A Simplectic Map For Plasmas With Poloidal Divertor

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ABSTRACT

We apply a procedure implemented in [1,2,3] to derive a new two-dimensional integrable symplectic map to describe the equilibrium magnetic field lines of plasmas in tokamaks with a single-null divertor. The invariant surfaces obtained by this map can reproduce a wide variety of magnetic surfaces with elongation (κ) and triangularity (δ) determined by the choice of free parameters, such as the hyperbolic point coordinates. The safety factor profile of the map can also be specified. These maps can be applied to simulate quite well plasma edge tokamak configurations with poloidal divertor in transport investigations. Resonant magnetic perturbations are introduced, replacing the map separatrix by a chaotic layer and allowing the study of open magnetic field line structure in the region between the plasma and the tokamak wall. The main aspects of transport, such as connection lengths and magnetic footprints on the divertor plate [1,2,3] are also presented.

1 – INTRODUCTION

An integrable two-dimensional symplectic map is developed to obtain magnetic field lines in tokamaks with a divertor, following the methodology described in references [1,2,3]. The proposed model consists of twelve parabolic branches jointed smoothly and it aims to eliminate the limitation imposed in the model [3], which does not provide magnetic surfaces with triangularity. The model allows representing surfaces with large sets of values of triangularity and elongation. In this map we apply a resonant perturbation to study the chaotic magnetic field line transport near the separatrix.

2 – THE METOD

The trajectory integration method can be summarized in the following steps [1,2,3]:

i –Choose an appropriate function V(x) in a Hamiltonian denoted by ψ:

$$\psi = \frac{x^2}{2} + V(y) \tag{1}$$

ii –Solve Hamilton´s equations:

$$\begin{cases} \frac{dy}{dt} = \frac{d\psi}{dx} \\ \frac{dx}{dt} = -\frac{d\psi}{dy} \end{cases} \rightarrow \begin{cases} x(x_i, y_i, t) \\ y(x_i, y_i, t) \end{cases} \tag{2}$$

iii – Discretize the continuous solutions by transformation:

$$\big(x(x_i, y_i, t), y(x_i, y_i, t)\big) \rightarrow \big(x(x_i, y_i, \Delta), y(x_i, y_i, \Delta)\big). \tag{3}$$

The parameter Δ is the discretization parameter and is related to the rotational transform of each invariant surface. In an equilibrium plasma this rotation is given by the safety factor. As topology of invariant surfaces independent of the Δ we can reproduce any safety factor associated magnetic surface we want to model through the appropriate choice of Δ(ψ) [3], which can be given by:

$$\Delta(\psi) = \frac{T(\psi)}{q(\psi)} \tag{4}$$

where $T(\psi)$ is the rotational period of the invariant curve ψ and $q(\psi)$ is the safety factor of the magnetic surface that we want to represent by the invariant curve ψ.

3 —EQUILIBRIUM MAGNETIC FIELD MODEL

The invariant curves as a diverted plasma shape are obtained by using two-dimensional potential (for x> 0 and x <0) in which each curve consists of six parabolic branches matching smoothly to preserve the integrability of Hamilton´s equations, as shown in Fig. 1. The positions of the local minima correspond to elliptic points and local maximum corresponds to the hyperbolic point (X point).

The main geometric parameters of the separatrix are shown in Fig 2, which can vary freely.

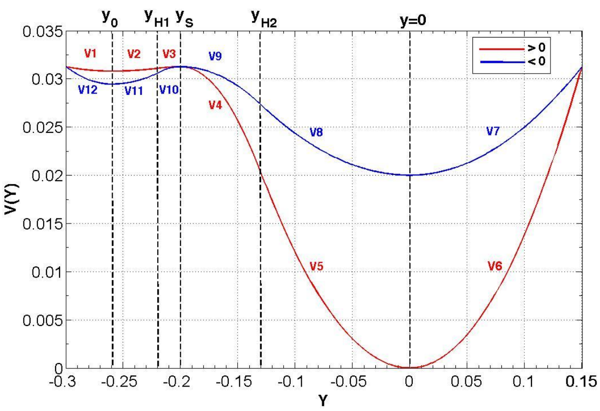


Fig 1. The potential V(y) for the proposed map. The curve plotted in red is the potential for x>0 and the curve plotted in blue is the potential for x<0. Six parabolic branches compose each curve.

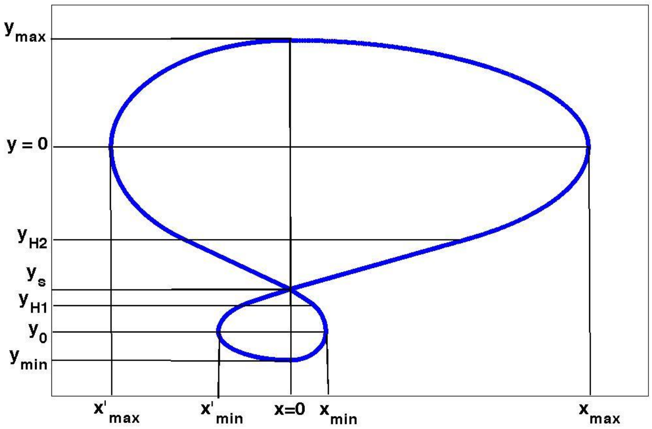


Fig 2. Schematic view of geometric parameters.

To illustrate the versatility of the model we show in Fig. 3 two configurations with different triangularity and elongation. A monotonic safety factor profile was used to reproduce these configurations.

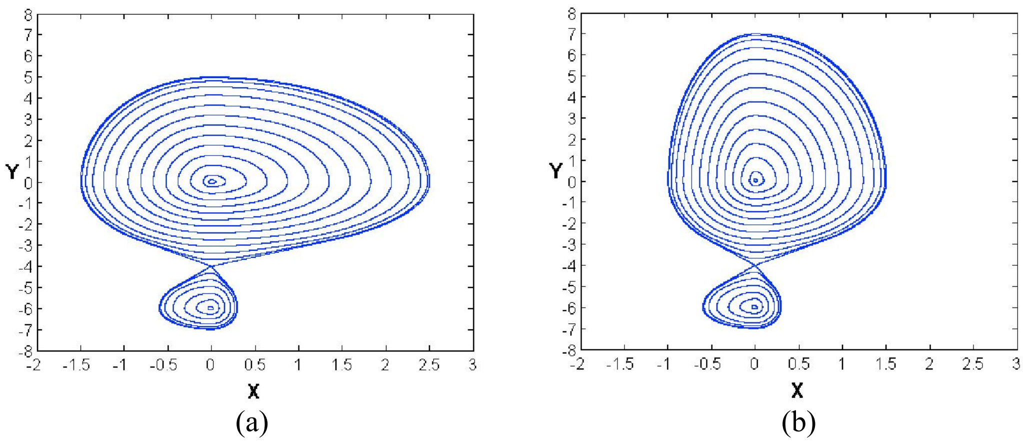


Fig 3. Invariant surfaces with parameters (a) $x_{MAX} = 2.5$, $x'_{MAX} = -1.5$, $x_{MIN} = 0.8$, $x'_{MIN} = -0.6$, $y_{MAX} = 5.0$, $y_{H2} = -2.75$, $y_S = -4.0$, $y_{H1} = -5.0$, $y_0 = -6.0$, $y_{MIN} = -7.0$ and (b) $x_{MAX} = 1.5$, $x'_{MAX} = -1.0$, $x_{MIN} = 0.8$, $x'_{MIN} = -0.6$, $y_{MAX} = 7.0$, $y_{H2} = -2.75$, $y_S = -4.0$, $y_{H1} = -5.0$, $y_0 = -6.0$, $y_{MIN} = -7.0$.

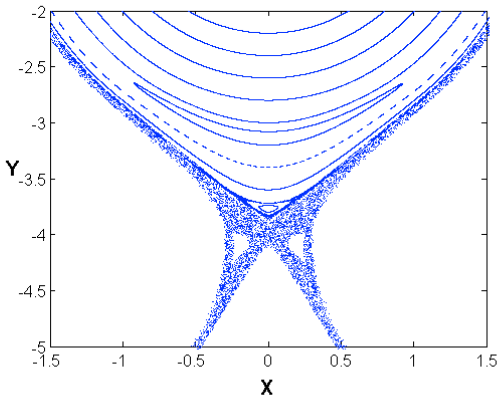
Thus, through the changes of free parameters we can obtain a wide variety of configurations of surfaces of equilibrium. We call attention that our map describes diverted magnetic fields without toroidal corrections. However, close the separatrix 1/x is nearly constant, so we can get a good approximation for the equilibrium field in large aspect ratio tokamaks.

4 – DIVERTOR MAP WITH MARTIN TAYLOR PERTURBATION

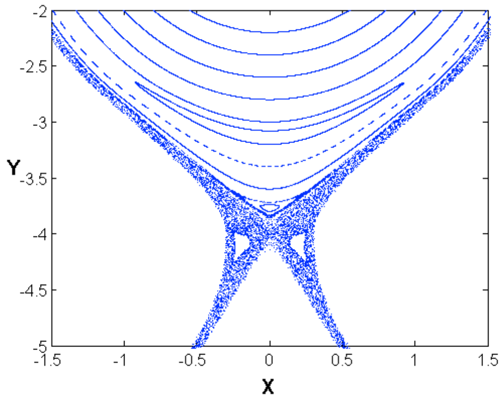
We use the non-integrable Martin Taylor map [4] to introduce an ergodic limiter perturbation. The total field line map $(x_n, y_n) \rightarrow (x_{n+1}, y_{n+1})$ considers integrable divertor map M_D, yielding$(x_n, y_n) \rightarrow (x^*, y^*)$, and the perturbing map M_P, which gives $(x^*, y^*) \rightarrow (x_{n+1}, y_{n+1})$ [3].

5 – RESULTS AND CONCLUSIONS

Initially we are considering a simpler case with only elongation (without triangularity) to include a perturbation in this proposed geometry (see Fig. 4). The model reproduces the results quite well compared to those obtained in [3].



(a)



(b)

Fig. 4. Perturbed Map for (a) the model described in ref. [3] and (b) The model proposed here both with shear = 1.80 and m = 3.

The next step we are going to consider the cases with triangularity and thus be able to reproduce more realistic configurations of magnetic field lines. The chaotic layer formed near the separatrix can be studied, as the transport of field lines and deposition patterns in the divertor plates.

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