Mechanisms of drift wave based zonal flow transitions and bifurcations

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Introduction

Numerical studies of the interaction between drift waves on the one hand and zonal flows on the other have been performed in a self-consistent nonlinear sheared-slab resistive Hasegawa-Wakatani-based drift wave system, with the goal to examine zonal flow behavior patterns in the according regimes, including their emergence above a certain threshold ρ_{crit} (with ρ defined as the ratio between ρ_s, the ion sound Larmor radius, and $L_\perp$, the length scale of maximal drift wave growth for a given parallel shear length). For the first time in such a system, transport bifurcations have been observed. These correlate both with density corrugations and a secondary zonal flow asymmetry - leading to steepened negative (those in the electron diamagnetic drift direction) and flattened positive flows. Not only has this bifurcation containing two transport regimes been explained qualitatively, but it has also been shown why a threshold parameter for the emergence of zonal flows has to exist.

The numerical basis of this work has been provided by NLET, a two-fluid code from IPP [3] simulating a turbulent cold-ion sheared-slab resistive Hasegawa-Wakatani drift-wave system:

$$d_t n = d_t \nabla_\perp^2 \phi \quad (1)$$

$$\rho^{-3} d_t \nabla_\perp^2 \phi = -\partial_\parallel^2 (\phi - n) \quad (2)$$

The examined system is not just of academic interest. Drift wave turbulence is the most important ingredient in the high-gradient tokamak edge (in the vicinity of internal transport barriers such as the H-mode) and it even features in a range of other systems, including the atmospheres of gas giants (in that case, geostrophic modes take the place of the drift waves).

Two transport regimes

The general drift wave growth rate in the non-adiabatic, shearless case can be determined from eqns. (1) & (2) as $\gamma = \Im(\omega) \propto \left[k_\perp^2 + k_\parallel^2 \left(\frac{1}{k_\perp k_y} + \frac{k_\perp}{k_y}\right)^2\right]^{-1}$ with the approximation $\gamma = \omega*^2 / \omega_\parallel = k_\perp^2 / \left(k_\parallel^2/(\rho^{-3} k_\perp^2)\right) = \rho^{-3} k_\perp^4 / k_\parallel^2$, leading to a mixing length estimate for the turbulent transport, with the resulting diffusion coefficient $D = \gamma / \vec{k}_\perp^2$ inclduing $k_\perp$, a quantity which can be determined either via ρ_s or via $L_\perp$. The ρ_s-dominated high-ρ-regime, for example, leads to $D_{L_\perp}|_{k_\perp \widehat{=} \rho_s} = \frac{\gamma_{L_\perp}}{k_{L_\perp}^2}|_{k_\perp \widehat{=} \rho_s} \propto \rho^{-2}$ and $D_{\rho_s}|_{k_\perp \widehat{=} \rho_s} = \frac{\gamma_{\rho_s}}{k_{\rho_s}^2}|_{k_\perp \widehat{=} \rho_s} \propto \rho^0$.

The same can be found for the $L_\perp$-dominated low-ρ-regime as well. The transition between both regimes is located at approx. $\rho \approx 0.5$, (coinciding well with the onset of flow formation, as will be shown below). A simple relation between both scales can be derived: $\frac{D_{L_\perp}}{D_{\rho_s}} = \rho^{-2}$.

Notably, there is a strong correlation with a D_{ρ_s}-plateau in the high-ρ-regime, which holds up well under high-resolution numerical test runs.

ρ-depence

It is a considerable numerical challenge to achieve high values of ρ - which are a requirement for fully developed flows - due to resource demand scaling with ρ^3 (the transport times scale with $k^{-2}D^{-1}$ - or ρ^4 - while the drift wave scale is just proportional to ρ) as well as due to increasing convergence issues requiring numerous parameter scans. These simulations have thus been performed on the powerful Helios and HPC-FF clusters, yielding a near-perfect plateau for the diffusion coefficient (and thus also turbulent transport) in units of ρ_s.

Bifurcations in transport

Crucially, the high-ρ-regime revealed transport bifurcations (using the definition of a forking of one steady state into two above a certain critical threshold parameter, even though the two states can co-exist at different locations at the same time), yielding two stable density gradients as a result of the nonlinear drift wave simulations.

These bifurcations are first and foremost associated with density corrugations - correlating to two different (stationary) transport states, including regions of high gradients and low diffusivity close to the flows in the electron diamagnetic drift direction (the sharply concentrated negative flows) as well as lower gradients with heightened diffusivity around the broader positive flows. Thus, the bifurcations are accompanied by a pronounced asymmetry in zonal flows.

This flow structure typically emerges on an order of $\sim O(10^1)$ for $\rho \approx 0.6$, increasing approx. ten-fold for every doubling of ρ, explaining why earlier studies [1] with inadequate numerical resources have not been able to find these drift wave based flows.

Interestingly, the zonal flow wavelength of drift wave based flows - differing from their ITG-based counterparts - is not prescribed by the system but rather can be changed with little effort by simply promoting a certain zonal flow wavelength, with no apparent countermeasures taken by the system, at least for sufficiently large amplitudes.

Qualitative mechanism

The interaction between the drift waves and the zonal flows can be better understood by looking at the general drift wave action invariant N of the wave packet intensity as introduced in [2],

$$\partial_t N_{\vec{k}} = -\nabla_{\vec{x}} \left(N_{\vec{k}} \cdot \vec{v}_{gr,\vec{k}} \right) - \nabla_{\vec{k}} \left(N_{\vec{k}}(x) \cdot \dot{\vec{k}}(\vec{x},\vec{k}) \right) \tag{3}$$

The second term describes the (local) influence of the shear flow on the wave number [5] via $\dot{\vec{k}} = -\vec{\nabla}_x \vec{v} \cdot \vec{k}_0$). This leads to the observation that the turbulence is capable of repulsing negative flows, while attraction occurs in the case of positive flows. Accordingly, the flows act similarly to forcefields, changing radial drift wave wavenumbers, thus reducing transport (and therefore turbulence levels) in the vicinity of the negative flows.

Still, any equilibrium has to maintain the transport balance $\partial_x \Gamma(x) = 0$, necessitating increased gradients around the negative flows to counterbalance this drift-wave-related reduction of transport. In the same manner, gradients around the positive flows are reduced, yielding stepped density gradients (with increased drift mode generation rates near the flow minima because of to the steepened gradients). But since the drift waves are being repelled by the negative flows, they exhibit Reynolds stresses which fuel the flow in return. The associated drift wave carry-off leads not only to a deepening of the negative zonal flows but also to a broadening of their respective positive counterparts, yielding the flow asymmetry described above.

Stress-mediated emergence of zonal flows

The actual emergence of zonal flows - no matter their secondary features - can be understood by examining the Reynolds stresses. If a flux surface average of the potential equation (2) is performed, followed by integrating over x, a zonal flow evolution equation

$$\partial_t v(x) = \partial_x < \partial_y \phi \partial_x \phi >= -\partial_x < v_x v_y >=: -\partial_x \Pi_{xy} = -\Pi'_{xy}[\partial_x v_0] \partial_x^2 (v - v_0) \tag{4}$$

based on the radial divergence of the Reynolds stress is reached.

The right-most side of the equation - an expansion around the equilibrium flow $v_0(x)$ - only holds true in the limit of large flow wavelengths (when compared to the turbulence length scale) when the stress is nothing but shearing rate dependent.

According to (4), extremal values of the shearing rate $\partial_x v = \pm u_0$ correspond to zero Reynolds stress. Similarly, $\pm \Pi'_{xy}[\pm u_0] > 0$ is a necessary condition for flow stability while only for $\Pi'_{xy}[0] < 0$ flows can be excited from random noise.

Wave-kinetic theory shows [7, 8] that zonal flow shear leads to (in-phase) turbulent stresses mostly by changing the x-wavenumber (k_y remains constant) of drift wave packets $\omega(\vec{k},\vec{r})$ via

$$\partial_t k_x = -\partial_x Re\omega = -\partial_x \omega_{Doppler} \tag{5}$$

where $\Pi_{xy} =< v_x v_y >= - < \partial_y \phi \partial_x \phi > \approx -k_x k_y |\phi|^2$.

This fulfills the above-mentioned condition for zonal flow growth, explaining the $\rho > 0.5$-case. However, the difference of the flowless regime below has to be motivated separately.

Flow transition

The zonal flow regime transition can almost solely be traced to two influence factors: A drift wave repulsion effect (independent of ρ) and the effect of the resonant surfaces. The overall result is a mixture of both repulsion and amplification of drift waves, where the repulsion grows weaker for increasing values of ρ.

Due to the repulsion effect, the drift waves get accelerated up the zonal flow gradient. For small values of ρ, the repulsion effect of the resonant surfaces increases (aided by dispersion broadening). Thus, the drift waves located at the low-v_y-side of these resonant surfaces spend more time in the close vicinity of them, yielding increases growth times for these waves, and thus larger overall amplitudes than those at the high-v_y-side. The resulting Reynolds stress is predominantly negative, leading to zonal flow damping.

Again, for high ρ - with the transition occurring somewhere around $\rho \approx 0.5$ -, the acceleration effect up the flow gradient dominates over the resonant surface effect, yielding predominantly positive Reynolds stress and thus the aforementioned amplification of zonal flows.

Summary

We have worked with drift wave turbulence in a sheared-slab system capable of producing self-consistent zonal flows. Under these conditions, we have been able to find a robust transport bifurcation phenomenon in the flow-yielding high-ρ-regime, with two distinctly different transport states correlating with a flow pattern asymmetry. A qualitative explanation of these bifurcations, the emergence of these flows and the transition to the flowless regime has been given, making use of transport balance arguments, wave-kinetic theory and the drift wave acceleration as well as resonant surface effects, respectively.

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