A Novel Representation Approach Applied to Flexible Design of Hohlruams in Laser Driven Inertial Confinement Fusion

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Hohlraum shape has drawn much more attention since there is no successful ignition as expected for laser driven inertial confinement fusion on the NIF. The holhraum is usually designed as cylindrical shape and extensively researched since it can be easily manufactured, and now is selected as the main shape of the ignition target for the National Ignition Facility (NIF). However, it has been reported that almost more than 80 percent of laser energy is absorbed by the wall of such cylindrical hohlraum， and only a small portion is absorbed by the capsule finally. In addition, inner beam propagation in cylinders with CH capsules requires high wavelength difference for strong cross-beam energy transfer (CBET) to achieve adequate symmetry. To improve the energy coupling efficiency, and reduce the need of CBET, a serial of hohlraums are proposed with spherical or rugby shapes such as ellipse, parabola, or arc with a large radius. The resulting drive temperature and symmetry have been significantly improved. Further, a new shape of hohlraum described by Lame curve, is recently proposed by varying the order of Lame curve to improve its drive performance, which has been reported that energy coupling efficiency and radiation symmetry can be improved. Nevertheless, the shape of designed hohlraums is still represented with explicit functions like $f(x,y)=\left(x/a\right)^k \pm \left(y/b\right)^k + C$, and only three parameters $\left(a,b,k\right)$ can be optimized, which still limits further improvement on the hohlraum performance. Therefore, a new shape representation is required to achieve an optimal hohlraum design with much higher performance.

In Computer Graphics and Computer-Aided Design (CG/CAD) field, Non-Uniform Rational B-Spline (NURBS) is now widely used to represent and model products, and is the unique representation form for most of modern CG/CAD softwares. The NURBS is defined with order, B-spline shape function, control points, and weights, which can be utilized to model any complex shapes and exactly represent conic curves and surfaces by setting proper values of order, B-spline shape functions, the locations of control points and their weights. Such uniform representation and shape definition with more parameters enable us more freedoms on flexible control of hohlraum shape for experiment design and optimization.

To build a free-form shape hohlraum and optimize its parameters, we first give the form of B-spline curves as

$$\mathbf{C}(u)=\sum_{i=0}^{n-1}B_{i,k}(u)\mathbf{P}_i\ ,\qquad (1)$$

where $\mathbf{C}(u)$ is a B-spline curve, u is a parameter defined over a parametric domain, k is the order of $\mathbf{C}(u)$) ($k=2$ for conic curves, and usually $k=3$ for most free-from curves in CAD/CAM), $\mathbf{P}_i$ is the control points defined by $[x\quad y\quad z]^T$, n is the number of control points $\mathbf{P}_i$, and $B_{i,k}(u)$ is the B-spline shape function with order k , which can be got by

$$\begin{cases}\mathrm{B}_{i,1}(u)=\begin{cases}1 & \mathrm{if}(\mathrm{u}_i\le u\le \mathrm{u}_{i+1})\\ 0 & \mathrm{others}\end{cases}\\ \mathrm{B}_{i,k}(u)=\dfrac{(u-\mathrm{u}_i)\mathrm{B}_{i,k-1}(u)}{\mathrm{u}_{i+k-1}-\mathrm{u}_i}+\dfrac{(\mathrm{u}_{i+k}-u)\mathrm{B}_{i+1,k-1}(u)}{\mathrm{u}_{i+k}-\mathrm{u}_{i+1}} \quad \mathrm{u}_{k-1}\le u\le \mathrm{u}_{n+1}\end{cases}.\qquad (2)$$

In Eq.2, $u_0,u_1,\ldots,\ u_n,\ u_{n+1},\ldots,$ and u_{n+k} are non-decreasing numbers, which form a knot vector $\mathbf{u}$ to define the B-spline shape function from 1 to k , or $\mathrm{B}_{i,1}(u)$, $\mathrm{B}_{i,2}(u),\ldots,B_{i,k}(u)$, and its dimension is $n+k+1$. A high-order B-spline shape function $B_{i,k}(u)$ can be iteratively computed from lower level B-spline shape functions. The distribute pattern of $\mathbf{u}$ can lead to various forms of curves, namely, uniform B-spline curves, quasi B-spline curves or non-uniform B-spline curves.

By giving different control point set $\mathbf{P}$, order k , and knot vector $\mathbf{u}$, we can construct complex curves for industrial applications. However, such form of curves cannot accurately represent conic curves used in shape design such as circles, ellipses, parabolas, or hyperbolas. Then, a weighted B-spline curves or NURBS is given as

$$\mathbf{C}(u)=\frac{\sum_{i=0}^{n-1}w_iB_{i,k}(u)\mathbf{P}_i}{\sum_{i=0}^{n-1}w_iB_{i,k}(u)}\qquad (3)$$

where w_i is the weight for the control point $\mathbf{P}_i$, which is a non-negative number, and enables the curve $\mathbf{C}(u)$ accurately represent conic curves. For the usual conic hohlraum shapes ($k=2$), we can represent them with NURBS as follows:

$$\mathbf{C}(u)=\frac{\sum_{i=0}^{2} w_i B_{i,2}(u)\mathbf{P}_i}{\sum_{i=0}^{2} w_i B_{i,2}(u)}. \tag{4}$$

For a hohlraum with an order $k=2$ and 3 control points, the NURBS based curve definition can be described in the following.

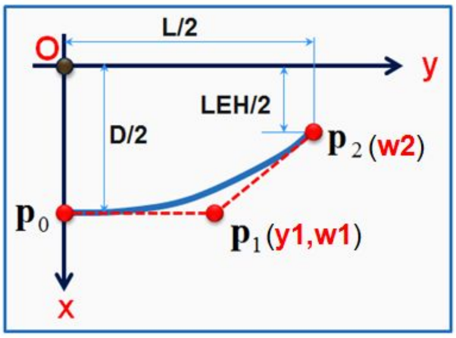


Fig.1 NURBS based the usual hohlraum shapes definition

As shown in Figure 1, D is the diameter of a hohlraum, LEH is the diameter of Laser Entrance Holes, L is the length. Since the usual shape of hohlraums is conic curves, only 3 control points $\mathbf{P}_0,\mathbf{P}_1,\mathbf{P}_2$ are required, $\mathbf{P}_0,\mathbf{P}_2$ are the end points of such curve, and $\mathbf{P}_1$ is the intersection point of two tangent vectors for $\mathbf{P}_0$ and $\mathbf{P}_2$ .The knot vector $\mathbf{u}$ should be $[0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1]$, since the curve is tangent with control polygon formed by $\mathbf{P}_0,\mathbf{P}_1,\mathbf{P}_2$ which can be used to define B-spline shape functions as

$$\begin{cases} B_{0,2}(u)=(1-u)^2 \\ B_{1,2}(u)=2u(1-u) \\ B_{2,2}(u)=u^2 \end{cases}. \tag{5}$$

Fix the weight $w_0=1$ for the control point $\mathbf{P}_0$, we can easily compute the weight w_1 and w_2 in the following table.

Table 1 NURBS defining parameters for various shape hohlraum

Hohlraum shapes	NURBS defining parameters		
	$\mathbf{P}_1$	w_1	w_2
Cylinder	$\begin{bmatrix} \frac{D}{2} & \frac{L}{2} & 0 \end{bmatrix}^T$	$+\infty$	1

Rugby-ellipse	$\begin{bmatrix} \frac{D}{2} & \frac{[L-LEH\cdot L/(D+LEH)]}{2} & 0 \end{bmatrix}$	$\sqrt{\frac{D+LEH}{2D}}$	1
Rugby-parabola	$\begin{bmatrix} \frac{D}{2} & \frac{L}{4} & 0 \end{bmatrix}^T$	1	1
Rugby-arc	$\begin{bmatrix} \frac{D}{2} & \frac{L^2+(D-LEH)^2}{4L} & 0 \end{bmatrix}^T$	$\frac{L}{\sqrt{L^2+(D-LEH)^2}}$	1
Sphere	$\begin{bmatrix} \frac{D}{2} & \frac{L^2-(D-LEH)\cdot LEH}{2L} & 0 \end{bmatrix}^T$	$\sqrt{\frac{D+LEH}{2D}}$	1

With estimated parameters above, we can compute a point on the curve with Eq. 3 for any parameter $u \in [0,1]$, and then draw the curve as below when $D = 2$ mm, LEH=1.1mm and $L = 1.67$ mm(spherical hohlraum), or 3.6 mm (rugby-cylinder, ellipse, parabola) for Shenguang III(SG-III) laser facility in Figure 2 and Figure 3.

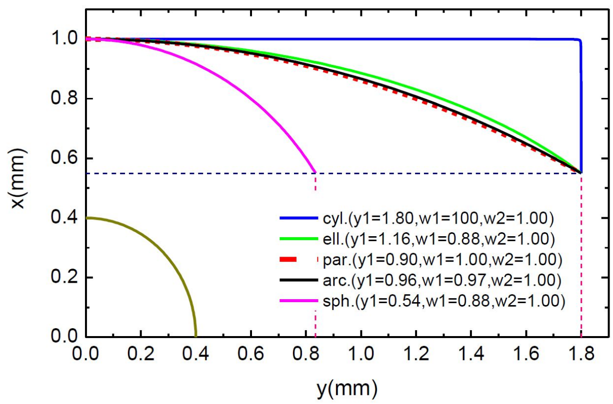
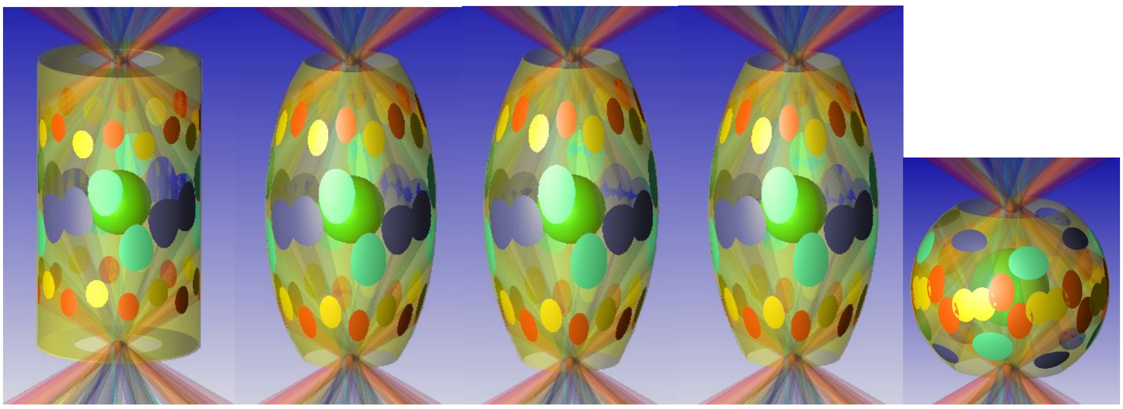


Fig.2 NURBS based conical curves representation



(a) Cylinder; (b) Rugby-ellipse; (c) Rugby-parabola; (d) Rugby-arc; (e) Sphere

Fig.3 NURBS based conical curves hohlraum shape representation for ShenguangIII laser facility

The representation of hohlraum described by Lame curve and shape optimization for SG-III laser facility will be investigated next.