Continuum SOC model of heat transport in magnetically confined plasmas

D. K. Oh[1], H. Jhang[1], R. Singh[2]

[1] *WCI Center for Fusion Theory, National Fusion Research Institute, Gwahangno 169-148, Yusung-Gu, Daejon, Republic of Korea 305-806*

[2] *Institute of Plasma Research, Bhat, Gandhinagar 382 428, Gujarat, India*

Self-organized criticality (SOC) has been dubbed a minimalist framework [1-2] among the theories of nonlinear transport in magnetic fusion devices. As this approach means a conceptual state of thermal flux ruled by the threshold of local instability [3], exact understanding of the physical ground is still lacking for avalanche-type transport, in spite of strong evidences, for instance, like scale invariance in the fluctuation data [4-5] and the propagating front in the numerical study [6]. Thus, it is essential to reconcile the avalanche transport in SOC with existing frameworks of the drift wave turbulence, especially related to the turbulent spreading with entrainment [7], non-Fickian constitutive relation of heat waves with weak non-locality [8]. In such a point of view, we investigate to the continuum SOC model [9] as a logical link for the physics-free SOC model to the description of plasma turbulence of avalanche-type energy transfer.

We show that a simple rearrangement of the continuum SOC model reveals the underlying principle of causality between the flux and its driving force (the gradient), where $Q = \chi_0$ or χ_1 *i.e.* $\chi_0 \rightarrow \chi_1$ if $|\nabla T| > g_c$, and $\chi_1 \rightarrow \chi_0$ if $|\nabla T| < \beta g_c$ ($\chi_1 > \chi_0$ and $0 < \beta < 1$).

$$\Gamma + \tau \frac{\partial}{\partial t}\Gamma - \tau\chi \frac{\partial^2}{\partial x^2}\Gamma = -Q[|\nabla T|]\ \nabla T \tag{1}$$

The uncovered constitutive relation implies the hyperbolic property in the governing equation, with finite length of spatiotemporal memory (a weak non-locality) against the critical condition imposed by Q, with which a numerical scheme can be developed to capture the propagating front of instability accurately. Accordingly, we can light the way of avalanche propagation in the continuum SOC as a “nonlinear wave”, in the context of “heat wave”. Moreover, the essential role of the nonlinear response against the threshold is discussed in such a wave propagation, showing the avalanche speed of as a function of characteristic speed $\upsilon_0 = \sqrt{(\chi_1 - \chi_0)/\tau}$ and the gradient along the wave-front g_{f} *i.e.* $\upsilon_{\mathrm{f}} = \upsilon_0\sqrt{g_{\mathrm{f}}/(g_{\mathrm{f}} - g_0)}$, and $\upsilon_0 < \upsilon_{\mathrm{f}} < \upsilon_0\sqrt{g_c/(g_c - g_0)}$.

We also present that the survival condition of the travelling wave implies the initial slope on the condition of multi-stable state against the threshold *i.e.* $\beta g_c < g_0 < g_c$. This gives not only an exact correspondence to the general assumption of meta-stability in SOC theories, but also a physical insight into an essential role of the subcritical turbulence to the avalanche-type thermal transport in the radial loss of confined energy.

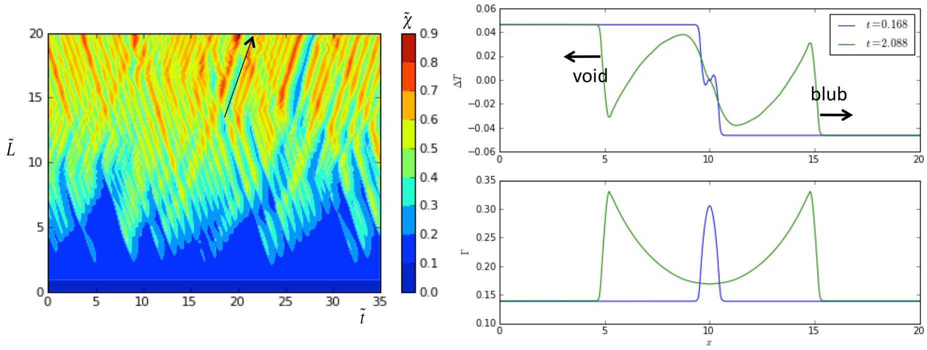

Fig. 1 The steady state profile of avalanches shown as a t-x diagram of instability indicated by the diffusion constant χ - the arrow is propagating direction of avalanche blub (left). The pair of single avalanche is generated from a stationary profile with respect to the step-wise perturbation at the center. The pair of packet can be described as travelling wave of nonlinear evolution. We showed perturbative profile *ΔT* (upper right) and the flux Γ (lower right)

The self-consistent interaction of the drift wave turbulence with the pressure gradient [10] has been considered as the simplest approach to the critical condition for the SOC transport of radial heat flux. In the analogy of $\chi(x,t)$ in the continuum SOC model, we can show that such an equation of turbulence energy $\dot{\mathcal{E}} = \gamma_0(R/L_T - (R/L_T)_c)\mathcal{E} - \mu\mathcal{E}^2$ can provide the constitutive relation similar to Eq. (1). Then, the characteristic speed of propagating front can be expressed as $\upsilon_0 = \sqrt{\mu\mathcal{E}(\chi_{neo} + \chi_1\gamma_0/\mu)}$, in which $(\chi_{neo} + \chi_1\gamma_0/\mu)$ represent the transport coefficient at steady state against the critical response, and $(\mu\mathcal{E})^{-1}$ has the meaning of time constant for retarded response, where χ_{neo} is neoclassical diffusivity and $\chi_1\mathcal{E}$ is turbulent one. However, even if such an analogy for the plasma turbulence can be supported by the recent study [8], we must pay attention to the survival condition of travelling wave, by which one can prove that the travelling wave of instability cannot be sustained because of the lack of multi-stability in the bifurcation characteristic of the turbulence equation. On this ground, we propose a possibility of avalanche propagation originated from an *ad hoc* component of multi-stability by nonlinear growth term as $\delta\,\mathcal{E}^{3/2}$, which is analogous to the Landau-Ginzburg approach of 1st order phase transition. To demonstrate the avalanche propagation from a localized instability (Fig. 2), we apply the subcritical turbulence model on the assumption of the critical gradient of the linear growth rate as $\gamma_L = \gamma_0(\frac{g - g_c}{g_c})$.

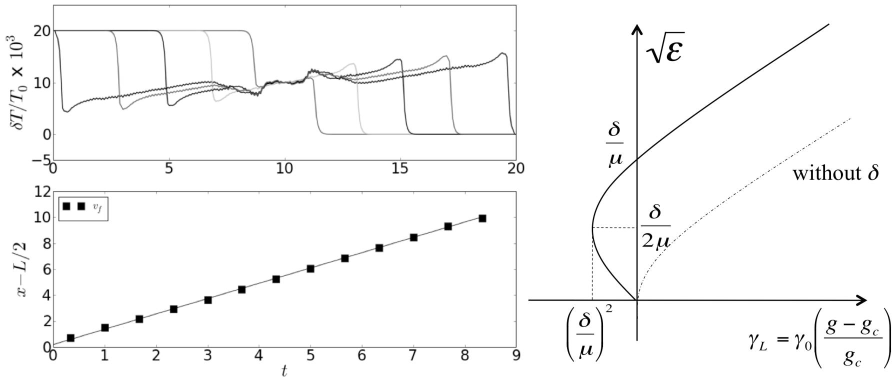

Fig. 2 Avalanche propagation in constant speed (left) is shown, which is ruled by non linear growth based on the model of turbulence evolution as $\dot{\varepsilon} = (\gamma_L + \delta\,\varepsilon^{1/2} - \mu\,\varepsilon)\varepsilon$. This model gives subcritical condition when the linear growth rate is $(\delta/\mu)^2 < \gamma_L < 0$ at steady state (right). The parameters are carefully chosen to achieve the condition of $\chi_t\varepsilon \sim 10\chi_{neo}$, letting the dimensionless quantity ε close to unity at the wave front. As shown in the right figure, corresponding hysteresis parameter 1-β can be determined as $(\delta/\mu)^2/\gamma_L$. For the simulation in the left figure, β is 0.96, and g_c = 0.295 (g_0= 0.29 so that $\beta g_c < g_0 < g_c$) so that the wave survives being consistent with our analysis.

Even if the nonlinear growth is introduced without physical ground, the idea of meta-stable condition for wave propagation implicates the relevance of the subcritical turbulence to the avalanche-type heat flux. We can discuss the possible origins of subcritical turbulence (or bifurcation of fluctuation level with the multi-stability) [11-13], which can play an essential role in the SOC paradigm of the magnetic plasmas.

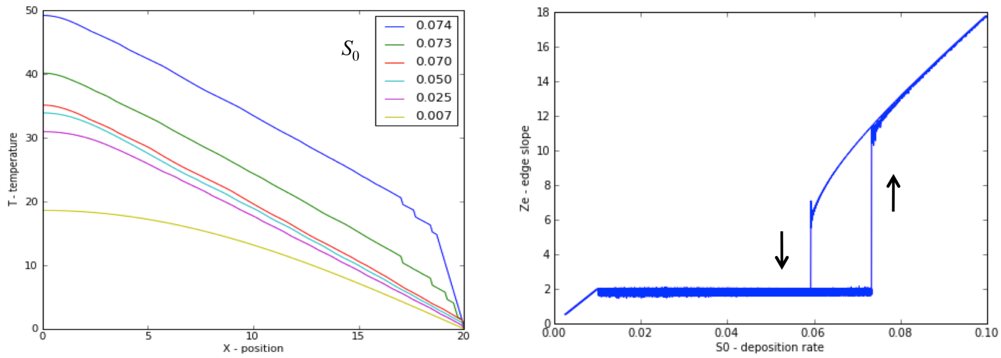

Fig. 3 The temperature profile (left) and the edge slope (right) with respect to the external heat load S_0 – all variables are normalized with the units t_0=100μs, x_0=2.5cm and T_0=250eV, where $\chi_1/\ \chi_0$=20, g_c=200eV/cm and S_0 is less then several MW.

As we validated the avalanche propagation mechanism of the continuum model, SOC paradigm of L-H transition was surveyed based on an early stage's approach of turbulence suppression by ExB shear flow. Being favourable on brewing a physical process in the model, because the model is continuum equation, the one-field continuum SOC model was applied to the temperature profile with L-H transition by adjusting the turbulent diffusivity according to the ExB mechanism depending on the gradient. Matching to the bi-stability of the gradient, which is the argument by Hinton, the result showed L-H transition successfully. The confinement time was estimated by simple analysis in which the Ohimic confinement just expresses gyro-Bohm scaling $\tau_E^{ohmic} = 4L^2/\pi^2\chi_0$, and the L-mode breaks it with the leading term of $g_c(1+\beta)L/4S_0$ [14]. According to the statistics of the confinement time, the energy confinement turned out to be determined by the occurrence of transport events *i.e.* avalanches. In spite the analysis was a preliminary attempt using the one-field approximation, our simplified approach gives light on the possibility to extend the continuum model to a multi-field description of L-H transition in fusion plasma. In consequence, the continuum model can be emerged as a proper theoretical approach for the SOC nature of fusion plasmas with many advantages.

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