Dynamic equilibria and MHD instabilities in toroidal plasmas with non-uniform transport coefficients

S. Futatani[1], J. A. Morales[1,2], W. J. T. Bos[1]

[1] *LMFA, CNRS UMR 5509, Ecole Centrale de Lyon, France*

[2] *CEA, IRFM, Centre de Cadarache, 13108 Saint-Paul-Lez-Durance, France*

Introduction

Plasmas confined by toroidal magnetic and electric fields are generally considered the best candidates to succeed sustainable nuclear fusion. Tokamaks and Reversed Field Pinches (RFPs) are two of such configurations which are currently investigated intensively. In the ideal case the plasmas in such reactors would remain quietly confined within the magnetic field in order to allow their core to reach the temperature needed for thermonuclear fusion.

Indeed, it has been common use to start a tokamak description considering a force-free static equilibrium. Such an equilibrium only exists if the pressure forces are balanced by the Lorentz-forces that originate from the imposed magnetic and electric fields. In straight cylinder configurations such force-free states can be easily defined, considering for instance z-pinch and θ-pinch devices [1]. Even though such geometries can be subject to magnetohydrodynamic (MHD) instabilities, a force-free state can be defined. This changes in toroidal geometry.

In the simplest case in which the toroidal electric field is generated by a central solenoid, without external current drive, and where the toroidal magnetic field is induced by the poloidally orientated coils, ignoring ripples and other details, the imposed electro-magnetic fields have a simple form. It was shown in previous studies [2, 3, 4, 5] that for this case, assuming uniform electric resistivity, such an equilibrium is not possible. These studies showed, following an increasing level of complexity, that the velocity can never be zero if the current density is linked to the electric field by Ohm's law. The present work builds upon the results of these studies, increasing by one step the complexity, considering spatially non-uniform electric resistivity and viscosity profiles. Indeed, in practice, strong pressure, density and temperature gradients will influence the local values of the viscosity and resistivity in the plasma. The present approach takes this into account in the coarsest way, by defining profiles as a function of the minor radius. In particular will we show how different types of dynamic equilibrium and MHD instabilities appear as a function of the strength and the spatial distribution of the transport coefficients.

MHD equations

In the magnetohydrodynamics approximation, plasma is modeled as a charge-neutral electromagnetic conducting fluid. In the incompressible description, the dynamics are given by

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{j} \times \boldsymbol{B} \tag{1}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \tag{2}$$

$$\boldsymbol{E} = \eta \boldsymbol{j} - \boldsymbol{u} \times \boldsymbol{B}, \tag{3}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{4}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{5}$$

with the current density $\boldsymbol{j} = \nabla \times \boldsymbol{B}$ and the pressure p. These equations consist of dimensionless values, using the toroidal Alfven speed $C_A = B_{ref}\sqrt{\rho\mu_0}$ as a reference velocity, ρ is the mass density and μ_0 is the magnetic permeability constant. The stress tensor σ_{ij} is given by

$$\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{6}$$

In the case of a spatially uniform viscosity and density and resisitivity, equations (1) (2) are simplified to

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu \Delta \boldsymbol{u} + \boldsymbol{j} \times \boldsymbol{B} \tag{7}$$

$$\frac{D\boldsymbol{B}}{Dt} = \boldsymbol{B} \cdot \nabla \boldsymbol{u} + \eta \Delta \boldsymbol{B} \tag{8}$$

In the following two cases will be considered:

1. Constant magnetic-resistivity η_{const} and kinetic-viscosity ν_{const},

2. Space dependent magnetic-resistivity $\eta(r)$ and kinetic-viscosity $\nu(r)$.

The profile of the magnetic-resistivity induces the profile of the current density through Ohm's law (Eq.3) if $\boldsymbol{u} = 0$ which assumes a scalar equilibrium state. The total toroidal current is fixed, i.e. $\int j_{\eta(r)} dS = \int j_{\eta(const)} dS$ with S a poloidal cross-section, so that the work allows us to investigate the influence of the profile of the magnetic resistivity η on the dynamics, for a given toroidal current.

Simulation results

The calculations are performed for several toroidal magnetic field values B_{tor}, ranging from 0.6 to 0.05. If the toroidal magnetic field B_{tor} is large enough, such as $B_{tor} = 0.6$, the evolution of the magnetic field fluctuations is small and the safety factor q does not change much either.

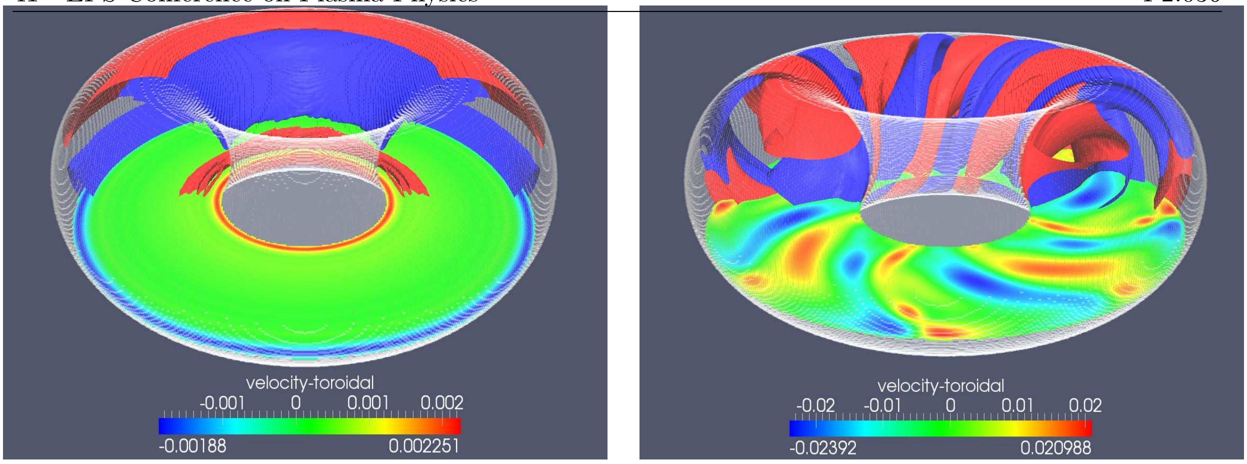

Figure 1: Contour lines of 1/5 of maximum toroidal velocity (red contours show positive velocity and blue contours show negative) at the steady state of the Lundquist number 2000 (M=2000) simulations. (Left) B_{tor} = 0.6 and (Right) B_{tor} = 0.05.

By decreasing the toroidal magnetic field B_{tor}, to 0.05, MHD instabilities induce an important poloidal flow, and the dynamo process increases the toroidal magnetic field B_{tor} in the core of the plasma. According to the evolution of B_{tor}, the profile of the safety factor at the steady state will be changed from the initial profiles. The profile of the safety factor becomes reversed-shaped, as is generally observed in Reversed Field Pinch (RFP) configurations. Figure 1 shows the contour lines of 1/5 of the maximum toroidal velocity at the steady for B_{tor} = 0.6 and B_{tor} = 0.05. If the toroidal magnetic field B_{tor} is large enough ($B_{tor} = 0.6$), the magnetic field and the velocity field are axi-symmetric along the toroidal direction. In the RFP-like regime, the plasma becomes unstable and the velocity field forms a helical structure around the toroidal axis.

Figure 2 shows the velocity profile of the 'uniform magnetic-resistivity η_{const} and kinetic viscosity ν_{const}' and the 'space-dependent magnetic-resistivity $\eta(r)$ and kinetic-viscosity $\nu(r)$' cases for their most unstable values. In the $\eta = \nu = const.$ case, a plasma instability is observed in the whole plasma domain, from the core to the edge. The clear helical structure of the velocity field is best observed in the core plasma. The helical structure is also to a lesser extent observed in the edge plasma. However, in the resistivity-profile case, the plasma instability is mostly concentrated in the core plasma. Furthermore, the velocity field in the core plasma is much stronger than for the case of uniform-resistivity. This is due to the non-uniformity of the resistivity profile, which leads to a modified current profile in the plasma. The larger current density in the core induces a stronger poloidal magnetic field, triggering instabilities which lead to large magnetic perturbations in the core plasma. As a consequence, there is a non-uniformity of the plasma instability in space.

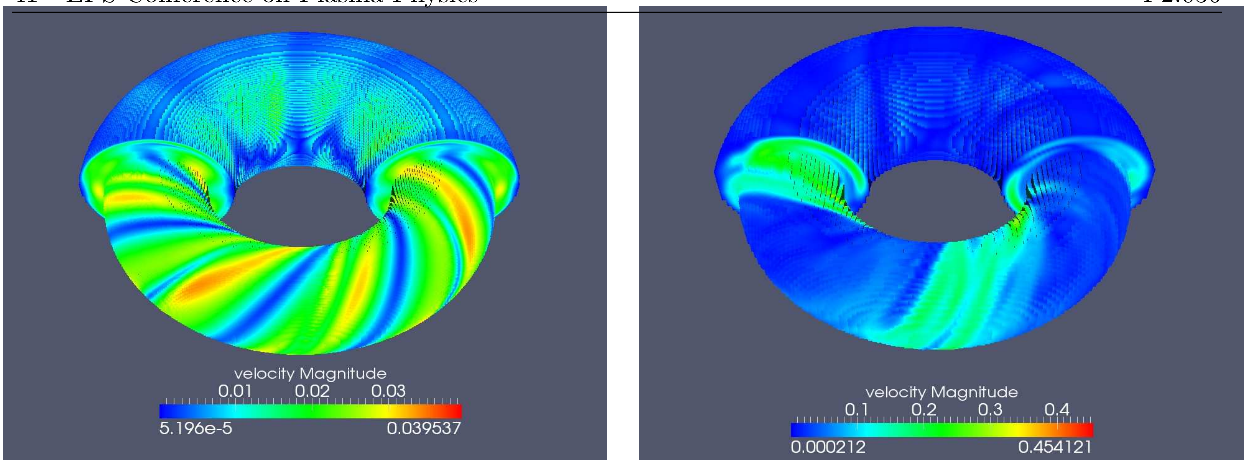

Figure 2: Velocity profile of (Left) the constant magnetic-resistivity η_{const} and constant kinetic-viscosity ν_{const} and (Right) the space-dependent magnetic-resistivity $\eta(r)$ and kinetic-viscosity $\nu(r)$ at their most unstable values.

Conclusions and perspectives

In the present work it is shown how a spatial inhomogeneity of the viscosity and resisitivity coefficients influences a toroidal equilibrium. Both parameters in the stable, tokamak-like regime and unstable, RFP-like regime are considered. In the full paper [6], simulations will be performed with high-resolution and longer simulation times to study the parametric dependence of Lundquist number (M) on the toroidal velocity, the poloidal velocity, etc. In particular, it will be shown that non-zero velocity fields are always present. These velocities increase as a function of the Lundquist number in the RFP regime, whereas they decrease in the tokamak regime.

References

[1] J. Freidberg, Rev. Mod. Phys. 54, 801 (1982).

[2] D. Montgomery and X. Shan, Comments on Plasma Physics and Controlled Fusion 15, 315 (1994)

[3] J. Bates and D. Montgomery, Phys. Plasmas 5, 2649 (1998).

[4] L.P. Kamp and D.C. Montgomery, Journal of Plasma Physics 70, 113 (2004)

[5] Jorge A. Morales, Wouter J. T. Bos, Kai Schneider, and David C. Montgomery, Phys. Rev. Lett. 109, 175002 (2012).

[6] in preparation.