Preliminary results from a strong-flow gyrokinetic PIC code

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Introduction

Usually, it is assumed that the E x B drifts in a tokamak are of the same order as the diamagnetic velocity (relatively small compared to typical thermal speeds). However, this assumption is false for certain tokamaks, such as MAST, where the effects of strong flows should be taken into account.

Large shearing rates on global scales, which are often suggested as a means to improve confinement, imply large flows. As such, investigations into these cases would benefit from the accurate treatment of large toroidal rotation. In addition, there are important contributions to the momentum flux from the strong flow terms which need to be retained (as the weak flow fluxes are close to zero).

The work covered in this paper introduces strong flow effects to an already existing gyrokinetic turbulence code, ORB5, which previously implemented the weak flow formalism.

ORB5 uses the gyrokinetic formalism and ordering. This averages out the high frequency gyration of particles around the magnetic field lines and allows for the 5-dimensional treatment of a plasma, $(\vec{R}, v_\parallel, \mu)$; making the gyro-angle an ignorable coordinate.

The gyrokinetic phase space Lagrangian containing strong flows, Γ, can be derived from the standard Lagrangian of a charged particle [1].

$$\Gamma = (q\vec{A} + m\vec{u}_E + mv_\parallel \vec{b})\cdot d\vec{R} + \frac{\mu B}{\Omega} d\theta - (H_0 + q\delta\phi)dt \tag{1}$$

$$H_0 = \varepsilon = \frac{1}{2}m\left(v_\parallel^2 + u_E^2\right) + \mu B + q\Phi_0(\psi)$$

$$\vec{u}_E = -(\vec{\nabla}\Phi_0 \times \vec{b})/B$$

With potential, $\Phi = \Phi_0 + \delta\phi$. ψ is the poloidal flux, $s = \sqrt{\psi/\psi_0}$.

From the Lagrangian, we can identify three constants of motion (along unperturbed orbits); energy, ε, magnetic moment, μ and toroidal canonical momentum, ψ_c. ψ_c is derived by taking the toroidal component of momentum, which is conserved due to the axisymmetric nature of the unperturbed system.

$$\psi_c = \psi + \frac{m}{q}\frac{F}{B}v_\parallel + \frac{m}{q}u_{tor} \tag{2}$$

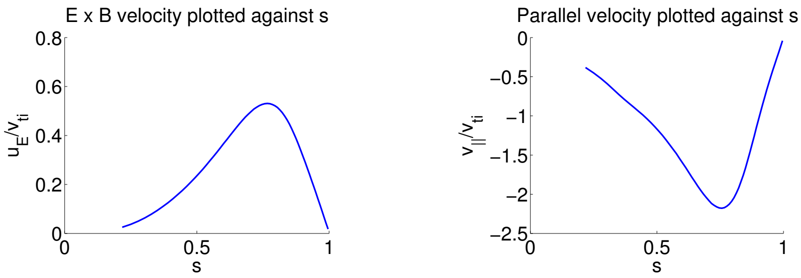

Figure 1: The background E×B and parallel velocity profiles used for testing.

The equations of motion used in the push of ORB5 follow the gyrocentre of the particles and are derived by substituting the gyrocenter phase space Lagrangian into the Euler-Lagrange equation. The non-trivial components can then be decomposed to give $\dot{R}$ and $\dot{v_{||}}$.

$$\frac{d\vec{R}}{dt} = v_{||}\frac{\vec{B}^*}{B^*_{||}} + \frac{\vec{b}}{eB^*_{||}} \times \left(e\vec{\nabla}\Phi + \mu\vec{\nabla}B + \frac{\mu B}{2\Omega}\nabla\left(\vec{b}\cdot\vec{\nabla}\times u_E\right) + \frac{m}{2}\vec{\nabla}\left(u_E^2\right)\right) \tag{3}$$

$$\frac{dv_{||}}{dt} = -\frac{\vec{B}^*}{mB^*_{||}}\cdot\left[e\vec{\nabla}\Phi + \mu\vec{\nabla}B + \frac{\mu B}{2\Omega}\vec{\nabla}\left(\vec{b}\cdot\vec{\nabla}\times\vec{u}_E\right) + \frac{m}{2}\vec{\nabla}\left(u_E^2\right)\right] \tag{4}$$

$$\vec{B}^* = \vec{B} + \frac{m}{q}\vec{\nabla}\times\left(\vec{u}_E + v_{||}\vec{b}\right)$$

These derivations, shown by Hahm [1], assume that the background velocity is of the order of the thermal velocity.

Defining the equilibrium distribution function

The background distribution function is chosen to be a function of the constants of motion, which ensures that it is a gyrokinetic equilibrium. The equilibrium distribution function has the form of a local Maxwellian, but with a temperature and density that vary according to ψ_c, which is a function of velocity and space.

$$f_0 = \left(\frac{m}{2\pi T_0(\psi_c)}\right)^{3/2} n_0(\psi_c)\exp\left[-\frac{1}{T_0(\psi_c)}\left(\varepsilon - q\Phi_0(\psi_c)\right)\right] \tag{5}$$

In the zero-flow and large system limit, $u_E = 0$ and $\psi_c \to \psi$, this distribution converges to a local Maxwellian with $n = n_0$ and $T = T_0$. However, for smaller systems, density and temperature calculated from the moments are not exactly n_0 and T_0, but can vary significantly, especially for large flows. This is inconvenient because we would like to accurately specify the initial profiles.

For background ExB and parallel flows as shown in figure 1 as well as $R/a = 2$, $\rho_* = 1/280$, N = 2 million markers and q varying between 0.854 and 1.23, the resulting profiles variations are as shown in figure 2.

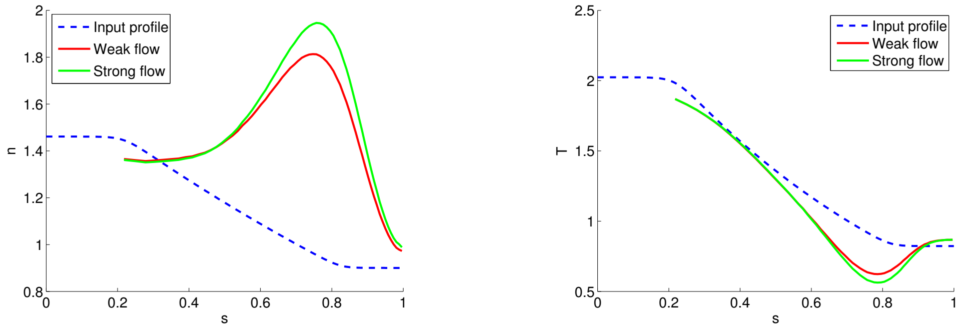

Figure 2: In the presence of toroidal rotation, the uncorrected f_0, equation 5, gives the reconstructed density (left) and temperature (right) shown. Plots show the profiles for both strong and weak-flow equilibria and are normalized to $T_0 = n_0 = 1$ at s = 0.67.

By expanding this distribution function in the large system limit, we can find an approximation for a local distribution.

$$f_0 \simeq \left(\frac{m}{2\pi T_0(\psi)}\right)^{3/2} n_0(\psi) \exp\left[-\frac{m}{2T_0(\psi)}\left(\left[v_\perp^2 + \left(v_\parallel - \frac{F}{B}\frac{\partial \Phi_0}{\partial \psi}\right)^2\right] - (R^2 - R_0^2)\left(\frac{\partial \Phi_0}{\partial \psi}\right)^2\right)\right] \quad (6)$$

The R_0 term is a correction we introduce to reduce the variation of the equilibrium from the input profiles. This local distribution is equivalent to a local distribution derived by Peeters [2]. The correction term is also used with the global distribution in ORB5.

For rotating systems, the density has a strong poloidal dependence. This is present in equation 6 via the term proportional to R^2 in the exponent. Although the potential should vary poloidally, the poloidal variation has yet to be implemented.

Conservation of the constants of motion in an unperturbed system

The best way to test the implementation of the strong flow terms in the equations of motion is by checking the conservation of the constants of motion. Though only ψ_c and ε require investigation, as μ will remain constant regardless.

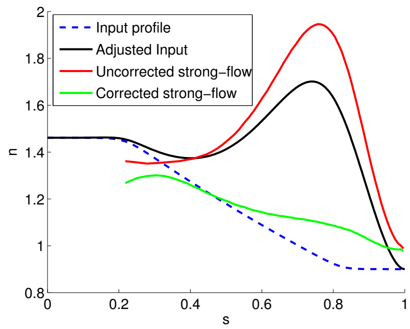

Figure 3: The corrected f_0 gives the density profile shown here. A modified input profile is also plotted which contains the expected density modification caused by a toroidal rotation.

Both ψ_c and ε should remain conserved in the absence of a perturbation. In addition, energy $(H_0 + q\delta\phi)$ should remain conserved with any time independent perturbed field and ψ_c should remain conserved in the presence of a radial or poloidal perturbed electric field.

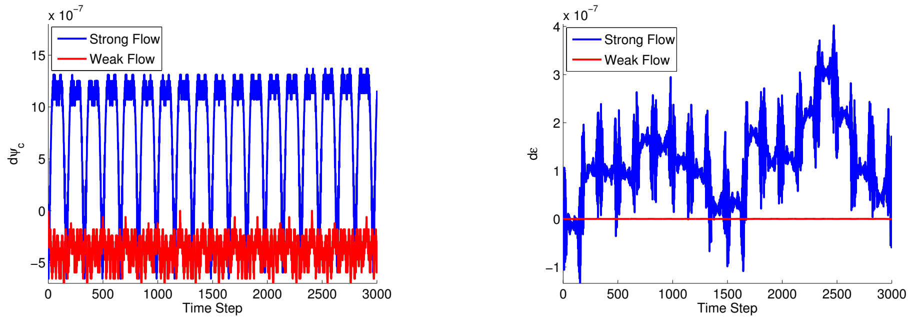

Figure 4: These graphs show the variation of toroidal canonical momentum and energy from their initial values with time, for one marker in an unperturbed system at s∼0.8. The time step size for the system is 10 Ω_{ci}, while the other parameters are the same as for the equilibrium case. $\psi_c \sim 0.66$ and $\varepsilon \sim$ -16.7 for the marker used here.

The good conservation properties, shown in figure 4, provide evidence that the equations of motion have been implemented accurately; this also required consistent evaluation of the derivatives. ψ_c is conserved to roughly the same order as in the weak-flow case and ε is also well conserved, although much more poorly than the equivalent weak flow case.

These constants of motion have also been checked under more extreme conditions ($u_E \sim \mathscr{O}(10 v_{Ti})$) than the conditions used for figure 4. This ensures the conservation holds for any realistic flows we may want to model.

Additionally, we split the distribution function as $f = f_0 + \delta f$. Conservation of f along orbits was checked to ensure δf evolves correctly in the strong flow simulations.

Preliminary strong-flow simulations

Several preliminary simulations have been run using a circular concentric equilibria in both linear and non-linear modes. There appear to be no additional numerical issues when including strong flows of the order of the thermal velocity, or obvious unphysical behaviours, but detailed benchmarking has not yet been performed.

References

[1] T.S.Hahm, Physics of Plasmas **3**, 4658 (1996)

[2] A.G. Peeters *et al*, Physics of Plasmas **16**, 042310 (2009)