

Analysis of particle tracer trajectories in resistive pressure-gradient-driven turbulence in cylindrical geometry

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Introduction

The use of particle tracers is very helpful in trying to understand turbulence-induced transport in magnetically confined plasmas. We apply the continuous random walk (CTRW) approach [1] to the analysis of the tracer trajectories. In doing so, we decompose the tracer trajectories in radial flights and use different criteria in order to classify those flights. Some correspond to trappings of the particle and some jumps between trappings. The next step is to understand the probability distribution of the trapping times of the tracers and of the flights between trappings.

Here, we study the distribution of the trapping times of the tracers. To interpret the results from the tracer calculations we start first with a resistive pressure-gradient-driven turbulence model in cylindrical geometry in which the basic picture of the instabilities and transport is simpler, but certainly not easy to understand.

In a previous paper [2], we have done a topological analysis of the flow structures for the same dynamical problem considered here. Now, we can relate these topological structures to the type of tracer trappings observed in the present calculations. Using the results of the topological analysis, we are able to build a probabilistic model to interpret the tracer results.

Topological analysis of the flow structures

We study the pressure-gradient-driven turbulence in cylindrical geometry by means of a reduced set of resistive MHD equations in the electrostatic limit. The geometry is that of a periodic cylinder, with minor radius a and length $L_0 = 2\pi R_0$. We use a coordinate system (r, θ, ζ) , in which r is the radius of the cylindrical surface, θ is the poloidal angle, and $\zeta = z/R_0$, where z is the coordinate along the axis of the cylinder, so ζ is an effective toroidal angle when the cylinder is bent in a torus. The $E \times B$ velocity is written in terms of the electrostatic potential: $\mathbf{V}_\perp = -\nabla_\perp \Phi \times \mathbf{z}/B_0$. The model consists of two equations, the perpendicular momentum equation for the electrostatic potential evolution, and the equation of state for the pressure evolution. Dissipative terms are included in both equations. The plasma considered here is a model of a configuration of the Large Helical Device (LHD) [3]. Details of the equations, configuration, numerical methods, and main parameters can be found in Ref. [2].

All the information on the turbulence flow is contained in the electrostatic potential Φ . We define a cubical space $N_r \times N_\theta \times N_\zeta$ covering the cylinder. At a fixed time t , we define a flow structure as the set of points such that $\Phi(r, \theta, \zeta, t) \geq \Phi_0 \max(\Phi)$, for a suitable constant Φ_0 , with $\max(\Phi)$ being the maximum value Φ at time t . Therefore, Φ_0 gives a fraction of the maximum value of Φ and $0 \leq \Phi_0 \leq 1$. The main finding of Ref. [2] was that the structure of the flow is filamentary. The filaments are vortices that are linked to the rational surfaces. Some of these filamentary vortices close on themselves forming toroidal knots. These are the cycles and they are normally located at the lowest rational surfaces. At the other low rational surfaces the filaments are broken and we characterise them by their length. Probably the most remarkable property that we have observed is the lognormal character of the distribution of filament lengths [2].

Relation between flow topology and tracer transport

To study the transport properties, we investigate the time evolution of pseudo-particle tracers. The equation of motion for the tracers is

$$\frac{d\mathbf{r}}{dt} = -\frac{\nabla_\perp \Phi \times \mathbf{z}}{B_0} + V_0 \mathbf{b},$$

where V_0 is a constant velocity along the field lines. As the tracer particles move by the turbulent flow, we consider the particles being trapped for certain periods of times in the flow filaments and then taking steps or flights between trapping times. This way of looking at the tracer particle motion allows us to connect with the CTRW transport approach.

To calculate the probability distribution of the trapping times and the flights we have to first define both of them in the context of our numerical calculations. Here, we are interested in the radial transport and for this reason we consider flights only in the radial direction. We say that a particle tracer performs a flight while it moves on a trajectory keeping the same sign of the radial component of the velocity. Then, we consider a tracer to be trapped if successive radial flights vary less than a given percentage. Normally a value of 30% has been used.

For the analysis of the flow structures [2], we considered cylindrical layers with constant radius. In the (θ, ζ) -plane, the filaments cross the area with the slope corresponding to $q(r)$. To visualise the topological structures and calculate their widths, we do first a transformation of the poloidal angle θ to $\theta + \zeta/q$. With this transformation, the filaments go in the vertical direction. Then, we project the structures to the $\zeta = 0$ plane. Fig. 1 shows the resulting projection of flow structures in the (r, θ) -plane. Also it is shown (in blue) the trajectory of a tracer. The tracer is most of the time trapped at different structures and occasionally jumps between them.

We have studied the evolution of tracers for three different values of V_0 . The initial tracer positions are randomly distributed in the cylinder, and we follow the trajectory of 10^5 tracers till the end of the calculation and accumulate the data. This data is analysed to identify the portion of the trajectories that the tracers remain trapped.

For each case, we have two sets of data on the trapping. There is one set for the trappings that do not reach the end of the calculation, that is, a set of data in which the trapping phase is completed. There is another set in which the tracers were still trapped the last step. In this last set we have tracers that are trapped practically during the full length of the calculation.

We have calculated the probability distribution function (PDF) of the trapping times for the different cases studied. If we re-scale the trapping times by multiplying by V_0 , we have practically the same dependence for the tail of the PDF, as can be seen in Fig. 2. This indicates that most of the trappings that are completed during the calculation correspond to tracers trapped on broken filaments, including possible multiple trappings (see Fig. 1). The PDF is then a function of the filament length (product of V_0 by the trapping time), and has a lognormal character, like the distribution of filament lengths. The sharp increase at the end of the distribution corresponds to the tracers which trapping period has not finished by the end of the calculation.

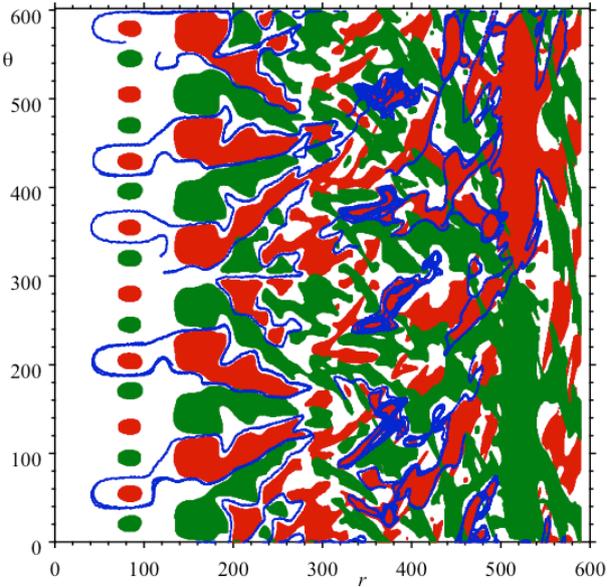


Figure 1: Projection of flow structures. Points for which $\Phi \geq 0.1 \max(\Phi)$ are in red and points for which $\Phi \leq -0.1 \max(\Phi)$ are in green. The trajectory of a tracer is shown in blue.

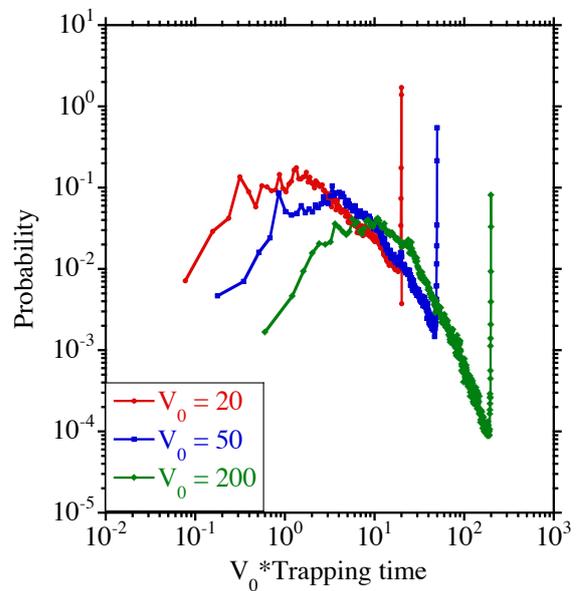


Figure 2: PDF of the trapping times multiplied by V_0 for the three cases studied.

Since the tracers are trapped on the flow structures, one expects that the radial flights of the tracers during trappings will be related with the radial width of the flow structures. Fig. 3 shows that the distribution of the radial width of the flow structures describes very well the distribution of the averaged radial flight of the tracers during a trapping period.

Probabilistic model for the trapping of tracers

We have developed a model in the line of the CTRW approach for simulating the trapping of tracers. Walks are defined along resonant field lines. For each walk, we take a step δr , which is the radial flight of the tracer when trapped.

For each tracer, the initial radial and poloidal locations are chosen randomly. For the radial step size, δr , we use the distribution of the radial width of the flow structures. For the filaments, the main parameter is the length of the filament along the tracer moves. In this case, we use the lognormal distribution of filament lengths.

The model has two parameters: 1) p_0 , the probability that a tracer on a filament jumps to another, and 2) p_1 , the probability of a tracer to detrap in a given step.

By choosing suitable parameters p_0 and p_1 we have a reasonable description of the distribution of trapping times and number of flights per trapping.

Conclusions

We have studied the trapping of tracers in a turbulent field in cylindrical geometry. The trapping of tracers, which do not remain trapped at the end of the calculation, seems to be due to trapping on finite size filaments including multiple possible trappings. The cycles seem to play a role for the tracers that remain trapped very large times, larger than the calculation time.

References

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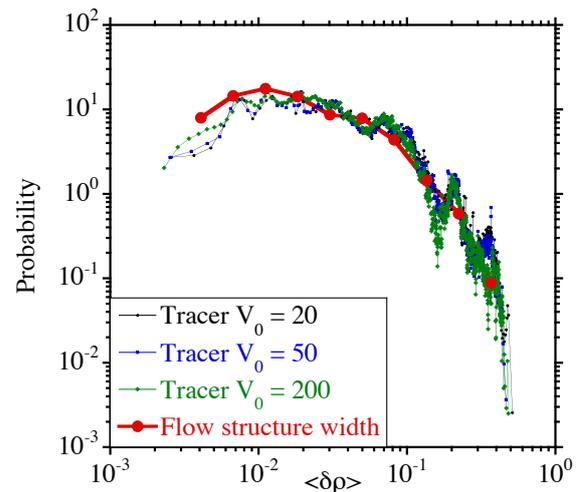


Figure 3: PDF of the averaged radial flight during trappings for the three cases studied, and of the radial width of the flow structures.