The rate of particle acceleration at oblique shocks

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Introduction

The diffusive acceleration of charged particles at collisionless shock fronts (DSA) is widely thought to be responsible for the acceleration of cosmic rays in supernova remnants. It is well-known, however, that this mechanism is slow [1, 2]. At a strong, parallel shock the timescale for acceleration can be estimated as $8\kappa_{\|}/v_{\rm sh}^2$, where $\kappa_{\|}$ is the spatial diffusion coefficient along the magnetic field in the upstream plasma, and $v_{\rm sh}$ is the velocity with which the shock advances into this plasma. It is generally assumed that $\kappa_{\|} \geq r_{\rm g}c/3$, where $r_{\rm g} = pc/|e|B$ is the Larmor radius of a particle of momentum p and charge e in a magnetic field B, in which case the estimated timescale has a lower limit. For protons at the required energy of 1 PeV, this exceeds the age of a typical supernova remnant unless the magnetic field is substantially larger than that encountered in the interstellar medium. Even allowing for magnetic field amplification, it seems only marginally possible for supernova remnants to accelerate protons to this energy (for a recent review see [3]).

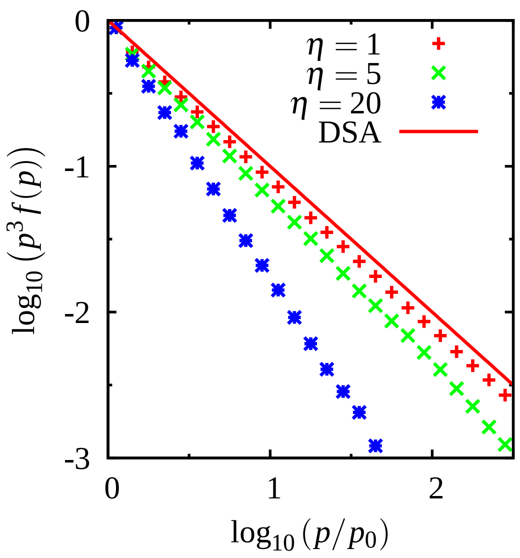

Figure 1: The time-asymptotic distribution of accelerated particles in momentum space at a perpendicular shock of compression ratio 4 and upstream speed $c/20$. Simulation results are shown for $\eta = 1$, 5, and 20 – see Eq. (2) – and compared to the prediction of DSA, which is independent of η.

In a seminal paper, Jokipii [4] noted that this difficulty is peculiar to parallel shocks. At an oblique shock, the diffusion coefficient controlling the timescale is that along the shock normal. Denoting by ψ the angle between the magnetic field and the shock normal, this coefficient is $\kappa = \kappa_{\|}\cos^2\psi + \kappa_{\perp}\sin^2\psi$. In weak turbulence, one expects $\kappa_{\perp} \ll \kappa_{\|}$ so that $\kappa \ll \kappa_{\|}$ for sufficiently oblique shocks, and the lower limit on the acceleration timescale is reduced.

This argument rests on the diffusion approximation, which holds when the particle distribution is almost isotropic. At non-relativistic, parallel shocks, the anisotropy indeed remains small, even for very low particle scattering rates. At relativistic shocks, on the other hand, it is well-know that the diffusion approximation fails. But it is less well-known that this can also occur in sufficiently oblique non-relativistic shocks, where the distortion of particle orbits caused by the jump in the magnetic field can cause a substantial anisotropy. This has recently been demonstrated by Bell et al [5], who found that the stationary power-law index softened as the obliquity of the shock increased, for shock speeds typical of young supernova remnants $\sim c/30$; in the diffusion approximation, the spectral index is independent of obliquity.

Here, we confirm this result for perpendicular shocks, using a different numerical technique. We also show that the failure of the diffusion approximation weakens the enhancement of the acceleration rate predicted by Jokipii [4]. Examination of the angular distribution at the shock reveals the strong anisotropy responsible for these results.

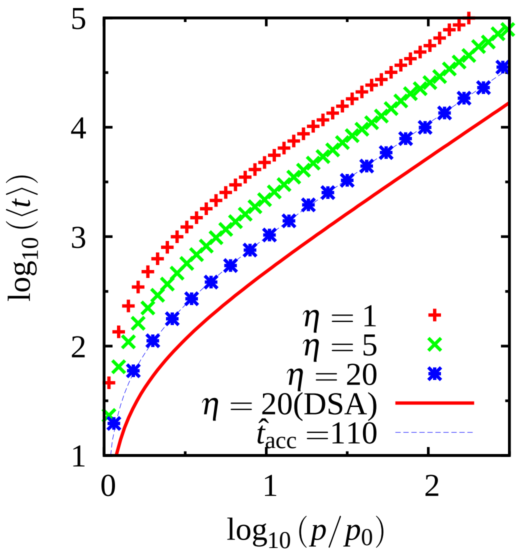

Figure 2: The average time taken for a particle to be accelerated at a perpendicular shock (same parameters as in Fig. 1) from p_0 to p, in units of $p_0/|e|B_{\mathrm{up}}$. The prediction of DSA for Bohm diffusion lies very close to the simulation results for $\eta = 1$. However, for $\eta = 20$, simulations indicate an acceleration time roughly double that predicted by DSA [4]

Method

We solve the fully relativistic kinetic equation for a phase-space distribution f of ultra-relativistic ($p \approx \gamma mc$) test particles undergoing continuous deflections by magnetic fluctuations that is modelled as isotropic diffusion in the direction of motion. In the local fluid frame (defined as that in which the scatterings are elastic and assumed to be an inertial frame) this reads

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla} f + \omega \frac{\partial f}{\partial \phi} = \frac{\nu_{\mathrm{coll}}}{2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}\right] \tag{1}$$

Here, $\vec{v}$ is the particle velocity, and θ and ϕ are angles in spherical polar coordinates. The magnetic field is assumed to be uniform and constant and lies along the z (polar) axis, $\omega = eB/\gamma mc$ is the angular velocity of the particle about the magnetic field and ν_{coll} is the the collision frequency. Note that the particle Lorentz factor γ and ω are both constant in the fluid rest frame. Using an expansion of the distribution in spherical harmonics, it is straightforward to show that the diffusion coefficients are

$$\kappa_{\parallel} = cr_{\mathrm{g}}\eta/3 \;\text{ and }\; \kappa_{\perp} = cr_{\mathrm{g}}\eta/\left[3\left(1+\eta^2\right)\right], \text{ with } \eta = \omega/\nu_{\mathrm{coll}} \tag{2}$$

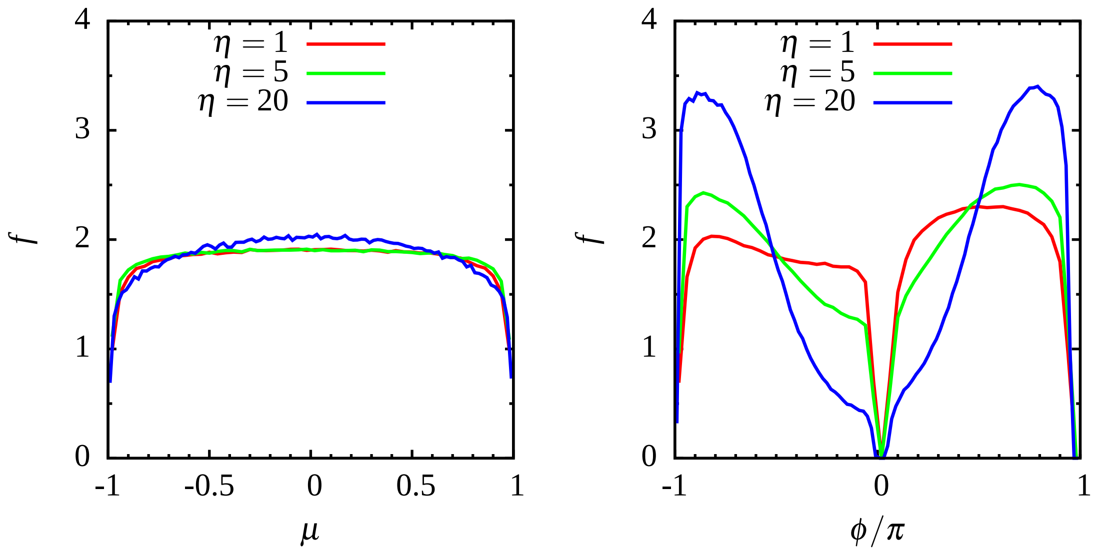

Figure 3: The angular distribution of particles at the shock front as measured in the upstream fluid frame. Whereas the pitch-angle distribution is reasonably uniform, there is a strong anisotropy in gyro-phase at larger η, which causes substantial deviations from the predictions of DSA.

In the following we assume $\eta \geq 1$ and independent of γ, i.e., the collision frequency scales with energy in the same way as the gyro frequency. In this case, the time-asymptotic distribution function at the shock contains no momentum scale other than those introduced by boundary conditions and is, therefore, a power-law $f \propto p^{-s}$ when these have little influence. The parameter η determines the degree to which particles are magnetized: $\kappa_{\parallel}/\kappa_{\perp} \gg 1$ for large η. For a perpendicular shock, κ is maximised for $\eta = 1$, when $\kappa = \kappa_{\perp} = \kappa_{\parallel}/2$. We shall refer to this case as Bohm diffusion.

Equation (1) is solved in the upstream ($B = B_{\rm up}$) and downstream half-spaces using a Monte-Carlo technique, allowing the sample trajectories to cross the shock front, without deflection or energy change. Trajectories start at the shock with momentum p_0 and propagate until they escape across a boundary placed far downstream. In common with [6], our method takes full account of anisotropy and non-conservation of the first adiabatic invariant (magnetic moment). It differs from [6], who considered large-angle scatterings, because Eq. (1) models the limit of small-angle scatterings, and because we include the effects of cross-field transport. Our approach is similar to that of [7, 8], but extends it to include time-dependence. It differs substantially from [9], who employ a guiding-centre approximation to the particle trajectory.

Results

For a perpendicular shock of compression ratio 4, propagating into the upstream at speed $v_{\rm sh} = c/20$, Fig. 1 shows the spectral index of the stationary (time-asymptotic) distribution. For $\eta = 1$ (Bohm diffusion) the spectrum is very close to the non-relativistic limit of $s = 4$. As η

increases, the spectrum softens, in agreement with [5], reaching $s = 4.8$ at $\eta = 20$.

When $\kappa \propto p$, the average time taken to accelerate from p_0 to p is, according to DSA, $\langle t \rangle = (p/p_0 - 1)\hat{t}_{\rm acc} p_0/|e|B_{\rm up}$, where the dimensionless timescale $\hat{t}_{\rm acc}$ is independent of p. As shown in Fig. 2, this functional behaviour holds also for oblique shocks. For $\eta = 1$, our simulations agree closely with the DSA value $\hat{t}_{\rm acc} = (4/3)\, c^2/v_{\rm sh}^2 \approx 533$. According to [4], for $\eta = 20$, DSA predicts $\hat{t}_{\rm acc} \approx 53$. However, a fit to our Monte-Carlo simulations gives $\hat{t}_{\rm acc} \approx 110$.

The reason for the failure of DSA is apparent from Fig. 3, which shows the dependence of f on the (cosine of the) pitch angle μ and the phase ϕ. Since the Monte-Carlo technique registers particles as they cross the shock front, it is not able to measure f at grazing incidence, $\phi = v_{\rm sh}/c$ and $\phi = v_{\rm sh}/c - \pi$. Nevertheless, it is clear that for $\eta = 20$ there is a large excess of particles moving predominantly in the negative x direction ($\phi = \pm\pi$). This is in the same direction as the drift caused by the increase in B across the shock front, and is consistent with an increase in particle density from upstream to downstream.

Jokipii originally speculated [4] that the diffusion approximation, and, hence, his prediction of the enhanced acceleration rate, should hold provided $\eta < c/v_{\rm sh}$. Subsequently, Achterberg & Ball [10] suggested the more restrictive condition $\eta < \sqrt{c/v_{\rm sh}}$. For a shock speed of $15{,}000\,$km/s, we find the acceleration rate to be a factor of two slower than Jokipii's prediction at $\eta = 20$ ($= c/v_{\rm sh}$), and a factor of 1.25 slower at $\eta = 5$ ($\approx \sqrt{c/v_{\rm sh}}$).

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