

Competition between trapped ion and trapped electron instabilities

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Introduction.

Low frequency turbulence developing from micro instabilities is responsible for the phenomenon of anomalous radial energy transport in magnetically confined fusion plasmas. Among these instabilities, ion temperature gradient (ITG), interchange instabilities, and trapped electron modes (TEM) may play an important role in explaining the anomalous heat and particle transport observed in tokamaks. These instabilities are driven by ion and electron equilibrium gradients [1][2][3][4]. ITG seems to be responsible of the anomalous ion heat transport whereas the TEM turbulence [5] drives electron particle and heat transport, and their interactions may play a non-negligible role in determining the whole properties of turbulent plasma transport [6]. In order to investigate this turbulent transport one must consider a gyrokinetic approach. Furthermore, capturing both micro-turbulence properties and self-organization properties on meso time and spatial scales including large scale transport events requires flux-driven (without scale separation between equilibrium and fluctuations) long simulation runs. In the following we consider a model with trapped kinetic ions and electrons which captures all these features in a reduced set of equations and which allows one to cover both TIM/TEM regimes simultaneously. The influence of ratio of the ion to the electron temperatures will be investigated through the linear analysis of the model. Finally, a first validation of the non-linear TERESA code will be presented. This work is a preparatory step towards full-f 5D gyrokinetic simulations including electrons and ions.

Model equations.

In order to derive the kinetic model describing the trapped electron and ion modes in a Tokamak, we assume an axisymmetric magnetic toroidal configuration and use the Boozer-Clebsch representation of the magnetic field [7] :

$$\vec{B} = \vec{\nabla}\psi \times \vec{\nabla}(\varphi - q\theta) \quad (1)$$

with q the safety factor defined by $q(\psi) = \frac{\vec{B} \cdot \vec{\nabla}\varphi}{\vec{B} \cdot \vec{\nabla}\theta}$, θ and φ are respectively the poloidal and the toroidal angular coordinates. ψ is the poloidal magnetic flux normalized to 2π . Hereafter, the radial coordinate and the poloidal flux coordinate are used indifferently : $\{r, \theta, \phi\} \leftrightarrow \{\psi, \theta, \phi\}$.

In such a configuration, trapped particle motions are characterized by very different time-scales ($\omega_c \gg \omega_b \gg \omega_d$), where ω_c is the cyclotron frequency, ω_b the back and forth frequency (or bounce motion) and ω_d the slow toroidal precession frequency. A dynamical system that shows different periodic motions is the ideal framework for using action and angular variables [9] [10][11] [12]: $(r, \theta, \phi, v_r, v_\theta, v_\phi) \rightarrow (\mu, E, \psi, \alpha_1, \alpha_2, \alpha_3)$ where μ (the magnetic moment), E (the kinetic energy) and ψ are invariants and α_i are angles linked to the cyclotron phase φ_c , the poloidal and toroidal angles (θ, φ) [8]. In order to derive a kinetic model describing TIM and TEM instabilities we use the gyro-average (defined by the \mathcal{J}_{0s} average operator¹) over both the cyclotron motion and the bounce motion to write the Vlasov equation for each species. This approach is justified provided the time scales associated with these two motions are much shorter than the one characterizing the precession drift. The final model accounts for two parameters (μ, E) in the 2D space (ψ, α) which is close to the usual (r, ϕ) space. After normalization [12] the Vlasov equation for ions and electrons then writes :

$$\begin{cases} \frac{\partial \tilde{f}_e}{\partial \hat{t}} - [\mathcal{J}_{0e} \hat{\Phi}, f_e] - \hat{E} \hat{\Omega}_d \frac{\partial f_e}{\partial \alpha} = 0 \\ \frac{\partial \tilde{f}_i}{\partial \hat{t}} - [\mathcal{J}_{0i} \hat{\Phi}, f_i] + \hat{E} \hat{\Omega}_d \frac{\partial f_i}{\partial \alpha} = 0 \end{cases} \quad (2)$$

with $\hat{\Omega}_d$ linked to the precession frequency which does not depend on ψ and on the electric charge of the species. The quasi-neutrality constraint ensures the self consistency of the model. This constraint reveals particularly delicate to derive analytically in the present model in the framework of gyro-bounce-averaged approach. After normalization, the quasi-neutrality constraint writes ² [12] :

$$\frac{2}{\sqrt{\pi} \hat{n}_{eq}} \left(\int_0^{+\infty} \mathcal{J}_{0i} f_i \hat{E}^{1/2} d\hat{E} - \int_0^{+\infty} \mathcal{J}_{0e} f_e \hat{E}^{1/2} d\hat{E} \right) = \frac{1}{\hat{T}_{eq,i}} [C_{ad}(\hat{\Phi} - \varepsilon_{\Phi} \langle \hat{\Phi} \rangle) - C_{pol}(\bar{\Delta}_i(\hat{\Phi}) + \tau \bar{\Delta}_e(\hat{\Phi}))] \quad (3)$$

C_{ad} and C_{pol} are constants linked to the passing particles which are assumed to respond adiabatically and the polarization due to the average.

Linear analysis of the system.

In order to study the linear behavior of the set of equation (2-3) for TIM and TEM modes we write the dispersion equation [12] :

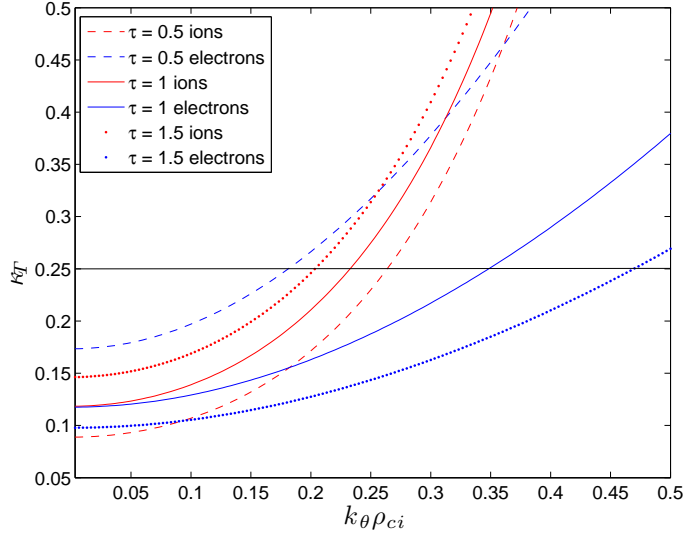
$$\mathcal{D} = C_n - \int_0^\infty \frac{[\kappa_n + \kappa_T (\xi - \frac{3}{2})]}{\Omega_d (\xi - x)} \mathcal{J}_{n,i}^2 e^{-\xi} \xi^{\frac{1}{2}} d\xi \tau - \int_0^\infty \frac{[\kappa_n + \kappa_T (\xi - \frac{3}{2})]}{\Omega_d (\xi + \tau x)} \mathcal{J}_{n,e}^2 e^{-\xi} \xi^{\frac{1}{2}} d\xi = 0 \quad (4)$$

¹ $\mathcal{J}_{0s} = J_0(n\rho_{c,s})J_0(k\delta_{b,s})$ with $\rho_{c,s}$ and $\delta_{b,s}$ are the normalized Larmor radius and the normalized banana width, n (resp. k) the toroidal (resp. radial) mode number and J_0 the 0^{th} Bessel function of the first kind.

²with the Poisson's brackets defined by $[f, g] = \partial_\alpha f \partial_\psi g - \partial_\alpha g \partial_\psi f$.

with $C_n = \frac{\sqrt{\pi}}{2} \left(C_{ad} + C_{pol} \left[n^2 (\rho_{c,i}^2 + \rho_{c,e}^2) + k^2 (\delta_{b,i}^2 + \delta_{b,e}^2) \right] \right)$ that contains the coefficients corresponding to the passing particles and the polarization term, $\kappa_{n,T}^{-1} = (\partial_\psi \log(n_{eq}, T_{eq}))^{-1}$ the density and the temperature gradient lengths and τ the ratio of the ion to the electron temperatures.

In Fig. 1 the linear instability threshold in terms of critical electron or ion temperature gradients $\kappa_{T,crit}$ (at vanishing density gradient $\kappa_n = 0$) is plotted against $k_\theta \rho_{c,i}$ for different electronic temperatures with the plasma parameters given in Table 1. Notice that TEM threshold exceeds TIM threshold as long as τ becomes smaller than unity.



$\delta_{b,i}$	0.1
$\delta_{b,e}$	2.10^{-3}
$\rho_{c,i}$	0.01
$\rho_{c,e}$	2.10^{-4}
c_n	0.1
$\hat{\Omega}_d$	1
$T_{i,eq}$	1
k	1
κ_n	0

Table 1: normalized parameters used for the linear analysis.

Figure 1: Linear instability threshold κ_T as a function of $k_\theta \rho_{c,i}$ for $\kappa_n = 0$ and $\tau = 0.5, 1, 1.5$.

Non-linear TERESA code.

In order to investigate the nonlinear effect of trapped electrons, a kinetic electron response has been implemented in the nonlinear semi-lagrangian TERESA code [13]. A first validation of this code is performed.

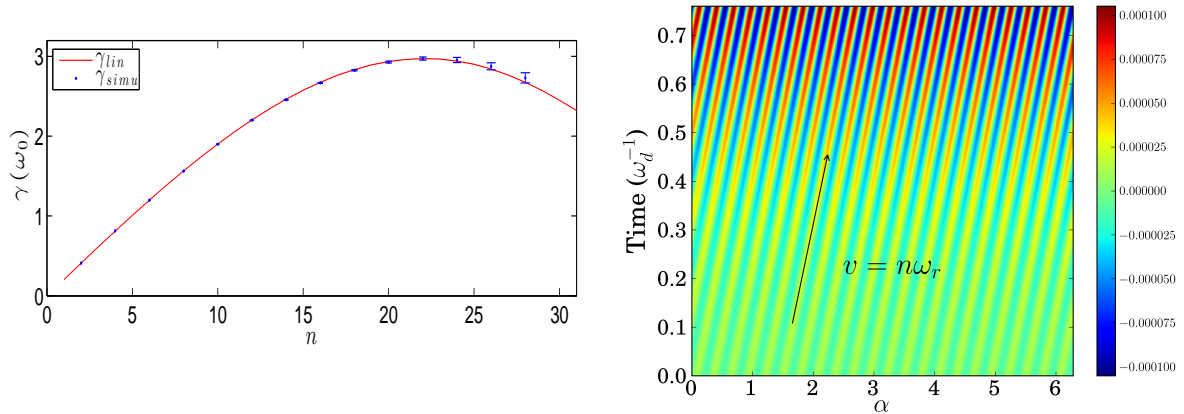


Figure 2: Comparison between linear growth rates given by the dispersion relation and TERESA code (left). Electric potential plotted against time and α (right).

In Fig.2 (left) the linear growth rate of 14 toroidal n modes obtained from TERESA linear simulations is found in perfect agreement with the one calculated from the dispersion relation, eq.(4). In Fig.2 (right) the electric potential is plotted against time and α . The phase velocity (real frequency) can be evaluated from this figure, and is found to be in a very good agreement with the one given by the relation dispersion.

Conclusion.

The trapped ion and electron driven modes have been studied by solving linearly a Vlasov equation averaged over the cyclotron and bounce motion of trapped particles. This model allows one to reduce the dimensionality of the dynamical system. The distribution function depends on the radial coordinate and the precession angle of trapped particles and is parametrized by the energy and pitch angle. The accuracy of the model has been verified with an exact solution in the marginal case. The validation of the non linear code TERESA has been performed for TIM modes in the linear phase. The same validation for TEM modes is in progress. The study of nonlinear interactions between TEM and TIM modes will be investigated.

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