Drift effect on geodesic mode instability driven by electron current in tokamak plasmas

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Geodesic Acoustic Modes (GAM) [1] discovered in the frame of a MHD plasma model have actively been studied theoretically including diamagnetic drift effects during last years [2-6]. This type of oscillations have been experimentally detected due to second poloidal harmonic magnetic fluctuations discussed in theory [5-7] under wide range of condition in various tokamaks, during Ohm, ion cyclotron resonance (ICR), or neutral beam (NB) heating [8-10].

Here, we present the kinetic treatment of the GAM type modes by fully taking into account parallel electron and ion dynamics and the ion diamagnetic drifts. The Ohm current is represented in the form of a diffused hot electron beam and the finite ion drift parameter, in relation to poloidal magnetic field, is considered in the calculation of the GAM instability. We investigate whether that the coupling of the standard GAM with the drift of the hot and cold ions may cause the spectrum modification. Then, a possible application of the results to tokamaks will be under discussion. We employ the quasi-toroidal set of coordinates (r, ϑ, ζ) in the large aspect ratio tokamak approximation [11] $R_0 >> r$, where the circular surfaces ($R = R_0 + r\cdot\cos\theta$, $z = r\cdot\cos\theta$) are formed by the magnetic field with toroidal and poloidal components, $B_\zeta = B_0 R_0/R$, $B_\vartheta = rB_\zeta/qR_0$, and $B_\theta << B_\xi$ in the standard drift kinetic equation

$$\frac{\partial f}{\partial \vartheta} - \frac{i\Omega f}{w} = \frac{eF}{mw}\left[\frac{(w - v_0)E_3}{k_0 v_T^2} + \frac{2+\eta(w^2+u^2-3)}{2k_0 v_T \omega_c d_r}E_2 - \frac{(2w^2+u^2)}{2v_T\omega_c k_0 R}E_1 \sin\vartheta\right] \qquad \textbf{(1)}$$

Here, $\Omega_i = \omega / k_0 v_{Ti}$ and $\omega_c = eB/mc$ are the normalized wave and cyclotron frequencies, E_k are components of wave field where the parallel field E_3 is potential part of electromagnetic field, $E_3 = h_\theta E_2$, $h_\vartheta = B_\vartheta / B_0$ is magnetic field inclination, $k_0 = h_\theta / r$ is the parallel wave vector, $v_{Te,i} = \sqrt{T_{e,i}/m_{e,i}}$ are thermal velocities, $\eta_{e,i} = \partial \ln T_{e,i} / \partial \ln n_{ei}$, $\partial n_{0i}/\partial r = -n_{0i}/d_i$ is density gradient, $w = v_\parallel / v_T$ and $u = v_\perp / v_T$ are normalized velocities for each specie and $k_0 = 1/qR$. A small fraction of the hot ions $f_h n_0$ is assumed in relation to the cold one n_0 ($f_{(i)} = 1$). Maxwell distribution function F is proposed for the hot and cold ions and the electron distribution is Maxwell distribution $F_e = F(1 + 2v_\parallel V_0 / v_{Te}^2)$ shifted by the parallel current velocity V_0.

Integrating Eq (1) for electrons in the limit [12] $\omega << v_{Te}/qR$ ($\Omega << v_{Te}/v_{Ti}$ in our notation) and $V_0/v_{Te} = \mu v_0 << 1$, we obtain the equations for the electron density perturbations $\tilde{n}_e$

$$n_{es} = \frac{e_i n_0 (1+f_h) R_0 q}{T_e}\left[\sqrt{\frac{\pi}{2}}\mu\left(v_0 + t_e\rho_i(1-\eta_e/2)\right)E_s - E_c\right],$$
$$n_{ec} = \frac{e_i n_0 (1+f_h) R_0 q}{T_e}\left[\sqrt{\frac{\pi}{2}}\mu\left(v_0 + t_e\rho_i(1-\eta_e/2)\right)E_c + E_s\right] \qquad \textbf{(2)}$$

where $\mu = v_{Ti}/v_{Te}$, $E_3 = E_s \sin\vartheta + E_c \cos\vartheta$, $\tilde{n}_e = n_{es}\sin\vartheta + n_{ec}\cos\vartheta$ and the contribution of electron drift and parallel current velocities are combined : $w_0 = v_0 + t_e\rho_i(1-\eta_e/2)$, where $\rho_i = v_{Ti}/d_r\omega_{ci}h_\theta$ is the normalized drift parameter and $t_e = T_e/T_i$. Then, we define the sum of the cold (i) and hot (h) ion density perturbations as follows

$$n_c = -\sum_{i=i,h}\frac{e_i n_0 f_i R_0 q}{2m_i v_{Ti}^2}\left[\left(2+\sqrt{2}\Omega_i Z_i\right)E_s + \mathrm{i}\left(\frac{\sqrt{2}}{2}(2+\Omega_i^2\eta_i - \eta_i)Z_i + \Omega_i\eta\right)\rho_i E_c\right]$$

$$n_s = \sum_{i=i,h}\frac{e_i n_0 f_i R_0 q}{2m_i v_{Ti}^2}\left\{\frac{\mathrm{i}v_{Ti}}{R_0\omega_{ci}}\left(\sqrt{2}(\Omega^2+1)Z_i/2+\Omega_i\right)E_1\right.$$
$$\left. + \left[\left(2+\sqrt{2}\Omega_i Z_i\right)E_c - \mathrm{i}\left(\sqrt{2}(2+\Omega_i^2\eta-\eta)Z_i/2+\Omega_i\eta\right)\rho_i E_s\right]\right\} \qquad \textbf{(3)}$$

$$\frac{E_s}{E_c} = \frac{\mathrm{i}}{2}\left\{\sqrt{2\pi}\mathrm{i}\mu w_0 - t_e\sum_{i=i,h} f_i\rho_i\left[\sqrt{2}\left(\Omega_i^2\eta_i + 2 - \eta_i\right)\frac{Z_i}{2} + \eta_i\Omega_i\right]\right\}\left[1+t_e\sum_{i=i,h} f_i\left(1+\sqrt{2}\Omega_i\frac{Z_i}{2}\right)\right]^{-1}$$

$$E_c = -\mathrm{i}\frac{t_e}{R_0 D}\left[1+\frac{\sqrt{2\pi}}{2}\mathrm{i}\mu\Omega + t_e\sum_{i=i,h}\left(1+\sqrt{2}\Omega_i\frac{Z_i}{2}\right)\right]\sum_{i=i,h}\frac{v_{Ti}}{\omega_{ci}}\left[\Omega_i + \sqrt{2}\left(1+\Omega_i^2\right)\frac{Z_i}{2}\right]E_1 \; ; \qquad \textbf{(4)}$$

where

$$D = \left[1+t_e\sum_{i=i,h} f_i\left(1+\sqrt{2}\Omega_i\frac{Z_i}{2}\right)\right]^2 - \frac{\sqrt{2}}{4}\left\{t_e\sum_{i=i,h} f_i\rho_i\left[\left((\Omega_i^2-1)\frac{\eta_i}{2}+1\right)Z_i + \frac{\Omega_i}{\sqrt{2}}\eta_i\right] - \mathrm{i}\sqrt{\pi}\mu w_0\right\}^2$$

Then, we get the radial component of the θ-averaged electron and ion current

$$j_r^{ei} = e\frac{v_{Te}^3}{2}\int_0^\infty u\,du\oint\frac{d\vartheta}{2\pi}\int_{-\infty}^\infty V_{re,i} f_{ei}\,dw; \quad j_r^e = \frac{e_i^2 q n_0}{m\omega_{ci}}\left[\mathrm{i}\sqrt{\frac{\pi}{2}}\frac{\mu}{2}\left(w_0 + t_e\frac{\rho}{2}(2+\eta_e\right)E_s - E_c\right]$$

$$j_r^{ion} = \sum_{i=i,h}\frac{e_i^2 f_i n_0 q}{4m_i\omega_{ci}}\left\{\mathrm{i}\rho_i\left[\sqrt{2}\left(\eta_i\Omega_i^4 + 2\Omega_i^2 + 2 + \eta_i\right)Z_i/2 + \eta_i\Omega_i^3 + \Omega_i(2+\eta_i)\right]E_s\right.$$
$$\left. - \left[\sqrt{2}\Omega_i\left(\Omega_i^2+1\right)Z_i + 2\Omega_i^2 + 4\right]E_c - \frac{\mathrm{i}v_{Ti}}{R_0\omega_{ci}}\left[\left(\Omega_i^4 + 2\Omega_i^2 + 2\right)Z_i + 2\Omega_i(\Omega_i^2+3)\right]E_1\right\} \qquad \textbf{(5)}$$

where $V_{re,i} = -(2w^2 + u^2)v_{Te,i}^2 \sin\vartheta/(2R\omega_{ce,i})$. The final equation for the geodesic continuum obtained from the quasi-neutrality condition is $j_r^e + j_r^{ion} + j_p = 0$, where j_p is the ion radial polarization current $j_p = -\mathrm{i}\omega c^2 E_1/4\pi c_A^2$, and $c_A = B/\sqrt{4\pi n_i m_i}$. The standard dispersion relation for the high frequency GAM may be found from that equation for large q in the limit $\Omega_i \to \infty$ and Ω_h <<1. Assuming small imaginary part ($\gamma << \Omega_G$), it gives

$$\frac{\Omega_G^2}{q^2} = \left(\frac{\omega_G^2 R_0^2}{v_{Ti}^2}\right) \approx \left[\frac{7}{2} + 2t_e + \frac{23 + 4t_e(4+t_e)}{q^2(7+4t_e)} + \frac{4\rho_i^2 t_e^2(1+\eta_i+2t_e)}{q^2(7+4t_e)}\right]\left(1 + q^2\frac{f_h}{2}\right)^{-1}. \quad \textbf{(6)}$$

The root corresponds to the GAM dispersion with the drift correction terms. Using the main term of the small ion Landau damping, the imaginary part of the frequency γ is found

$$\gamma \approx \sqrt{\frac{\pi}{2}}\frac{v_{Ti}q}{2R_0}\left[\frac{v_{Ti}}{v_{Te}}t_e\left(\frac{\rho_i w_0}{\Omega_G^2}(2+2\eta_i+3t_e) - \frac{\Omega_G^4}{2t_e}\mathrm{e}^{-\Omega_G^2/2}\right) - f_h\frac{v_{Th}}{v_{Ti}}\left(t_e^2\frac{\rho_i^2 d_i}{d_h}(1+\frac{\eta_h}{2}) + \Omega_G^2\right)\right].$$

In this case, the electron current may drive the GAM instability due to the term produced by the current velocity and density gradient for $d_i>0$, $f_h=0$,

$$\frac{V_0(1+\eta_i+3t_e/2)}{\omega_{ci}d_i} > \frac{\omega_G^6 R_0^6 q^6}{2t_e v_{Ti}^6}\mathrm{e}^{-\Omega_G^2/2},$$

and/or for $d_h<0$, $\mu \approx 0$,

$$-(1+\eta_h/2)\,d_i/d_h > \Omega_G^2 q^2/t_e^2\rho_i^2$$

To accurately obtain the frequency of geodesic continuum and the threshold velocity for moderate values of q and ρ_i, we calculate numerically the dispersion equation and the results of the frequency and instability threshold calculations are shown in the figure.

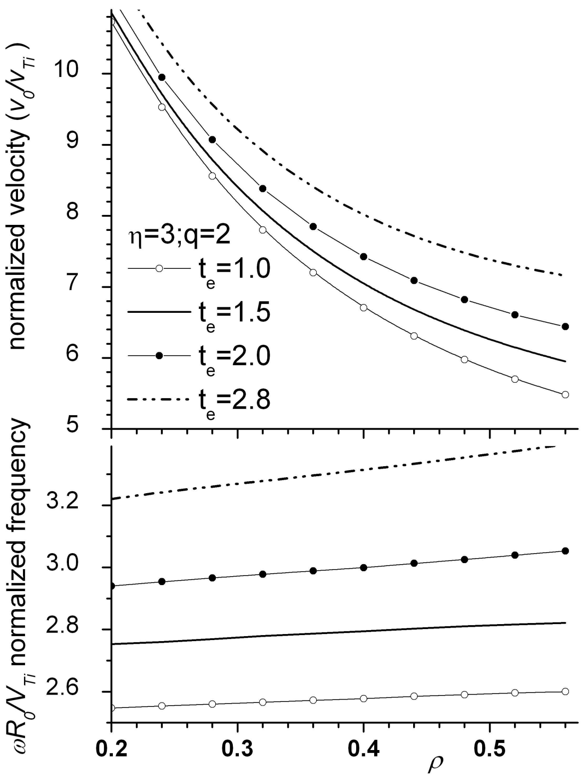

Fig1. Plot of the threshold velocity and the geodesic continuum as a function of ρ_i for f_h=0, η_i =3, q=2, t_e=1.0, 1.5, 2 and 2.8.

The threshold velocity decreases fast as an inverse function of the drift parameter as demonstrated in Fig 1. The GAM frequency follows the GAM dispersion (6) and increases slowly with the ρ_i-parameter when the electron temperature is in the range (1≤ t_e ≤2.8). The real frequency is weakly dependent of the η_i-parameter. The threshold velocity grows roughly linear with q-parameter and the dependence on η_i is more pronounced. These results concerned with the GAM continuum. The eigenmode solutions are possible near the maximum or minimum of the continuum. This condition may be realized in the case of reversed shear configurations observed in the experiments [8, 10]. The mode radial structure $k_r^2 \rho_{Li}^2 \approx \left(1 - \omega_{G\min}^2 / \omega^2\right)$ is determined by finite Larmor radius effects above continuum minimum due to modification of the polarization current by the factor $\left(1 - (3/4) k_r^2 \rho_{Li}^2\right)$ or due to a finite orbit effect taking into account the second poloidal harmonic effect [5].

Recently, the geodesic modes have been observed during plasma current ramp up with counter injection of the NB heating that forms reversed shear configuration in JT-60 [10]. These modes were not reported during co-NB injection. The observed modes have a smaller frequency by half of the value of the core GAM frequency, and approximately coincide with the local GAM frequency at q-minimum. We suggest that this geodesic mode is the above-discussed unstable GAM driven by the electric current and localized at the minimum of the continuum that formed due to reversed shear.

Finally, we conclude that our analytical and numerical calculations show that the standard geodesic mode may be unstable when the electron velocity of the Ohm current is above the GAM phase velocity $v_0 >> \omega_G Rq$ or in the case of the sharp hot ion density profiles due to ICR heating [8,13]. The ohm current instability may be observed in the ramp up current phase in tokamaks with counter-NB injection heating, when the inversed shear profile is formed. Even if the instability is very weak ($\gamma \propto v_{Ti}\omega / v_{Te}$), it may serve as some indicator for diagnostics of plasma parameter.

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