Stability of a magnetic island within a sheared flow

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I. Introduction. It has been observed in the TJ-II heliac a close correlation between rational surfaces and transport barriers as well as a persistent MHD activity that has a dynamical evolution [1]. Rational surfaces are usually the locus of magnetic island formation because the magnetic perturbations are resonant there; therefore it is the most likely region for the origin of the observed MHD activity. On the other hand, transport barriers are commonly associated with the presence of sheared flows which, in the case of the TJ-II experiments, would appear at or around the magnetic resonant surfaces. A possible sequence of events that may explain the observations in TJ-II is that the non-ambipolar transport processes in the vicinity of a rational surface give rise to a sheared radial electric field which reduces the anomalous transport creating a barrier. Then the magnetic island that may be present at the resonant surface is affected by the sheared flow according to the stability properties of the tearing mode. The island evolution is observed as MHD activity and may disrupt the transport barrier. Here we study the stability of the magnetic islands under the influence of the sheared flow around them. We start with the island of the vacuum magnetic field at a low order rational surface and analyze the effect of the plasma through the polarization drift on the stability parameter Δ', which determines the nonlinear evolution of the island width. Δ' depends on the velocity profiles near the island separatrix which are calculated from a neoclassical transport model that contain the shear.

II. Island dynamics with shear flow.

The qualitative picture for the events in TJ-II is that the islands enhance the electron radial transport thus creating a larger radial electric field across the island and the resulting sheared flow creates a transport barrier. This flow modifies also the stability and dynamics of the island. There are also fast particles detected at the wall, which may arise during the reconnection process or as a result of the inductive parallel electric field due to island motion. To study island stability at a rational surface, the tearing mode evolution has to be considered, including the presence of shear flows. An important effect to take into consideration when the island width is as small as the ion Larmor radius is the polarization drift. This effect is important when the island begins to grow and it has been variously described in the literature with some discrepancies as explained in [2]. The relevant effect that determines the contribution to the growth is the velocity profile across the island.

Magnetic islands in stellarators often appear even in the vacuum field at rational surfaces due to small coil errors. We consider then the evolution of a vacuum magnetic island under the action of plasma response in presence of a sheared $E\times B$ flow. The equilibrium magnetic field is $\mathbf{B} = \nabla\psi\times\nabla\theta + \iota(\psi)\nabla\zeta\times\nabla\psi$. The perturbed magnetic field is $\tilde{\mathbf{B}} = \nabla\tilde{\psi}\times\nabla\zeta$ and a vacuum island at the rational surface $\iota = n/m$ is represented by a single harmonic approximation $\tilde{\psi}_v = \psi_{v0}\cos(m\alpha+\Delta\phi)$, in terms of the helical variable $\alpha = \theta-(n/m)\zeta$ and a phase $\Delta\phi$, from which the helical flux function that describes the island surfaces is $\psi^* = \iota' x^2/2 - \psi_{v0}\cos(m\alpha)$ ($x = \psi-\psi_s$ is a local radial coordinate). The island has a width $w = 4\sqrt{\psi_{v0}/\iota'\psi'}$. Then, in presence of plasma the island structure may be modified in amplitude and phase, so it can be represented by $\tilde{\psi} = \psi_0\cos(m\alpha)$ with the width $w = 4\sqrt{\psi_0/\iota'\psi'}$. The usual treatment is to separate the solution of the equations in a narrow region around the rational surface and an external region where ideal MHD is used, and then match the solutions. The external solution is singular at the rational surface with a discontinuity in the derivative of ψ denoted by Δ'. The constraint imposed by the external coils enters through a boundary condition term as $\Delta'+\Delta'_{BC}$ and the matching condition can be split in sine and cosine terms as

$$\int dx\int\frac{d\alpha}{\pi}\int\frac{d\zeta}{2\pi}\frac{\sqrt{g}}{|\nabla\psi|}J_{\|}B\cos m\alpha = \Delta'_c\psi_0,\qquad \int dx\int\frac{d\alpha}{\pi}\int\frac{d\zeta}{2\pi}\frac{\sqrt{g}}{|\nabla\psi|}J_{\|}B\sin\alpha = \Delta'_s\psi_0$$

where $\Delta'_c = \Delta'(1-k_v\frac{\psi_{v0}}{\psi_0}\cos\Delta\phi)$, $\Delta'_s = \Delta' k_v\frac{\psi_{v0}}{\psi_0}\sin\Delta\phi$ and $J_{\|}$ is current parallel to the B-field in the inner region. Separating the inductive part of $J_{\|}$ (using Ohm's law $\eta^{-1}\partial\psi/\partial t$), the first condition leads to the equation for the island width evolution

$$\frac{dw}{dt} = \frac{\rho_s^2}{\tau_R}(\Delta'_c+\Delta_{pol})\quad\text{where,}\quad \Delta'_{pol} = \int\frac{dx}{\psi_0}\int\frac{d\alpha}{\pi}\int\frac{d\zeta}{2\pi}\frac{\sqrt{g}}{|\nabla\psi|}J_{pol}B\cos m\alpha. \tag{1}$$

On the other hand, the sine contribution gives the flux-surface averaged electromagnetic torque [4] due to the action of the exterior region through the error field

$$N_{EM} = \int dx\int\frac{d\alpha}{\pi}\int\frac{d\zeta}{2\pi}\hat{\alpha}\cdot\mathbf{J}\times\mathbf{B}\frac{\sqrt{g}}{V'} = \frac{2\pi^2\psi'}{\mu_0 b_t}m\psi_0^2\Delta'_s \tag{2}$$

Thus, the sign of Δ'_{pol} determines if the shear flow is destabilizing ($\Delta'_{pol}>0$, island growth) or not; while the torque competes with the viscous torque given by neoclassical viscosity to determine the rotational state of the island. In order to know this effects it is necessary to obtain the noniductive parallel current solving the transport problem. The combined action of the two effects may lead to the following sequence of events: the island grows due to shear flow and as it grows the viscous torque increases making the island rotate with the flow, which in turn reduces the shear in the vicinity of the island.

The first thing is to determine the $\mathbf{E}\times\mathbf{B}$ plasma velocity in the inner region starting from the ion momentum balance equation in steady state: $\mathbf{J}\times\mathbf{B}-\nabla\cdot\mathbf{P} = 0$. The stress tensor is

$\mathbf{P} = p\mathbf{I} + \Pi$. This is averaged over the helical flux surfaces described by ψ^* that define the island structure

$$-\frac{\psi'}{\iota'}\langle \mathbf{J}\cdot\nabla\psi^*\rangle_I - \langle \hat{\alpha}\cdot\nabla\cdot\mathbf{P}\rangle_I = 0$$

requiring charge neutrality across island flux surfaces $\langle \mathbf{J}\cdot\nabla\psi^*\rangle_I = 0$ and using $\langle \hat{\alpha}\cdot\nabla\cdot\mathbf{P}\rangle_I = -\psi'\sum_s q_s\Gamma_s^{\psi^*}$ the ambipolarity condition results

$$\Gamma_i^{\psi^*}(E) = \Gamma_e^{\psi^*}(E). \quad (3)$$

The helical flux surface average is defined as $\langle f\rangle_I = (d\psi^*/V)\int d\zeta\int d\alpha\sqrt{g_I}f$ and thus the transport described represents flows across the island. Here $\Gamma_s^{\psi^*}$ are the cross field fluxes that are assumed to have the same dependence of the neoclassical radial fluxes with just different values of the coefficients. So we can use, for instance, Kovrizhnykh's analytical model which interpolates across collisionality regimes [5].

The solution of Eq. 3 gives the ambipolar electric field which in turn yields the $\mathbf{E}\times\mathbf{B}$ plasma velocity in the direction of α, v_α. There are in general multiple solutions for E but for the model in [5] a cubic equation results which has been found to have a single root that changes from the electron root ($E > 0$) at low density to the ion root ($E < 0$) for high n [6]. This sheared flow can give rise to turbulence suppression with the consequent formation of a transport barrier. This flow enters in the friction term $\nabla\cdot\mathbf{P}$ and drives a perpendicular current that, according to momentum balance equation, is $\mathbf{J}_\perp = \frac{c}{B^2}\mathbf{B}\times\nabla\cdot\mathbf{P}$. This current creates a parallel current in order to maintain the quasineutrality condition $\nabla\cdot\mathbf{J} = \nabla_\perp\cdot\mathbf{J}_\perp + \nabla_\parallel J_\parallel = 0$, which is the one responsible for the island evolution in eqs. (1), (2).

III. Island evolution.

The evolution of the island width w due to the polarization drift can be written as

$$\frac{dw}{dt} = \frac{\rho_s^2}{\tau_R}[\Delta' + g\omega(\omega-\omega_{*i})]$$

where g depends on the velocity profile and its sign determines the stability. Depending on the frequency range, it can be stabilizing or destabilizing [2]. The asymmetry in the velocity profiles due to shear flow determines the stability; as shown in [3] the shear contribution is stabilizing for low and high shear but it destabilizes the mode for intermediate values of the shear. This may lead to oscillations of the island width. According to Connor et al. [2], for $w \le \rho_i$, $g \sim \omega - \omega_{*e}$ giving

$$\frac{dw}{dt} = \frac{\rho_s^2}{\tau_R}\Delta' + \frac{4\mathscr{I}}{\tau_R k^2 v_a^2 w}L_s^2(\omega-\omega_{*e})(\omega-\omega_{*i}). \quad (4)$$

where $\mathscr{I} > 0$ is a coefficient that depends of the density profile. This produces a polarization contribution that is destabilizing for frequencies larger than ω_{*e}. The island rotation velocity in the plasma frame, is determined by the electron or ion diamagnetic velocities as: $\omega = f\omega_{*e} + (1-f)\omega_{*i}$, where f is the flattening parameter that measures the degree of temperature profile flattening inside the island. $f = 1$ means there is no flattening. Since in TJ-II NBI plasmas there is no flattening detected during off-axis sawteeth, one would expect $\omega \approx \omega_{*e}$. This would give $\Delta'_{pol} \approx 0$ for for TJ-II, so that the island should be stable (if $\Delta' \leq 0$) in the frame of reference where $E_r = 0$. Moving to a frame with $E_r = -3$ kV/m, Eq. 4 predicts there can be instability for frequencies outside the range $0 < \nu < 20$ kHz for a resonance 8/5 near the edge, for NBI plasma of moderate density. The corresponding growth rate is of order $10^5 s^{-1}$ when $I/(kW)^2 \sim 1$. This seems to be consistent with the experimental data.

On the other hand, the viscous torque has been calculated in [4] finding (with no cross-field viscosity)

$$N_V = -\frac{2eV'}{\sqrt{6}} w\Gamma_i \frac{\nu_\alpha}{\nu_\alpha - \nu_{\alpha,amb,i}}$$

which is proportional to the radially integrated neoclassical friction $\int dx\langle\hat{\alpha}\cdot\nabla\cdot\mathbf{P}\rangle \sim w\langle\hat{\alpha}\cdot\nabla\cdot\mathbf{P}\rangle \sim w\Gamma_i$. This indicates that as the island grows the torque gets larger and the torque balance $N_{EM} + N_V = 0$ cannot be maintained for large enough w. Then the island is dragged across the plasma, balancing the flows on both sides, that is, the shear flow is reduced. Therefore, the transport barrier can disappear. At the same time, the island rotation produces eddy currents at the rational surface which produce magnetic activity as observed in TJ-II around rational surfaces.

For a selfconsistent evolution of the island width and frictional drag the radial electric field in the island should be solved and used to compute the parallel current, which would then give Δ'_{pol} and N_V. Here a first approximation was developed assuming that the fluxes are aproximated by their neoclassical expressions.

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References

[1] D. López-Bruna et al., Nuclear Fusion **53**, 073051 (2013)

[2] J.W. Connor, F.L. Waelbroeck and H.R. Wilson, Phys. Plasmas **8**, 2835 (2001).

[3] R. Fitzpatrick and F.L. Waelbroeck, Phys. Plasmas **16**, 052502 (2009).

[4] C. Hegna, Phys. Plasmas **19**, 056101 (2012).

[5] L.M. Kovrizhnykh, Plasma Phys. Rep. **31**, 14 (2005).

[6] C. Gutiérrez-Tapia et al., IAEA Tech. Meet. RUSFD, J. Phys. Series, in press (2014).