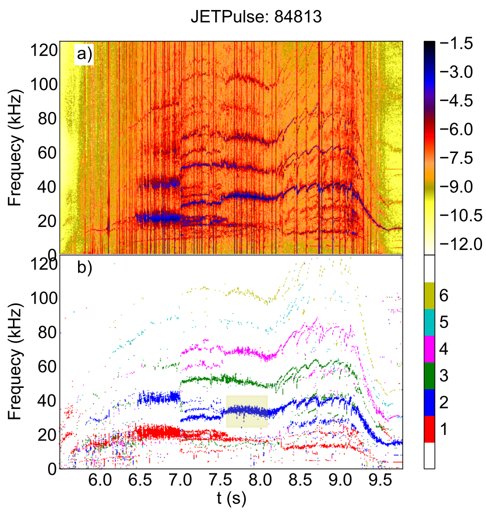

Figure 1. a) mode amplitudes for JET pulse: #84813; b) toroidal mode number. The mode (n=2) in the green box will be taken as example in the following figures.

Mode detection

The coils typically used on JET for the toroidal mode number reconstruction are: H302, at $\phi = 2.94^o$; H303, at $\phi = 13.11^o$; H304, at $\phi = 18.74^o$; H305, at $\phi = 20.38^o$; T002, at $\phi = 312.15^o$; T009, at $\phi = 200.37^o$. All coils have a radial position of 3.88 m but the nominal vertical position is between 1.00 m and 1.04 m with respect to the JET midplane.

These small differences can give a residual dependence on the poloidal mode number that in the following as a first approximation, we are going to ignore. The coil signals are first divided in short segments which are multiplied for a window function (Hanning) and then Fourier transformed. In the following the elaboration is carried on at a single frequency and a single time (the average time of the segment) so that a unique index "j" is used to identify the time and the frequency. In the elaboration we are assuming this model, where only one mode is present:

$$S_{j,k} = A_j R_k e^{in\phi_k} + \varepsilon_{j,k}$$

where j is a global index of frequency and time, A_j is the mode amplitude, n is the toroidal mode number, R_k is a systematic error in the coil response which we assume to be close to one, and $\varepsilon_{j,k}$ a normal random complex variable of zero mean.

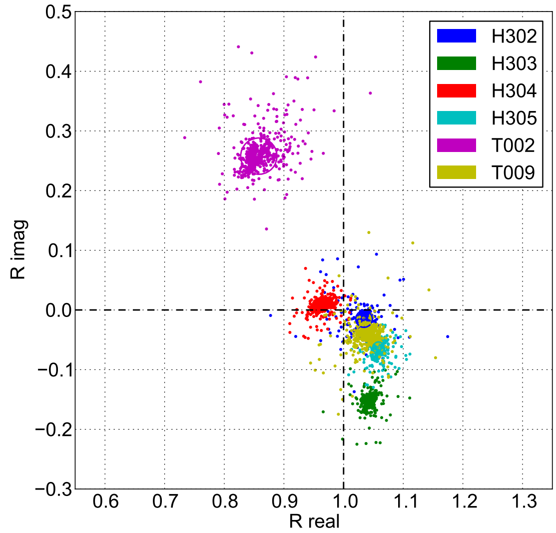

Figura 2. the correction factors (real and imaginary part) for the n=2 mode in the yellow box fig. 1.

We also assume that $N_{coil} = \sum_k R_k$, which is trivially satisfied when $R_k = 1$. The mode number is determined by multiplying the signal by a tentative factor $e^{-i\tilde{n}\phi_k}$, a rotation in the complex plane. In absence of systematic errors, when $\tilde{n} = n$, the rotation would take all coil signals (at a single frequency and time) close to a single point in the complex plane. We Define:

$$Z_{j,k} = S_{j,k} e^{-i\tilde{n}\phi_k}$$

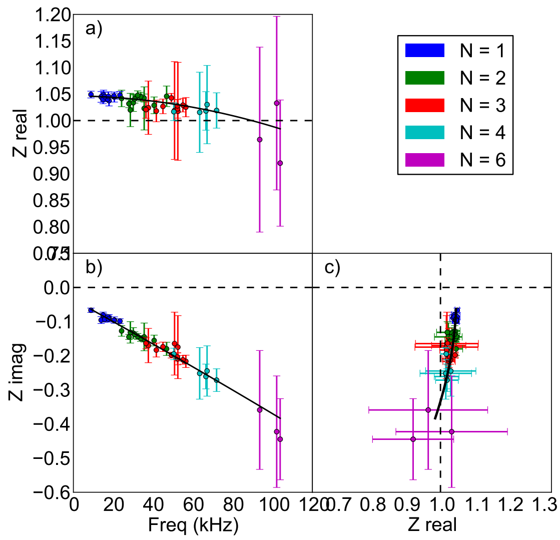

Figura 3. The correction factor for coil H303, the lines are the polynomial fitting curves: a) and b) real and imaginary part, while c is the imaginary part vs the real one.

And the average:

$$\bar{Z}_j = \frac{1}{N_{coil}} \sum_k S_{j,k} e^{-i\tilde{n}\phi_k} \cong \frac{A_j}{N_{coil}} \sum_k e^{i(n-\tilde{n})\phi_k}$$

The absolute value of $\bar{Z}_J$ will get its maximum when the tentative mode number corresponds to the actual mode number, that is $\tilde{n} = n$. That would also be the case if the systematic errors are close enough to one. In the following we are assuming that the correct toroidal mode number has been found so that $\tilde{n} = n$, being this the case we have $\bar{Z}_J = A_j + \bar{\varepsilon}$ where $\bar{\varepsilon}$ is the actual average of the noise, which is different from its expected value, which is zero. In the situation where the mode amplitude is larger than the noise, that is when $|A_j| \gg |\bar{\varepsilon}|$ it is possible to estimate the systematic errors:

$$\breve{R}_{j,k} = \frac{Z_{j,k}}{\bar{Z}_J} \cong R_k + \frac{\varepsilon}{A_j}$$

The error on the determination of the systematic errors scales as the inverse of the mode amplitude, an average value can be calculated:.

$$\bar{R}_k = \frac{\sum_{j\,selected} \breve{R}_{j,k} |\bar{Z}_j|^2}{\sum_{j\,selected} |\bar{Z}_J|^2}$$

The average is done on a selected subset of j inside a frequency and time window where the amplitude of the selected mode is higher than a given threshold (between 10% and 20% of the maximum mode amplitude).

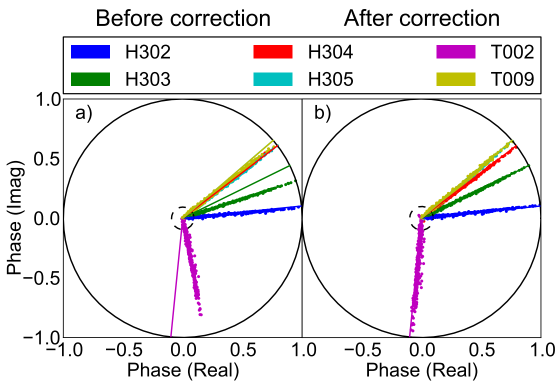

Figura 4. Signal from the different coils normalized for their expected phase for the mode (n=2) in the green box of fig 1. The first panel is without the correction factor, the second panel is with the correction factor. The coils H305 and T009 are one opposite to the other in the toroidal location ($\boldsymbol{\phi_{T009} - \phi_{H305} = 180}$), so that they are one on top of the other for an even mode.

A selection of 14 discharges has been chosen from the August to September 2013. In each discharge some clearly visible modes are selected. For each coil the average $\bar{R}_k$ together with its standard deviation and its frequency range are saved in a database. A weighted polynomial fit on the real and imaginary part of $\bar{R}_k$ is carried out as function of frequency. The fits are saved and can be applied back in the mode detection procedure. On fig. 4 it is shown the effects of the correction on the coil signals.

Preliminary statistical analysis

The detection of a mode is a classical problem of the falsification of the null hypothesis, one possibility is to use the complex t-student distribution, where the hypothesis to falsify is that no mode is present.

Given a random variable:

$$t = \frac{\bar{Z}_J}{\sqrt{\sum_k \frac{\left|Z_{j,k} - \bar{Z}_J\right|^2}{N_{coil}(N_{coil}-1)}}}$$

if $Z_{j,k}$ are independent identically distributed (i.i.d.) complex variables with zero mean then the random variable t follows the complex t-student distribution with $N_{coil} - 1$ degree of freedom [2]. The cumulative distribution function is:

$$F_t(|t|) = 1 - \frac{1}{\left(1 + \frac{|t|^2}{N_{coil} - 1}\right)^{N_{coil}-1}}$$

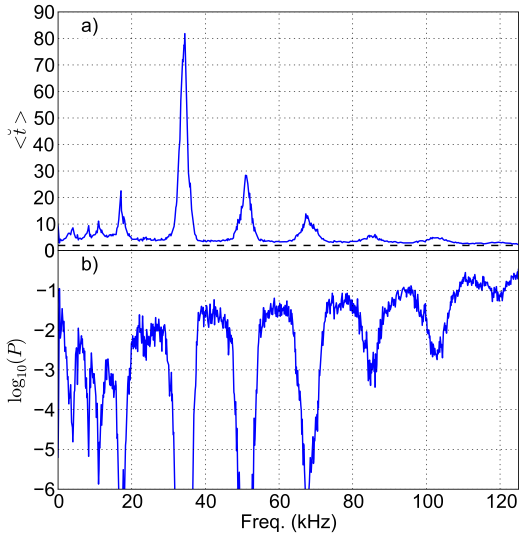

Figura 5. a) average value of $\check{\boldsymbol{t}}$, the dashed line correspond to the expected value. b) Complementary Cumulative Distribution Function of $\check{\boldsymbol{t}}$ according to the approximation of independent $\boldsymbol{t}(\tilde{\boldsymbol{n}})$.

Actually the random variable t depends on $\tilde{n}$ by $\bar{Z}_J$ which also depends on $\tilde{n}$. If a mode is absent and there is no correlation between the coil signals the random variables $t(\tilde{n})$ should be distributed like the complex t-student distribution. The $\tilde{n}$ selection is obtained looking for the maximum of $t(\tilde{n})$ over a set of $\tilde{n}$ in our case for $-7 \leq \tilde{n} \leq 7$ ($\check{t} = \max_{\tilde{n}}|t(\tilde{n})|$). A good approximation for the cdf of $\check{t}$ for $\check{t} > 4$ is $F_{\check{t}}(\check{t}) \cong F_t(\check{t})^{15}$ where 15 is the number of different $\tilde{n}$. The mean value of $\check{t}$ obtained from a Montecarlo is about 1.96. The actual value of the mean is about two three times bigger indicating residual correlation between the coil signals (fig. 5).

Conclusions

The systematic deviation from the expected value of the coil signals can be measured and depends mainly on the frequency. Further statistical analysis are needed in order to understand these residual deviations.

References

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