Species separation produced by laminar shocks in inertial fusion targets

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Observations on laser compressed targets have shown the existence of very strong electric fields, of the order of 10^9 - 10^{10} V/m, localized over distances of the order of 100 nm [1, 2, 3, 4] . Recently we have suggested that these might be readily explained as weak laminar collisionless shock structures [5], rather than dissipative shocks as suggested elsewhere [4]. Here, after a brief recapitulation of the theory, we concentrate on the effect that the existence of such structures in fusion targets may have on species concentrations, showing that reflection of different fractions of deuterium and tritium at the shock front, combined with different slowing down of the unreflected parts of the distribution, may produce significant deviations from an initial equal concentration of the species.

If we consider a single ion species flowing into a region where the potential increases monotonically from zero to $\phi_{\max}$, in the rest frame of the shock, then the ion density in the upstream region, normalized to the density of the incoming flow, is

$$\begin{aligned} n_i(\phi,\phi_{\max}) &= \frac{1}{\sqrt{2\pi}}\int_0^\infty \exp\left[-\frac{\left(\sqrt{v^2+2\phi}-V\right)^2}{2}\right]dv \\ &+\frac{1}{\sqrt{2\pi}}\int_0^{\sqrt{2(\phi_{\max}-\phi)}} \exp\left[-\frac{\left(\sqrt{v^2+2\phi}-V\right)^2}{2}\right]dv, \end{aligned} \tag{1}$$

with velocities normalized to the thermal velocity V_i of this species and the potential ϕ to $\frac{m_iV_i^2}{e}$, assuming the ions to be singly charged. Here V is the incoming flow velocity, assumed to be well above the thermal velocity so that the Maxwellian thermal spread around this velocity has no significant backward flowing part. The first term is the density of ions flowing into the shock, while the second is the reflected ion component, upstream of the potential maximum. The latter cannot, of course, be chosen arbitrarily, but must be consistent with the system dynamics, as discussed below. If we take this initial ion species to be deuterium, then a similar expression for tritium is readily obtained. The temperature of the two ion species is taken to be the same, so the normalized thermal velocity of the tritium is $\sqrt{\frac{2}{3}}$. With a given flow velocity we can then find the D and T charge densities $n_{D,T}(\phi,\phi_{\max})$ as a function of the local potential and the potential maximum.

For the electrons we make the assumption that on the time scales involved in the shock they take up a thermal distribution, so that the electron density is given by

$$n_e = n_0 \exp\left(\frac{\phi}{T}\right)$$

with T the ratio of the electron to ion temperatures. We also assume that the electrons flow so as to neutralize the ion charge far upstream where ϕ tends to zero, so that $n_0(\phi_{\max}) = n_D(0,\phi_{\max}) + n_T(0,\phi_{\max})$. It is convenient to introduce a Mach number in terms of the ion sound speed $c_s = \sqrt{\frac{4T_D}{5m_D}}$ or $\sqrt{\frac{4T}{5}}$ in our normalized units. This assumes that there are equal concentrations of the two ion species and neglects the contribution of the ion pressure to the sound speed. Since the structures that concern us only exist when the electron temperature is well in excess of the ion temperature this definition of the Mach number is never very far from the true Mach number.

We can now obtain Poisson's equation in the dimensionless form

$$\frac{d^2\phi}{dx^2} = [n_e(\phi,\phi_{\max}) - n_D(\phi,\phi_{\max}) - n_T(\phi,\phi_{\max})] \tag{2}$$

where the length scale is $\frac{V_D}{\omega_{pD}}$ with V_D the deuterium thermal velocity and ω_{pD} the deuterium plasma frequency, calculated using the a density equal to the total electron density in the incoming flow. To determine the value of $\phi_{\max}$ consistent with the dynamics of the system we introduce the Sagdeev potential (or pseudopotential) Ψ_s such that Eqn. (2) takes the form

$$\frac{d^2\phi}{dx^2} = -\frac{\partial\Psi_s(\phi,\phi_{\max})}{\partial\phi}. \tag{3}$$

and the problem becomes analogous to that of the motion of a particle in the Sagdeev potential. Our assumption of a monotonically increasing potential reaching a maximum at $\phi_{\max}$ requires that Ψ_s has a maximum at $\phi = 0$ (where we can take $\Psi_s = 0$) and returns to zero at $\phi = \phi_{\max}$, forming a potential well. This means that we must have

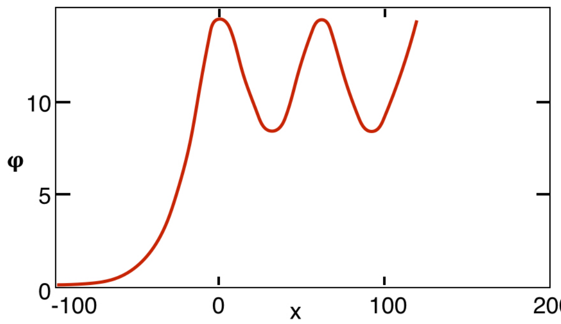

Figure 1: *Potential for T=20, M=1.2*

$$\Psi_s(\phi_{\max},\phi_{\max}) = 0 \tag{4}$$

with $\Psi_s < 0$ in the interval $0 < \phi < \phi_{\max}$. This is only possible within certain ranges of the electron/ion temperature ratio and the Mach number. If the Sagdeev potential was the same in the downstream region then we would obtain a solitary wave solution but it differs because of the absence of the reflected ion component. To obtain the potential profile we find a solution to Eqn (4), then integrate Poisson's equation upstream and downstream starting with the initial conditions $\phi = \phi_{\max}, \frac{d\phi}{dx} = 0$.

A typical solution is shown in Figure 1. The oscillatory profile downstream is typical of the behaviour we find, the amplitude of the downstream oscillations decreasing as the Mach number increases until the latter reaches a critical value beyond which a solution no longer exists. This is consistent with behaviour found by Forslund and Freidberg in computer simulations many years ago [6], where a laminar structure of the type found here, with only a few reflected ions, was found to go over to a more complex structure with almost all upstream ions reflected.

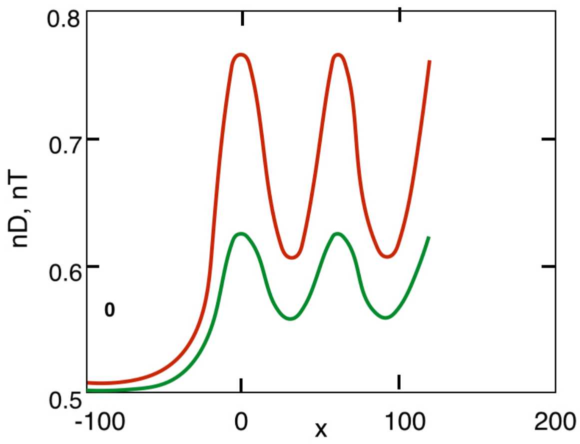

Figure 2: *Deuterium (red) and Tritium (green) densities for T=20, M=1.2*

If we assume an ion temperature of 100 eV then the maximum electric field for this case is around 3×10^{10} V/m while the length of the potential ramp is approximately 100 nm. With equal densities of incoming D and T ions from upstream, each normalized to 0.5, the ion densities in the shock are shown in Figure 2.

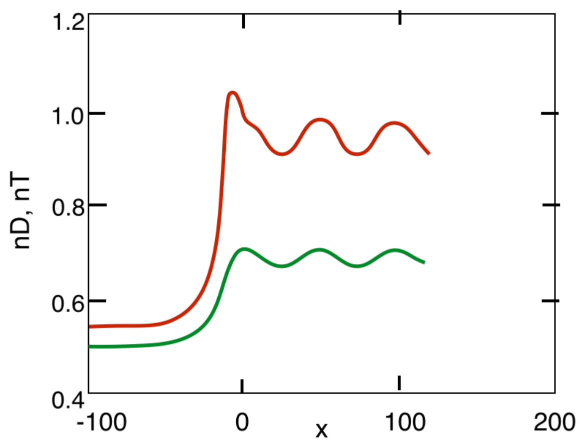

Figure 3: *As Fig.2 with M=1.3*

Upstream there is a small density difference resulting from higher reflection of the lighter deuterium ions. There is a much more important difference downstream, where the tritium ions are slowed down less than the deuterium and so have a lower density. If we increase the Mach number to 1.3, with the same temperature ratio we get the density profiles of Figure 3.

With this higher Mach number the downstream oscillations are smaller and the density difference in this region appreciably bigger. This results from the fact that the normalized $\phi_{\max}$ in this case is around 23 as compared to 15 in the previous case

In this paper we have shown that low Mach number collisionless laminar shock waves in ICF targets can generate large electric fields, of order 10^9 - 10^{10} V/m localized over distances of the order of 100 nm. With an initial mix of equal deuterium amd tritium fuel we show that such weak shocks can produce significant species separation. A density difference of the order of 25-30 percent is readily produced in the downstream region with shocks of Mach number not much above one, simply because the electric field produces a higher flow velocity, in the laboratory frame, of the lighter deuterium ions . The existence of such shocks in ICF targets could, therefore, have a significant effect on fusion yields.

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