Reliability of BGK Model to ions drift in parent gas

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Ion flux characteristics can be determined by solving the Boltzmann kinetic equation for the ion distribution function $f(\mathrm{v})$

$$\frac{\partial f}{\partial t} + \mathrm{v}\nabla f + \frac{eE}{m}\frac{\partial f}{\partial \mathrm{v}} = I_{st}(f)\,, \qquad (1)$$

where e is the ion charge, m is the ion mass, $I_{st}(f)$ is the collision integral.

In weakly ionized plasma, elastic collisions of ion with atoms, electrons, and ions can often be neglected. Since in the case of ion collisions with atoms of own gas the cross section of resonant charge exchange of ions is usually the largest, let us consider kinetic equation (1) in the spatially uniform case at a dc electric field, considering only resonant charge exchange of ions,

$$\frac{eE}{m}\frac{\partial f}{\partial u} = \int [\, f(\mathrm{v}')\varphi(\mathrm{v}) - f(\mathrm{v})\varphi(\mathrm{v}')]\left|\mathrm{v}-\mathrm{v}'\right|\sigma_{res} n_a d\,\mathrm{v}'\,, \qquad (2)$$

where u is the velocity component along the electric field, σ_{res} is the cross section of resonant charge exchange, na is the density of atoms; ion □ and atom distribution functions are normalized to unity: $\int f(\mathrm{v})d\,\mathrm{v} = \int \varphi(\mathrm{v})d\,\mathrm{v} = 1$

Equation (2) describes the relay-race ion transport; this model was proposed by L. A. Sena [1, 2]. According to the model, the ion velocity after collision is equal to the velocity of the atom with which it collided. This model neglects a change in the atom velocity during collision.

To consider the effect of collisions, the Bhatnagar, Gross, Krook (BGK) model integral is often used to describe the relaxation of the ion distribution to the equilibrium distribution function of atoms with a characteristic relaxation time, which is assumed constant:

$$I_{st} = \frac{\varphi - f}{\tau_0}\,, \qquad (2)$$

which describes the ion distribution function f relaxation to the equilibrium atom distribution function φ with characteristic relaxation time τ_0 which is set constant. The BGK integral qualitatively accurately describes plasma relaxation to equilibrium in the case of a small deviation from it. However, this integral is inapplicable, if the ion–atom collision frequency depends on the relative ion velocity or the deviation from equilibrium is large.

The BGK model integral qualitatively correctly describes the process of plasma relaxation to equilibrium only in the case of small deviations from it. But BGK integral can not account for, in particular, the following factors:

1) collisions with constant cross section (resonance charge transfer, gas-kinetic collisions), as it does not consider the dependence of the probability of collision of speed;

2) for a constant-time collision (polarization collisions) it ignores ion velocity after scattering.

These factors are determining at a drift velocity comparable to the thermal velocity of atoms. Hence, the BGK integral is inapplicable to the problem of determining characteristics of ion drift in parent gas.

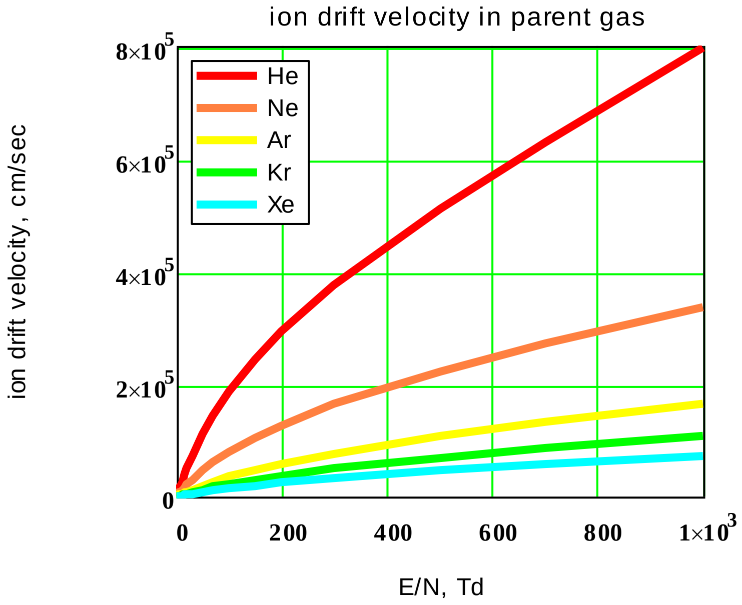

Fig. 1. Results of calculations of the ion drift velocity in own gas as a function of the reduced electric field strength.

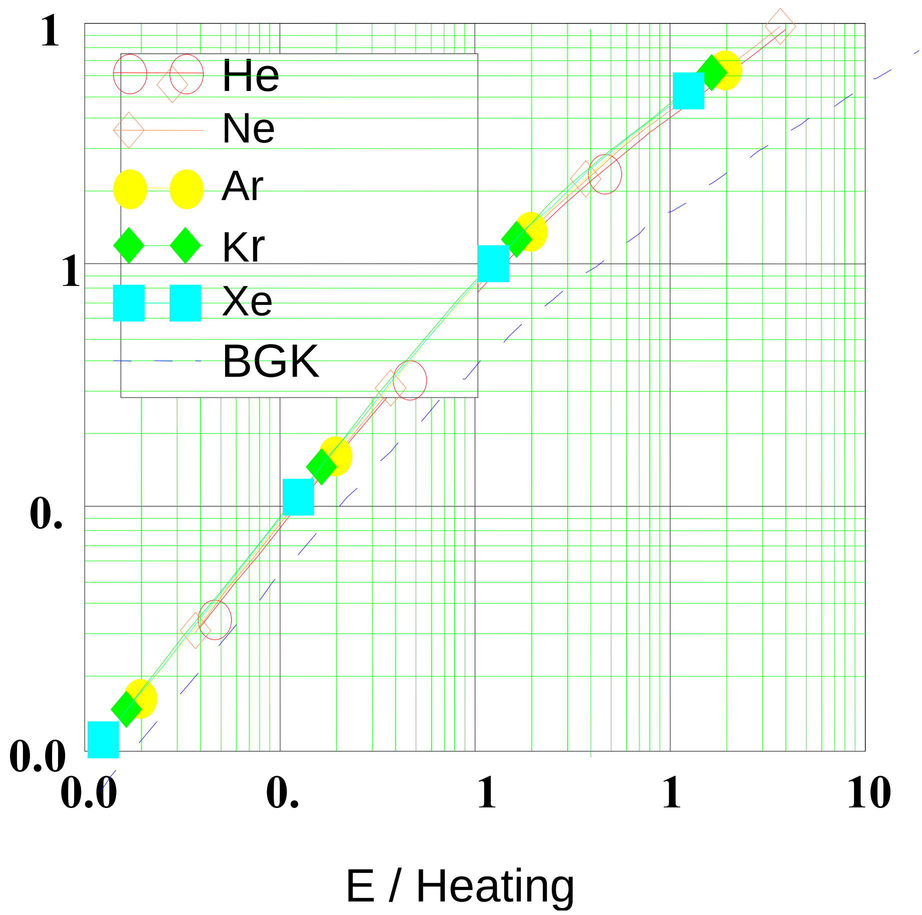

Fig. 2. Results of calculation of the ion drift velocity in own gas as a function of the electric field strength in dimensionless units. The drift velocity is normalized to the thermal velocity (the ion velocity with the energy equal to the atom temperature), the field is normalized to the characteristic "heating" field in which the energy equal to the atom temperature is gained in the mean free path. The dashed curve is the solution to the Boltzmann equation with the BGK collision integral.

Figure 1 shows the results of calculation of the ion drift velocity in own gas as functions of the electric field strength for all inert gases [5].

Figure 2 shows the same results in which the drift velocity is normalized to the thermal velocity (the velocity of the ion with energy equal to temperature) $W = u_d / V_T$, and the field is normalized to the characteristic "heating" field $F = E / E_T$ in which the energy equal to the atom temperature, $eE_T < \lambda_{st} > = T_a$ is gained in the mean free path. The dashed curve is the solution to the Boltzmann equation with the BGK collision integral from [6],

$$W = \frac{F(F^{1/2}+1)}{\pi^{1/2}(1+F^{1/2}+F)} . \quad (3)$$

An analysis showed that the approximately twofold difference is caused by the fact that backscattering collisions are not dominant even in the strong field.

Figure 3 shows the dependence of the fraction of backscattering collisions in the center-of-mass system in relation to the total number of collisions (small-angle scattering collisions are not included) on the reduced field strength.

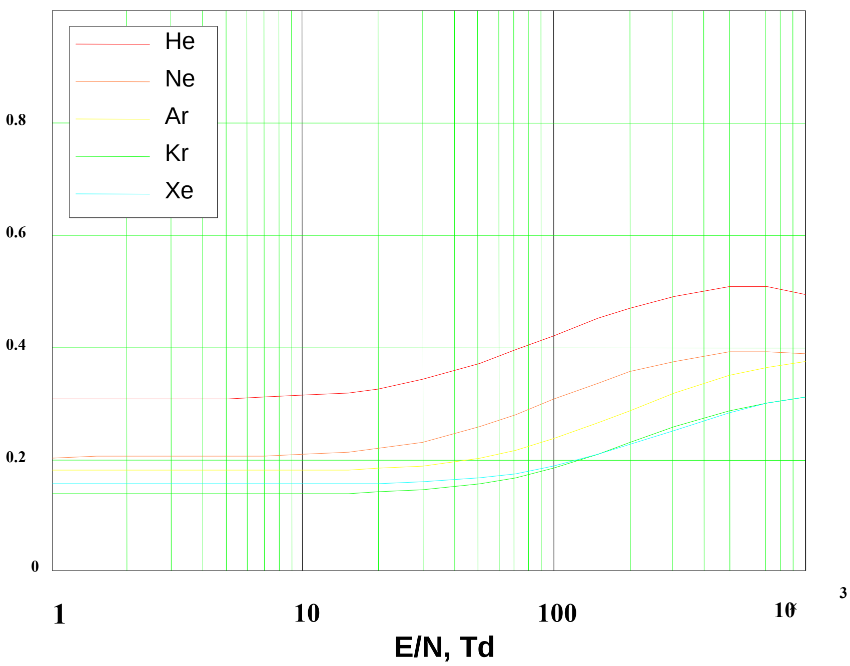

Fig. 3. Fraction of collisions with backscattering in relation to the total number of collisions as a function of the reduced field strength.

The presented figures allow the following conclusions.

(i) Introduction of dimensionless units makes it possible to reduce characteristics of different gases to universal curves.

(ii) The BGK collision integral for the problem of ion drift in own gas leads to significant errors, which does not allow description of actual processes even at the qualitative level (see, e.g., [7–9]).

(iii) An unexpected and nontrivial fact takes place: although the charge-exchange cross sections are the largest, the fraction of backscattering collisions is only 15–45% for noble gases at 300 K (in this connection, see [10], where it was attempted to approximate collisions as a sum of isotropic scattering and backscattering collisions).

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