# High mode number kink test of local gyrokinetic simulations

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**Introduction:** Development of kinetic modeling capabilities for stability and transport in turbulent tokamak plasmas is crucial for making reliable predictions for future fusion devices. Gyrokinetic simulation codes normally only consider gradients of a Maxwellian distribution as drives for instabilities and assume that the electron and ion distribution functions are Maxwellians with the same drift velocity. Therefore, the parallel electron current driven by the induced electric field in a tokamak that can drive or modify instabilities is normally not retained in these simulations. To retain the current gradient as a drive for instabilities the electrons must be allowed to have the proper self-consistent parallel flow with respect to the ions. When this electron flow becomes comparable to the ion thermal speed in an inhomogeneous, finite beta tokamak plasma the shear Alfvén wave solutions of the electromagnetic gyrokinetic equation can become nearly purely growing kink modes, but do so outside the regime of validity of a local analysis assuming a linear variation of the safety factor over the radial domain of interest. Nonetheless, this spurious ideal kink instability can be a useful test of local gyrokinetic codes when a current gradient drive term is implemented as long as one is aware of the simplifications going into these models regarding magnetic geometry.

Using the new "low-flow" version of the local gyrokinetic code GS2 developed for momentum transport studies [1], we model the effect of the induced parallel electric field on the unperturbed electron distribution to study the destabilizing influence of current on stability. We identify the spurious high mode number kink modes in GS2 simulations and make comparisons to analytical theory in sheared magnetic geometry. We demonstrate good agreement with analytical results both in terms of parametric dependences of mode frequencies and growth rates, and regarding the mode structure.

**Induced field effects to GS2:** The gyrokinetic code GS2 is radially local and it assumes a scale separation between the parallel and perpendicular length scales of the perturbed quantities, otherwise it does not make any further simplification to the gyrokinetic-Maxwell system. To study intrinsic rotation, $\rho_{*p} = m_i c v_i / e_i B_p a$ corrections to the lowest order gyrokinetic equations were implemented. These terms are normally assumed to correspond to neoclassical effects. Here, $v_\alpha = (2T_\alpha / m_\alpha)^{1/2}$, $T_\alpha$, $m_\alpha$, and $e_\alpha$ are the thermal speed, the temperature, the mass and the charge of species $\alpha$ respectively ($\alpha = e$ $(i)$ for electrons (ions)), $a$ is the plasma minor radius and $B_p$ is the poloidal magnetic field. In particular, the non-fluctuating distribution functions are allowed to deviate from co-drifting Maxwellian. This departure represents the effect of the induced electric field on the electron distribution function; for details see [2].

**Analytic Kink Stability Condition:** Some of the properties of strongly driven kink modes can be understood from analytical expressions derived in a shearless magnetic geometry. Assuming a parallel electron flow, the gyrokinetic-Maxwell system (with electrostatic and shear magnetic perturbations) can be simplified in the $k_\perp \rho_i \ll 1$ limit when the parallel electric field is close to zero. This yields a local dispersion relation (also found previously in slab geometry in [3])

$$\omega = \omega_{*i}^p / 2 \pm \left[ (\omega_{*i}^p / 2)^2 + (v_i k_\parallel)^2 / \beta_i - 2\omega_{*e}^j k_\parallel u / (\tau k_\perp^2 \rho_i^2) \right]^{1/2}, \tag{1.1}$$

where $\omega_{*i}^p = (ncT_i / e_i)\partial_\psi (\ln p_i)$, $\omega_{*e}^j = (ncT_e / e_e)\partial_\psi (\ln j_0)$ and $\rho_i = m_i c v_i / e_i B$, with $n$ the toroidal mode number and $\psi$ the poloidal flux, $p_i = n_i T_i$, $j_0 = e_e n_e u$. Furthermore, $n_\alpha$ is the species density, $\beta_i = 8\pi p_i / B^2$, $-u$ is the electron flow speed. When the mode is strongly driven, so that the diamagnetic correction is negligible, a finite parallel wave number, $k_{\parallel o} = u\beta_i \omega_{*e}^j / (\tau k_y^2 \rho_i^2 v_i^2)$, maximizes the growth rate $\gamma_o = u\sqrt{\beta_i}\omega_{*e}^j / (v_i k_y^2 \rho_i^2 \tau)$.

The mode structure and the stability condition for these local kink modes can be obtained from a sheared slab calculation, where the unperturbed magnetic field is assumed to have the form $\mathbf{B} = B(\hat{z} + \hat{y}x / L_s)$ corresponding to a parallel wave number $k_\parallel = k_y x / L_s \equiv X / L_s$, where $L_s$ is the magnetic shear length (in toroidal geometry $L_s = qR / s$ with $s = r(d_r q) / q$). We obtain the following dispersion relation

$$X(\partial_{XX} - 1)\hat{B} - \lambda(\partial_{XX} - 1)(\hat{B} / X) - \sigma\hat{B} = 0, \tag{1.2}$$

where $\sigma = -2L_s \beta_i u \omega_{*e}^j / (\tau k_y^2 \rho_i^2 v_i^2)$, with $k_y = nq / r$, $\lambda = \omega(\omega - \omega_{*i}^p) L_s^2 \beta_i / v_i^2 \approx \omega^2 L_s^2 \beta_i / v_i^2$ and $\tau = (-e_i / e_e) T_e / T_i$. The radial component of the perturbed magnetic field $B_x$ is given by

$B_x = \hat{B}(x)\exp(-i\omega t + ik_y y)$. We find that within this local description at sufficiently high current gradient high mode number kink modes are destabilized. At zero ion pressure gradient the stability criterion is $|\sigma| < 2$. The stability limit $|\sigma| = 2$ coincides with the magnetohydrodynamic stability limit for high mode number kink modes [4]. A new unstable mode appears at $\sigma = 2N$ for each non-zero integer $N$, and for a given $\sigma$ the modes with lower growth rates exhibit a more oscillatory radial structure. In a sheared slab magnetic geometry we find that the mode is strongly asymmetric, being localized on one side with respect to a resonant surface. The parallel wave number corresponding to the radial location of the highest amplitude is close to $k_{\|o}$, which maximizes the growth rate in the shearless magnetic geometry.

**Limitations of the local modeling:** In the derivation of the stability criterion $|\sigma| < 2$ it is assumed that both the current density $j_0$ and the poloidal magnetic field $B_p$ can be expanded linearly about their values at the resonant surface, and this truncated expansion is sufficient over the radial extent of the mode. However, the radial variations of $B_p$ and $j_0$ are related through Ampère's law. In cylindrical geometry $4\pi R j_0/(cB) = r^{-1} d_r (r^2/q)$ with the safety factor $q = rB_0/(RB_\theta)$. The current gradient is then given by $(4\pi R)/(cB) r d_r j_0 = (2s^2 - 3s - w)/q$ with $w = (r^2/q) d_{rr} q$. Inserting this relation for $d_r j_0$ into the stability criterion, $4\pi r\,|d_r j_0|/(cB_p) < 2m\,|q'/q|$, where $m = nq \gg 1$ at the rational surface, we find that the ideal kink mode would be unstable when $2s^2 - 3s - w > 2ms$. This implies a strong (nonlinear) profile variation of $q$, thus the assumption of $(k_\| \propto) B_y \propto x$ over the radial extent of the mode (which is $X \sim 1 \Rightarrow x \sim 1/k_y$) is broken. When the current and the safety factor profiles are calculated consistently, the stability condition must always be satisfied when only the linear term in the expansion of $q$ about the rational surface is retained [5].

**Simulation Results:** We find good agreement between GS2 simulations and analytical estimates in terms of mode frequencies, eigenmode structure. Fig. 1a shows that the $\beta$ dependence of the growth rate indeed follows the $\gamma \propto \beta^{1/2}$ behavior, as expected from the shearless slab result for the optimal growth rate $\gamma_o = u\sqrt{\beta_i}\,\omega_{*e}^j/(v_i k_y^2 \rho_i^2 \tau)$. Similarly, linear dependence on $u$, $d_r \ln j_0$, and $1/k_y$ are also observed. The $q$ and the $s$ dependences are not captured by the local calculation, however, the linear $L_s = qR/s$ dependence of $\sigma$ suggests that close to marginality decreasing $q$ or increasing $s$ have a stabilizing impact. In the strongly driven limit, $|\sigma| \gg 1$, the solution is asymptotically independent of $L_s$ and approaches the local limit, $\gamma_o$. By comparing kink modes assuming a Maxwellian electron distribution with a parallel flow and alternatively a Spitzer function departure from a Maxwellian as a drive (Fig. 1a) we demonstrate that the exact velocity structure of the non-fluctuating electron distribution function is unimportant.

For modes which are electrostatic in nature an electron flow – even when comparable to the ion thermal speed – is not expected to significantly modify their stability. We have found practically no effect on ion- and electron temperature gradient modes for typical plasma parameters even when $\beta$ and $u$ exceed their experimentally relevant range in the simulations.

The large aspect ratio and $k_y\rho_i \ll 1$ limit of high mode number kink modes may be used as a simple test case for linear validation of local electromagnetic gyrokinetic codes when current drive is to be modeled. As a result, we can say with certainty that GS2 is correct in the stable lower current gradient, weaker $q$ variation limits where the theory and code are completely valid.

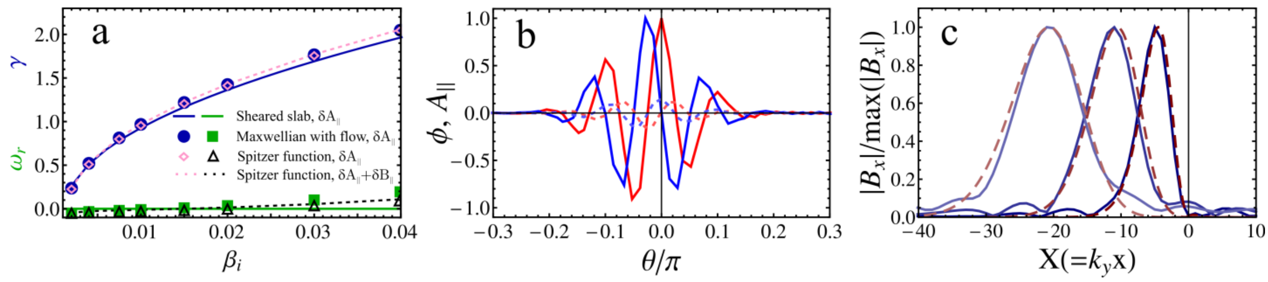


Figure 1. a) $\beta_i$-dependence of mode frequency $\omega_r$ and growth rate $\gamma$ in a sheared slab model and in GS2 simulations using models of different sophistication in terms of unperturbed electron distribution and electromagnetic physics. b) Typical parallel mode structures of $\phi$ (solid) and $A_{||}$ (dashed) for a strongly driven high mode number kink mode (GS2 simulation). c) Radial mode structures of the perturbed magnetic field from the sheared slab model (dashed) and inferred from GS2 simulations (solid) for three different values of $\beta_i$.

## References

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