Effects of the geodesic acoustic mode driven by energetic ions on transport and cooling of these ions

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Introduction. The geodesic acoustic mode, GAM, [1] and the corresponding energetic-particle mode, E-GAM, [2,3] are mostly electrostatic with $m = n = 0$ (m and n are the poloidal and toroidal mode numbers, respectively), so that the mode electric field is $\tilde{\mathbf{E}} = (\tilde{E}_r, 0, 0)$ (we use the radial, poloidal, and toroidal coordinates r, θ, φ). This statement is true, unless the plasma has high β, in which case the $m \neq 0$ harmonics of the perturbation are important and the perturbed magnetic field is not negligible [4,5]. When $\tilde{\mathbf{E}} = (\tilde{E}_r, 0, 0)$, the radial component of the $\tilde{\mathbf{E}} \times \mathbf{B}$ drift vanishes (**B** is the equilibrium magnetic field). For this reason, a widespread point of view is that GAM / E-GAM does not affect the energetic ion transport and that the only direct effect of GAM / E-GAM on the energetic ions is the change of their pitch angles (which can lead to the transformation of passing ions to trapped ones and the concomitant increase of the particle radial displacement). However, in this work we will show that there is a direct mechanism of the transport of the energetic ions across the magnetic field. This mechanism is associated with the decrease of the energy of fast ions driving the GAM instability.

Fast ion motion and cooling in the presence of GAM / E-GAM. Let us consider a passing energetic ion with a standard orbit in the presence of a GAM / E-GAM mode described by $\tilde{E}_r = \hat{E} \sin \omega t$. Its guiding center motion is governed by the following equations:

$$\dot{r} = -V_D \sin\theta, \quad \dot{\theta} = \omega_t - \omega_D \cos\theta + \omega_E \sin \omega t, \qquad \dot{\in} = e\dot{r}\hat{E}_r \sin \omega t, \tag{1}$$

where $\in$ is the particle energy, ω_t is the transit frequency, ω_E is the frequency of the poloidal motion due to the $\hat{\mathbf{E}} \times \mathbf{B}$ drift, $\omega_D = V_D / r$, V_D is the velocity of the toroidal drift, dot over letters means the time derivative. Assuming that the mode amplitude is sufficiently small, we can make the transit time averaging. Then

$$\langle \dot{r} \rangle = -\frac{V_D}{\tau_t} \oint \frac{d\theta}{\dot{\theta}} \sin\theta \approx -V_D \frac{\omega_E}{\omega_t} \oint \frac{d\theta}{2\pi} \sin\theta \sin[\omega t(\theta)], \quad \langle \dot{\in} \rangle = -e\hat{E}_r V_D \oint \frac{d\theta}{2\pi} \sin\theta \sin[\omega t(\theta)], \tag{2}$$

where $\langle \cdots \rangle$ means the transit time averaging, τ_t is the transit period. It follows from (2) that

$$e\hat{E}_r\langle\dot{r}\rangle=\sigma\frac{\omega_E}{|\omega_t|}\langle\dot{\in}\rangle\,, \tag{3}$$

where $\sigma=\operatorname{sgn}V_\parallel$, $V_\parallel$ is the particle velocity along the magnetic field. We conclude from here that (i) $\langle\dot{r}\rangle\neq 0$ when $\langle\dot{\in}\rangle\neq 0$ and vice versa; (ii) the fast ions driving the GAM / E-GAM instability are displaced outwards in the case of counter injection ($\sigma<0$) and inwards in the contrary case ($\sigma>0$) because these ions give their energy to the mode ($\langle\dot{\in}\rangle<0$).

Taking into account that fast ions interact with GAMs through the resonance $\omega=\omega_t$ and that $\theta\approx\omega_t t+\theta_0$ (due to small ω_E), we can write the integral in (2) as

$$I\equiv\oint d\theta\sin\theta\sin[\omega t(\theta)]=\oint d\theta\sin\theta\sin(\theta-\theta_0)]=\frac{1}{2}\cos\theta_0\,. \tag{4}$$

It follows from (4) and (2) that the mode-particle interaction leads either to the decrease of the particle energy or to its increase (provided that $\theta_0\neq\frac{\pi}{2}+l\pi$, with $l=1,2,3...$). This is not surprising: in the phase space $(\in,\theta)$ the time derivative of the energy of particles trapped in the wave is either positive or negative in all points of the exact resonance $\omega=\omega_t$, except for X-points and O-points of the resonance islands.

Because GAMs do not change the particle magnetic moment and $\omega_t=V_\parallel/(qR)$, $\omega_E=-c\hat{E}/(\kappa rB_s)$ [q is the safety factor, R is the distance from the major axis of the torus, κ is the elongation of the plasma cross section, $B_s=B(r=0)$], Eq. (3) can be written as

$$\langle\dot{r}\rangle=\frac{qR_s}{\omega_B\kappa r}\langle\dot{V}_\parallel\rangle, \tag{5}$$

where ω_B is the gyrofrequency, R_s is the radius of the magnetic axis. Noting that $\langle V_\parallel\rangle\approx V_\parallel(\theta=\pi/2)\equiv u$ and neglecting the magnetic shear, we obtain from (5):

$$\Delta\left(r^2\right)=\frac{2qR_s}{\omega_B\kappa}\Delta u\,. \tag{6}$$

This agrees with the more general relation

$$MR_s\Delta u=\frac{e}{c}\Delta\psi_p(\pi/2) \tag{7}$$

[ψ_p is the poloidal magnetic flux on the particle orbit, $\psi_p(\pi/2)\equiv\psi_p[r(\theta=\pi/2)]$, which immediately follows from the conservation of the canonical angular momentum, $P_\varphi=MV_\parallel R-e\psi_p/c=const$.

In order to demonstrate the change of the energy and location of the passing resonant ions driving a GAM / E-GAM instability, we solved numerically the following equations:

$$\frac{dx}{d\tau} = -\frac{1+\chi^2}{1+\chi_0^2}\bar{\omega}_D W \sin\theta, \qquad \frac{d\theta}{d\tau} = \frac{\chi}{\chi_0}\sqrt{W} - \bar{\omega}_D \frac{W}{x}\frac{1+\chi^2}{1+\chi_0^2}\cos\theta - \frac{\bar{\omega}_E}{x}\sin(\theta-\theta_0),$$

$$\frac{dW}{d\tau} = -\frac{1+\chi^2}{1+\chi_0^2}\bar{\omega}_D W \bar{E}\sin\theta\sin(\theta-\theta_0), \tag{8}$$

where $x = r/a$, a is the plasma radius in the equatorial plane of the torus, $\tau = t\omega_{t0}$, $W = \in / \in_0$, the subscript "zero" labels magnitudes at $t = 0$, $\chi = \sigma\sqrt{1 - W_\perp / W}$ is the particle pitch, $\sigma = \mathrm{sgn}\, V_\parallel$, $W_\perp = \in_\perp / \in_0$, $\in_\perp = const$ is the transverse energy, $\bar{\omega}_D = \omega_{Da} / \omega_{t0}$, $\bar{\omega}_E = \omega_{Ea} / \omega_{t0}$, $\bar{E} = \dfrac{e\hat{E}a}{\in_0}$, $\omega_{Ea} = \dfrac{c\hat{E}}{aB_s\kappa}$, $\omega_{Da} = \dfrac{V_{D0}}{a} = \dfrac{\rho_0}{R_s}\dfrac{V_0}{a\kappa}\dfrac{1+\chi_0^2}{2}$, ρ is Larmor radius. These are actually Eq. (1) modified to take into account the finite resonance width. They are written in the assumption that the fast ion remains resonant in spite of the fact that its longitudinal energy and the radial location change. This can be the case when the width of the resonance in the phase space and the radial mode width are large enough. Note that the mode width depends on the fast ion orbit width [3] and the magnitude of β **(**$\beta = 8\pi p / B^2$**)** [5].

The results of calculations for a deuteron with $\in_0 = 75$ keV, $\chi_0 = 0.7$, and DIII-D parameters [2] ($R_s = 170$ cm, $a = 60$ cm, $\kappa = 1.3$, $T = 1.5$ keV, and *q=4*), are shown in Fig. 1. The GAM amplitude was taken from the condition $\hat{E}_r = const = e\Phi / L$=5 kV/m, with $e\Phi = T$ and $L = 0.3$ m, which seems realistic [5,6] (the local $\hat{E}_r$ can be much higher, up to *28* keV/m [7]). Figure 1 confirms our consideration above. In addition, it shows that a considerable effect of the mode takes place for a few tens of the transits ($\tau = 200$ corresponds to 32 transits, about 1 ms), which is less than the instability bursts (several ms) in the experiment.

The particle energy after the slowing down ($\in^f$) can be evaluated as $\in^f = \in^i + \Delta \in^{res}$, where $\in^i$ is the initial energy, $\Delta \in^{res}$ is the resonance width, and [6]:

$$\frac{\in^f}{\in^i} = 1 - 4\sqrt{2S\frac{\hat{n}_e}{n_e}}\left(\sqrt{1-\lambda} - \sqrt{2S\frac{\hat{n}_e}{n_e}}\right), \qquad \text{with} \qquad \sqrt{1-\lambda} > 2\sqrt{2S\frac{\hat{n}_e}{n_e}}, \tag{9}$$

$\lambda = \mu B_s / \in$ is the pitch angle parameter, $S = 1 - c_s^2 / \left(q^2 R_s^2 \omega^2\right)$, c_s is the sound velocity, n_e is the electron density, $\hat{n}_e$ is the amplitude of the electron density perturbation. In particular, in

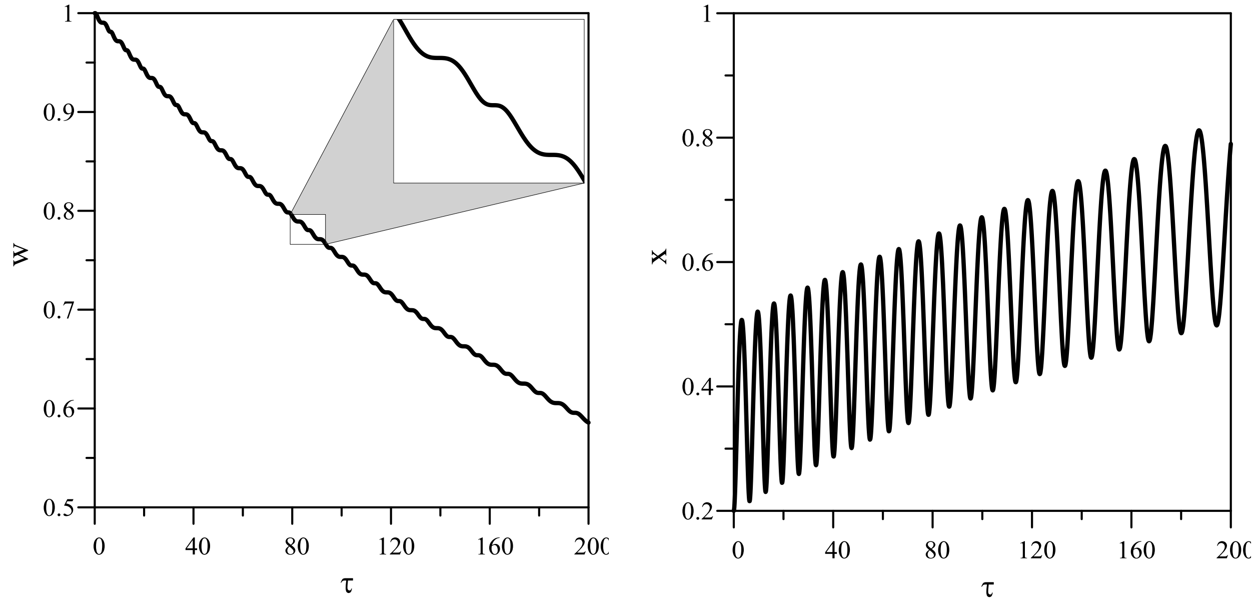

Fig.1. W and x versus τ for fast ions driving a GAM / E-GAM instability.

the DIII-D experiment [2] $\frac{\hat{n}_e}{n_e} = 1.5\%$, f =25 kHz, and q=3.5 at $r/a \sim 1/2$. For these parameters, (9) yields $\in^f / \in^i = 0.5 - 0.6$ when $\lambda = 0 - 0.3$.

Summary. The resonant interaction of the passing energetic ions and a global GAM / E-GAM mode can lead to the transfer of a large fraction of the energy of these ions to the mode (large $\Delta \in / \in^i$) and to a considerable radial displacement of these ions. Note that the large $\Delta \in / \in^i$ is a necessary condition for the high efficiency of the alpha channelling [8] and the spatial channelling of the fast-ion energy and momentum by the mode [9].

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