Semi-analytic modeling of shock ignition

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Abstract

In the shock ignition scheme for inertial fusion confinement [1] the target is first imploded at low velocity and then, a high power laser spike launches a shock that increases in pressure during convergence and ignites the fusion reactions in the center of the target. The self-similar solution of Guderley is extended to describe the shock coupling to the hot-spot in the case of a finite shock Mach number. To fulfill the ignition conditions, the shock pressure must be higher than 3 Gbar at the inner face of the shell. We present also a quantitative analysis of the shock pressure amplification processes needed to reach such a pressure.

Introduction

Shock ignition (SI) is an alternative scheme for inertial fusion confinement [1]. First the shell is imploded at low velocity ($250-300$ km.s^{-1}) to compress the fuel. Then a shock is launched by a laser spike to bring the supplementary energy source needed to ignite fusion reactions in the hot spot. We present paper a theoretical analysis of the ignitor shock. In a first part we present a model describing the flow behind the ignitor shock in the the hot spot. An ignition criterion is expressed. We present in a second part the processes involved in the shock pressure amplification in the shell.

Shock coupling to the hot spot

We consider the converging phase, the rebound and the diverging phase of a shock wave of position $R_s(t)$ in a 1D spherical geometry. The flow is characterized by the density $\rho(r,t)$, the fluid velocity $u(r,t)$ and the local sound speed $c(r,t)$. These quantities obey the ideal gas equation of state $c^2 = \gamma p/\rho$, where γ is the specific heat ratio and p is the pressure. We use the Euler's equation to express the conservation laws of the mass, the momemtum and the energy without diffusive effects. The initial conditions correspond to a gas at rest $u = 0$, $\rho = \rho_0$, $c = c_0$. The discontinuities of the flow quantities at the shock front are described by the Rankine–Hugoniot relations. We impose a continuity condition on the flow quantities at the collapse time. Finally, we add the condition of a spherical symmetry at the center, requiring the fluid velocity to vanish there $u(0,t) = 0$.

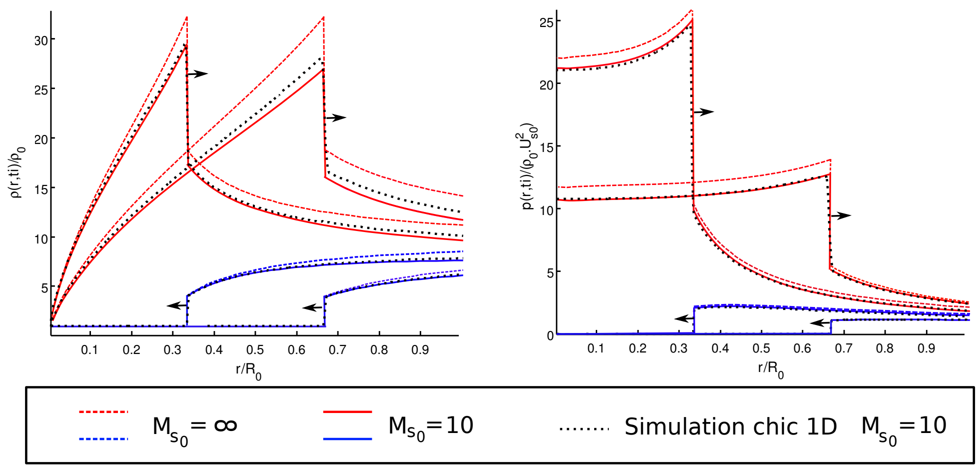

Figure 1: Pressure en density radial profiles

We introduce new independent variables defined by $x(r,t) = \frac{r}{R_s(t)}$, $\quad y(t) = \frac{c_0}{U_s(t)}$ and new dependant variables $\rho = \rho_0 G(x)\left[1+y^2G_1(x)\right]$, $\quad u = c_0\frac{x}{y}U(x)\left[1+y^2U_1(x)\right]$, $\quad c = c_0\frac{x}{|y|}C(x)\left[1+y^2C_1(x)\right]$. This transforms the partial differential equations of Euler into two systems of ordinary differential equations. The zeroth order solution G,U,C is the self similar solution of Guderley [2] valid for an infinite shock Mach number. The first order solution G_1,U_1,C_1 is a linear correction [3, 4]. The shock trajectory $R_s(y) = \kappa^{\pm}|y|^{-1/\lambda}\left[1+\frac{\lambda_1^{\pm}}{2\lambda}y^2+o(y^4)\right]$ is part of the problem. The parameters $\lambda,\lambda_1^{\pm}$ are iteratively calculated so that the solution does not undergo any singularity apart from the position of the shock. The correction for finite Mach number affects mainly the density and pressure of the flow after the rebound of the shock as it can be seen on the Figure 1.

This solution allows to define an analytical criterion for SI which takes into account the initial temperature of the hot spot. The ignition criterion is $\partial_t E_\alpha > \partial_t E_r + \partial_t E_c$ with E_α the energy deposited by the α particles, E_c the energy lost by electronic conduction and E_r the energy lost by radiation. We obtain a relation between the initial areal density of the hot spot and the initial shock velocity (see Figure 2). The shock velocity must be higher than $U_{s0} = 800$ km.s^{-1} with $<\rho R>_0 = 25$ mg.cm^{-2} to ignite the fuel. If we consider a hotspot of 50 µm this corresponds to a shock pressure $P_{s\text{fuel}} \approx \frac{3}{4}\rho_0 U_{s0}^2 = 1.6$ Gbar. Then the shock pressure at the inner face of the shell is $P_{s\text{shell}} = P_{s\text{fuel}} \times 2-3$ =3-5 Gbar.

Shock pressure amplification in the imploding shell

According to numerical simulations, the ignitor shock is seeded by an ablation pressure of 300 Mbar with a spike laser intensity of $5-10\times10^{15}$W.cm^{-2} [5]. Thus an amplification of the shock pressure by a factor higher than 10 is needed when it propagates into the imploding shell.

The total pressure amplification of the shock is given by $\chi = p_{sf}/p_{sini} = \chi_{imp}\chi_{shell}\chi_{coll}$. The

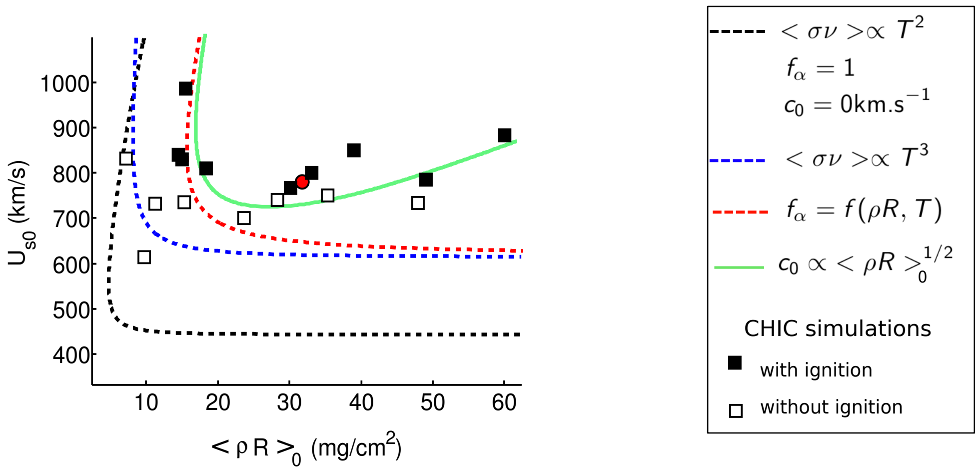

Figure 2: Ignition threshold

amplification factor χ_{imp} is equal to the overall shell pressure amplification during the shock propagation. The second amplification factor χ_{shell} is the shock pressure amplification in the shell comoving frame. It depends on the spatial profile of the shell, on the initial strength of the shock and on the shell thickness. The shock pressure decreases in a accelerated shell whereas it increases in a decelerated shell. As soon as the shell is decelerated, a returning shock is diverging in the shell and the ignitor shock collides with it. The shock pressure amplification through this collision is the third amplification factor χ_{coll}.

The figure 3 presents the total amplification factor in a HiPER implosion [5] depending on the shock timing and on the spike intensity. The unity isocontour is close to the dashed line delimiting the domain where the ignitor shock undergoes a collision with the returning shock. For earlier spike times the shock propagates only in an accelerating medium which induces a decrease of the shock pressure. For time later than the isocontour of unity, the shock pressure amplification increases quickly and reaches 500. This huge modification of the amplification is visible on a variation of 400 ps of the spike time. In this domain, the shock collides with the returning shock and propagates into a decelerated medium. The main reason why the total shock amplification factor reaches several hundred is that the overall shell pressure increases quickly in this time domain. Indeed, near the stagnation, several shocks and compression waves coming from the hot spot are transmitted to the shell and the mean pressure of the shell increases quickly.

Conclusions

We present here a new semi-analytic hydrodynamic model to describe the ignitor shock from its generation until fuel ignition. At the end of the implosion, the ignitor shock converges and diverges into the target hot-spot. The ignition of the fusion reaction is expected when the shock

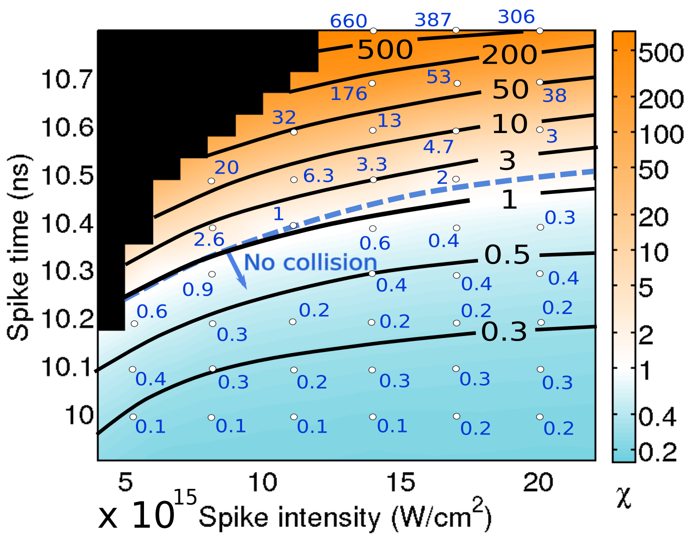

Figure 3: Shock pressure amplification depending on the spike timing and the laser intensity. The white dots represent simulation results.

exits the hot-spot. The self-similar solution of Guderley is extended to describe the shock coupling to the hot-spot in the case of a finite shock Mach number. To fulfill the ignition conditions, the shock velocity must be higher than 800 km.s^{-1} when it enters in the hot spot with an areal density higher than 25 mg.cm^{-2}. Considering a hot-spot of 50 μm, this means that the ignitor shock must reach a pressure higher than 3 Gbar at the inner face of the shell. According to numerical simulations, the ignitor shock is seeded by an ablation pressure of 300 Mbar with a spike laser intensity of $5-10\times10^{15}$W.cm^{-2} [5]. Thus an amplification of the shock pressure by a factor higher than 10 is needed when it propagates into the imploding shell. The shock propagates into a non inertial medium with high density and pressure radial gradient and an overall pressure increase with time. The collision with a returning shock coming from the assembly phase of SI enhance furthermore the ignitor shock pressure. The amplification of the shock pressure is very sensitive to the shock timing. It is higher than 10 on a window of 200 ps in a HiPER implosion.

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