On the Formation of Phase Space Holes and Clumps in Kinetic Systems

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Signals with rapidly changing carrier frequencies are often attributed to the formation and subsequent evolution of phase space holes and clumps in the fast particle distribution [1, 2]. However, their origin has never fully been explained. In the following, we show that hole-clump formation follows from the presence of a nearly unmodulated phase space plateau in the fast particle distribution, which gives rise to a pair of shifted *edge modes* that destabilize due to dissipation in the background plasma and nonlinearly evolve into holes and clumps.

For simplicity, we consider electrostatic oscillations in a one-dimensional, uniform plasma equilibrium comprised of immobile ions, cold electrons and a low density beam of energetic electrons. The cold electrons respond linearly to the electric field $E(x,t)$, so their perturbed velocity $\delta v_e(x,t)$ satisfies the linear fluid equation

$$\frac{\partial \delta v_e}{\partial t} = \frac{e}{m_e} E - 2\gamma_d \delta v_e \ , \tag{1a}$$

where a linear friction term has been added in order to introduce dissipative wave damping. The fast electrons interact resonantly with the electric field, and so are described kinetically in terms of their distribution $F(x,v,t) = \delta f(x,v,t) + F_0(v)$, which evolves according to the Vlasov equation

$$\frac{\partial F}{\partial t} + v\frac{\partial F}{\partial x} - \frac{e}{m_e} E \frac{\partial F}{\partial v} = 0 \ , \tag{1b}$$

and whose unperturbed part $F_0(v)$ is assumed to have a constant, positive velocity gradient throughout the wave-particle resonance. The system is closed by means of Ampére's law, which relates the perturbed currents in the two electron species to the electric field, i.e.

$$\frac{\partial E}{\partial t} = \frac{e}{\varepsilon_0} \left[n_{e0} \delta v_e + \int \delta f \, dv \right] \ , \tag{1c}$$

where n_{e0} is the unperturbed density of cold electrons. On their own, the cold electrons support a mode that oscillates at the electron plasma frequency ω_{pe}. In the presence of the fast electrons, that mode is driven at linear rate $\gamma_L \propto dF_0/dv$ and linearly damped at rate γ_d. A numerical algorithm that solves Eqs. (1a) – (1c), given values for γ_L and γ_d, was described in [3], where also frequency chirping due to hole-clump formation and evolution was thoroughly investigated. That same tool was employed for all simulations presented in this contribution (i.e. those on display in Figs. 1, 2 and 5).

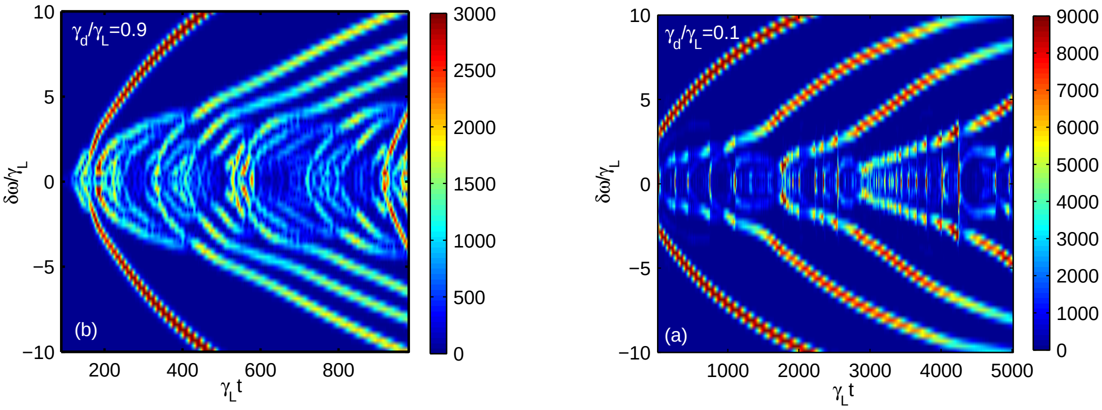

Figure 1: *Frequency chirping close to (left) and far from (right) the instability threshold.*

It was previously widely accepted that hole-clump formation and, therefore, frequency chirping only occurs when the mode is weakly driven close to the instability threshold, [1], i.e. when $0 < \gamma_L - \gamma_d \ll \gamma_L, \gamma_d$. On the contrary, Fig. 1 displays frequency chirping for both $\gamma_d/\gamma_L = 0.9$ and 0.1, i.e. close to *and* far from the threshold. Moreover, these simulations reveal two additional important features of hole-clump generation, namely that a) the chirping components initiate noticably shifted from the original resonance and b) each hole-clump launch is preceded by the formation of a nearly unmodulated phase space plateau with a velocity width slightly larger than the observed initial shifts in phase velocity (cf. Fig. 2).

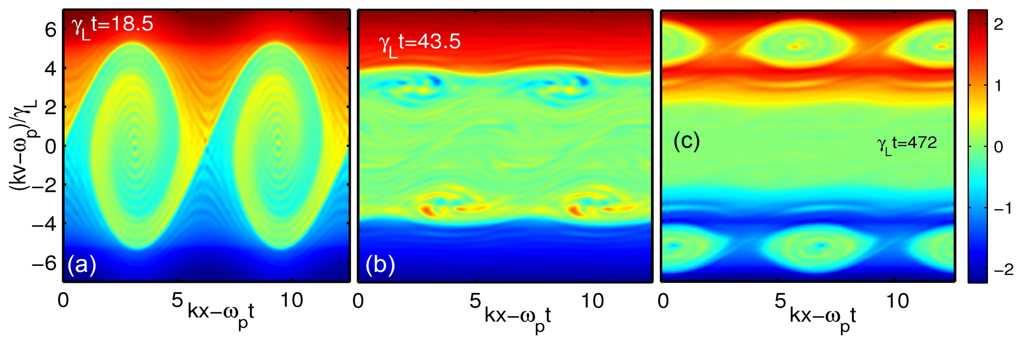

Figure 2: *Phase space snapshots of the fast electron distribution during hole-clump generation when* $\gamma_d/\gamma_L = 0.1$, *i.e. far from the instability threshold.*

A natural first step is to assess the linear stability of the plateau state on display in Fig. 2. To do so, we adopt a simplified, entirely unmodulated shelf distribution with discontinuous edges (cf. Fig. 3). The dispersion relation according to Landau, cf. [4], i.e.

$$1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_c)} = \frac{e^2}{m_e \varepsilon_0 k} \int_{-\infty}^{\infty} \frac{dF_P/dv}{kv - \omega} dv \, , \tag{2}$$

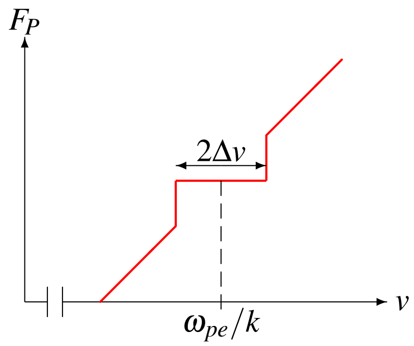

Figure 3: *Simplified fast electron distribution used for linear stability analysis of the plateau state observed in the simulations.*

becomes then

$$\varepsilon\left(z;w,\gamma\right)=wz+i\gamma+\log\left[1-z\right]-\log\left[1+z\right]+\frac{1}{1+z}-\frac{1}{1-z}=0\ , \tag{3}$$

where $z\equiv\left(\omega-\omega_{pe}\right)/k\Delta v$, $w\equiv\pi k\Delta v/\gamma_L$, $\gamma\equiv\pi\gamma_d/\gamma_L$ and it has been assumed that $z_R\equiv\mathrm{Re}\left[z\right]\in(-1,1)$, which corresponds to phase velocities inside the plateau, while $k\Delta v$, γ_d, $\gamma_L\ll\omega_{pe}$.

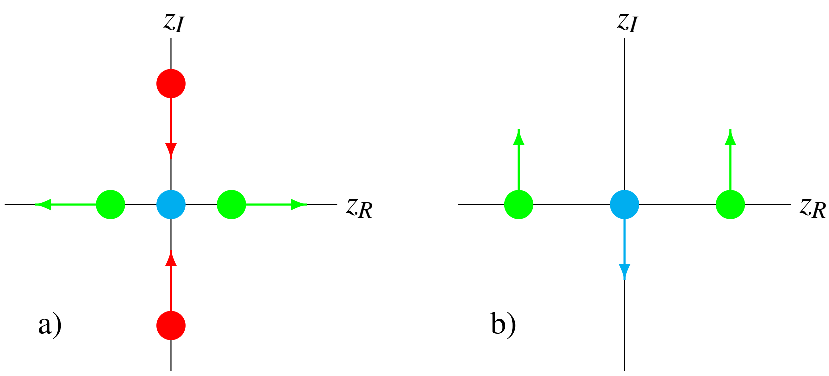

Figure 4: *Roots of the dispersion relation in the absence a) and presence b) of dissipation. In a), the arrows indicate the root movement with increasing plateau width w, whereas the arrows in b) indicate that of increasing γ from a plateau with fixed $w>4$.*

As indicated by the cartoons in Fig. 4, there are always three roots of Eq. (3). When $\gamma=0$, i.e. in the absence of dissipation, one always sits at $z=0$ (the blue dot in Fig. 4a). The complimentary two bifurcate mirrorwise according to the following: For $w<4$ they form a stable/unstable conjugated pair (shown in red) on the imaginary axis that approaches the origin with increasing w. At $w=4$, all three roots coalesce at the origin. For $w>4$, they diverge from $z=0$ along the real axis, tending asymptotically to $z=1$ as $w\rightarrow\infty$. The latter type (shown in green) are dubbed *edge modes*, and it is important to note that all plateaus observed in the simulations during hole-clump launches have $w > 4$. Interestingly, whereas the central mode damps in the presence of dissipation, when $\gamma > 0$, the edge modes actually destabilize (cf. Fig. 4b). E.g., for a large plateau with $|z_R| \lesssim 1$, an expansion in $1 - |z_R| \ll 1$ yields

$$z_R \approx \pm\left[1 - w/\left(w^2 + \gamma^2\right)\right] \;, \quad z_I \approx \gamma/\left(w^2 + \gamma^2\right) \;, \tag{4}$$

where $z_I \equiv \mathrm{Im}\left[z\right]$.

The reason that the edge modes are driven rather than damped by dissipation is that they are negative energy waves, which simply means that their presence lowers the total system energy. As the dissipative friction term in Eq. (1a) removes energy, it therefore drives such oscillations. Indeed, the wave energy (cf. [5]) of the plateau modes is proportional to $\partial\varepsilon/\partial z \propto w\left[1 - z^2\right]^2 - 4$, so the sub-critical stable/unstable pair has positive energy whereas the edge modes have negative energy.

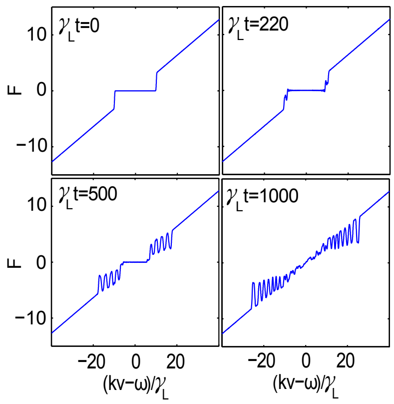

Figure 5: *Nonlinear evolution of prescribed shelf with initial width* $w = 10\pi$.

Finally, the conversion from dissipation-driven, linear edge modes into holes and clumps is demonstrated via nonlinear simulations of a prescribed shelf (cf. Fig. 5). The modes are found at the predicted phase velocities and grow with the predicted growth rates as they acquire their separate trapping regions and eventually peel of the plateau in the form of hole-clump pairs. The remaining plateau is somewhat smaller, but still unstable, so the process continues, thereby gradually eroding the plateau towards the critical width $w = 4$.

References

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