Period-doubling bifurcation path to chaos due to oscillating external heating rate in a zero-dimensional model in fusion plasmas

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1. Introduction Zero-dimensional models[1-6], which are sets of coupled nonlinear differential equations that have time as the single independent parameter, can assist in interpreting fusion plasma phenomenology, both global and local. The model variables typically denote key macroscopic quantities such as, here, the temperature gradient, the micro-turbulence level, and the amplitudes of zonal flows and geodesic acoustic modes which are mesoscale structures. The model typically embodies Lotka-Volterra or predator-prey dynamics, which can be strongly nonlinear. Zero-dimensional models can generate configurations that are proxies for enhanced confinement states, together with transitions to and from them. Here we investigate the impact of oscillatory external heating rate in the framework of the zero-dimensional model of Zhu *et al*.[5], hereafter ZCD, which couples the four variables introduced above, and is driven by the heating power $q(t)$. We find that when the external heating rate in the ZCD model includes a component that oscillates sinusoidally in time, a period-doubling bifurcation path to chaos exists. The amplitude A of the oscillatory component of heating rate is the control parameter. The micro-turbulence level E bifurcates with increasing A, and we find that Feigenbaum's first constant[7]characterises successive bifurcations to high accuracy.

2. Model description The ZCD model variables are[5] drift wave microturbulence level E, temperature gradient N, zonal flow velocity V_{ZF}, geodesic acoustic mode velocity V_{GAM}. The equations embody both linear effects and nonlinear couplings. The mesoscale structures V_{ZF} and V_{GAM} are induced by the micro-turbulence level E. The growth of microturbulence E, driven by N, is suppressed by mesoscale structures V_{ZF} and V_{GAM}, as well as being self-suppressed. External heating $q(t)$drives the system and the heating rate acts as a control parameter. Normalising time t to τ as in [5], the ZCD model equations are [5]

$$\frac{dE}{d\tau} = \left(N - a_1 E - a_2 d^2 N^4 - a_3 V_{ZF}^2 - a_3 V_{GAM}^2\right) E$$
$$\frac{dV_{ZF}}{d\tau} = \left(\frac{b_{1Z} E}{1 + b_{2Z} d^2 N^4} - b_{3Z}\right) V_{ZF}$$
$$\frac{dV_{GAM}}{d\tau} = \left(\frac{b_{1G} E}{1 + b_{2G} d^2 N^4} - b_{3G}\right) V_{GAM}$$
$$\frac{dN}{d\tau} = -\left(c_1 E + c_2\right) N + q(\tau)$$

Numerical solutions of this system have recently been studied[5] for parameter regimes maximally close to those investigated in the three-variable model of [1,2].

3. Consequences of oscillating heating We represent the external heating rate by[8]

$$q(t) = q_0 + A \sin(\omega t)$$

where $q_0 = 0.47, \omega = 0.05$ and all other coefficients and initial conditions take the values that were used to generate in Fig.2 of [5]. The oscillatory timescale is fast compared to the duration of the quasi-stationary phases in Fig.2 of [5]. Specifically, the period of the oscillating heating rate is approximately half that of the limit cycle in Fig.2 of [5].

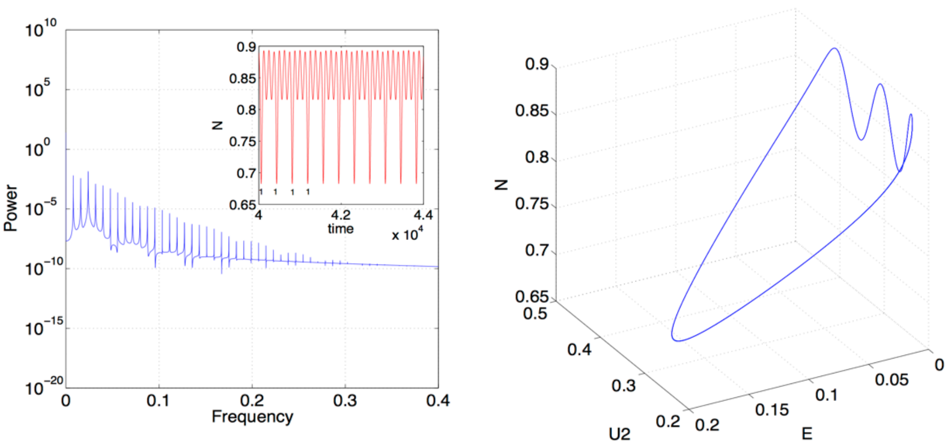

Figure 1. Period-1 oscillation in ZCD system with oscillating external heating rate with amplitude A = 0.0215. Left panel: power spectrum of temperature gradient N. Right panel: the attractor in (N, U_2, E) phase space, where U_2 is normalised V_{GAM}. Inset on left, a segment of the time series of N. Reproduced from [8].

Figures 1 to 5 show the initial period-doubling path from period-1, via period-2 and higher, to a chaotic attractor as the value of A is increased from 0.0215 to 0.0295. These values of A correspond to a few per cent of the steady heating rate q_0 = 0.47.

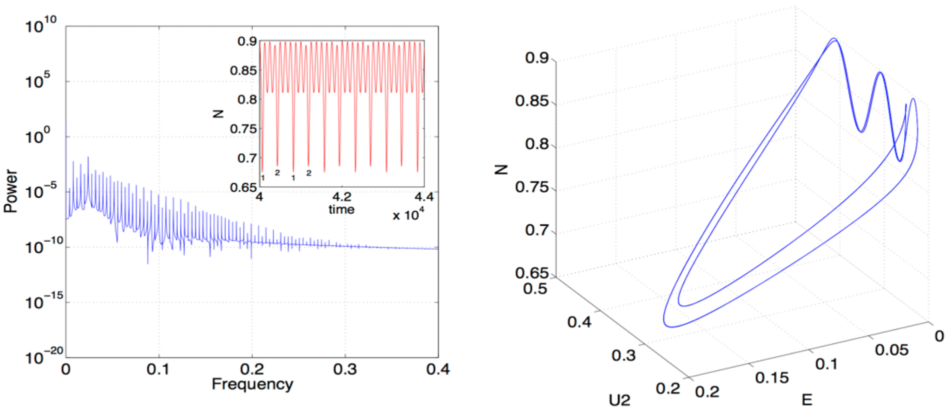

Figure 2. As Fig.1, demonstrating period-2 oscillation in ZCD system with amplitude A = 0.0240. Reproduced from N[8].

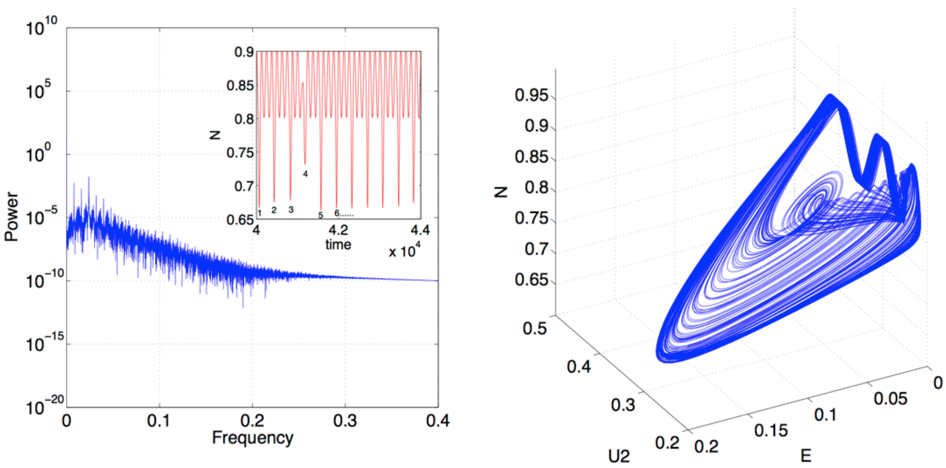

Figure 3. As Fig.1, showing chaotic attractor of the ZCD system with amplitude A = 0.0295. The time series of N has become erratic. Reproduced from [8].

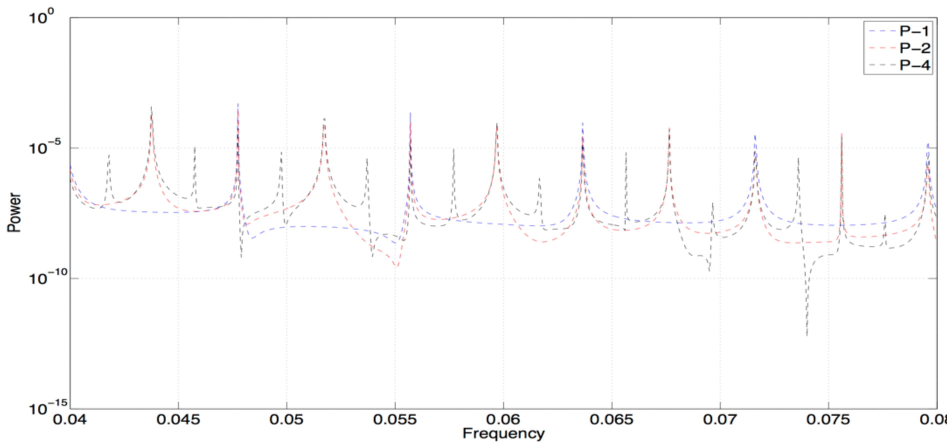

Figure 4 Period-doubling demonstrated by over-plotted power spectra of N in the frequency range from0.04 to 0.08. Periods 1, 2 and 4 are denoted by blue, red and black dash lines. Reproduced from [8].

Figure 5 is a bifurcation diagram of the period-doubling path to chaos in the value of E as the parameter A increases from 0.0215 to 0.0295 in the ZCD model. We have obtained the values of A_n at which the nth period-doubling bifurcations occur, from period-1 to period-8, giving the first Feigenbaum's constant 4.666 which is within 0.05% of the universal asymptotic value 4.669[7].

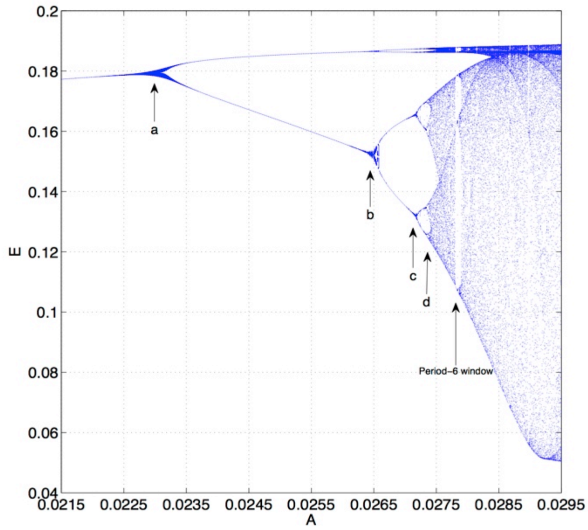

Figure 5.Period-doubling bifurcation path to chaos of ZCD system dynamics. Microturbulence level E is plotted versus amplitude A of oscillating heating, in the range 0.0215 to 0.0295. Arrows a to d mark successive bifurcation points, which occur at the values A = 0.0230, 0.0265, 0.0272 and 0.0273. The last arrow shows a period-6 window within the chaotic region. Reproduced from [8].

4. Conclusions When a small oscillatory external heating component, whose amplitude A is a few per cent of the steady state heating value, is included in the ZCD model, a classic period-doubling bifurcation path to chaos is found in the micro-turbulence level E as A increases. This phenomenology has not previously been observed, so far as we are aware, as a response to oscillatory heating in other zero-dimensional models e.g.[6]. It may assist future experiments testing the assumptions of zero-dimensional models, and perhaps distinguish between them.

This work was part-funded by the support by KAKENHI (21224014, 23244113) from JSPS and the RCUK Energy Programme under grant EP/I501045 and the European Communities under the contract of Association between EURATOM and CCFE. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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