Ferromagnetic destabilization of resistive wall modes in tokamaks

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1. Introduction. Some dangerous instabilities in tokamaks can be suppressed or at least partially stabilized by a nearby conducting wall. The wall stabilization plays an important role in the steady-state advanced scenarios developed for large tokamaks [1–4]. At the same time, theory predicts that a ferromagnetic wall may act as a destabilizer [5–7], though its negative effect cannot be dramatically strong at typical tokamak parameters. The most pessimistic prognosis [5] was up to 20% reduction in the stability limit at $\hat{\mu} \equiv \mu/\mu_0 = 4$, where μ is the magnetic permeability of the wall and μ_0 is the vacuum one. This can be counteracted by the stability control system [2] if properly accounted for in the feedback algorithms. The problem should be considered in relation with experimental plans on the JT-60SA tokamak where high-β operation is expected in combination with the use of ferritic materials [4]. Here we calculate the downward shift of the stability boundary of resistive wall modes (RWMs) in the presence of a resistive wall with $\hat{\mu} > 1$. The analysis is based on the dispersion relation for RWMs at $\mu \neq \mu_0$, which is obtained [6] by solving the external task with linear plasma response and hence can be coupled with any plasma model. This makes our approach essentially different from that in [5] and more universal.

2. Formulation of the problem. We consider a cylindrical plasma surrounded by a coaxial resistive ferromagnetic wall with radius r_w, thickness d_w, uniform conductivity σ and magnetic permeability $\mu \neq \mu_0$. The plasma-wall gap and space behind the wall are treated as a vacuum. In this case, the dispersion relation for the external kink modes is [6, 8]

$$\Gamma_m = \frac{y_i}{\hat{\mu}} \frac{hK_{m-1}(y_i) - gI_{m-1}(y_i)}{hK_m(y_i) + gI_m(y_i)} - m\frac{\hat{\mu}-1}{\hat{\mu}} \quad \text{with} \quad \frac{g}{h} = \frac{y_e K_{m-1}(y_e) - m(\hat{\mu}-1)K_m(y_e)}{y_e I_{m-1}(y_e) + m(\hat{\mu}-1)I_m(y_e)}, \tag{1}$$

where I and K are the modified Bessel functions, $y_i \equiv r_w\sqrt{\mu\sigma\gamma}$ and $y_e \equiv y_i(1 + d_w / r_w)$. This is derived for the (m,n) mode of the magnetic perturbation $\mathbf{b}$ depending on time as $\exp(\gamma t)$ with a growth rate γ, assuming also $nr_w/(mR) << 1$, where R is the major radius. The parameter Γ_m is determined by the plasma properties [6] through the boundary conditions for b_m (the radial component of $\mathbf{b}$) at the inner side of the wall, $r = r_w - 0$:

$$\Gamma_m = \Gamma_m^0 + m - 1 - (rb_m'/b_m)\big|_{in} \quad (2)$$

with $\Gamma_m^0 = -2m$ the value of Γ_m without plasma. In the analysis below, we consider Γ_m and γ real as in [8]. For slow RWMs, viz. for $\gamma\tau_w < r_w/(d_w\hat{\mu})$, Eq. (1) reduces to [6, 8]

$$\Gamma_m = \gamma\tau_w + \Gamma_m^c \quad (3)$$

with $\tau_w \equiv \mu_0\sigma\, r_w d_w$ and $\Gamma_m^c < 0$ determining the stability threshold $\gamma = 0$. Next we compare

$$\Gamma_N \equiv \frac{\Gamma_m^c}{\Gamma_m^0} = \frac{\hat{\mu}^2 - 1}{2\hat{\mu}} \frac{1-\varepsilon_w}{\hat{\mu}+1-\varepsilon_w(\hat{\mu}-1)} \quad \text{with} \quad \varepsilon_w \equiv \left(\frac{r_w}{r_w + d_w}\right)^{2m} \quad (4)$$

at various combinations of $\hat{\mu}$, d_w/r_w and m which are the key parameters here.

3. Computation results. It is clear that the downward shift of the stability boundary must increase with $\hat{\mu}$ from $\Gamma_N = 0$ at $\hat{\mu} = 1$ to $\max \Gamma_N = 0.5$ at $\hat{\mu} = \infty$. In between, for the $m = 2$ mode at $d_w/r_w = 0.02$ roughly corresponding to parameters of the DIII-D tokamak, Eq. (4) gives $\Gamma_N = 0.06$, 0.14, 0.34 and 0.49 (or 12, 28, 68 and 98% of the maximal Γ_N), respectively, for $\hat{\mu} = 4$, 10, 100 and 1000. With a thicker wall, Γ_N is larger. The normalized shift Γ_N is plotted versus $\hat{\mu}$ at several values of d_w/r_w in Fig. 1 and versus d_w/r_w in Fig. 2. At $\hat{\mu} = 4$ we have $\Gamma_N = 0.01$, 0.06 and 0.27 for $d_w/r_w = 0.002$, 0.02 and 0.2.

It follows from (4) that $\partial\Gamma_N/\partial\varepsilon_w < 0$, which means larger Γ_N at higher m and fixed d_w/r_w. The dependence of Γ_N on $\hat{\mu}$ for $m = 2$ (solid), 4 (dashed) and 8 (dashed-dotted lines) at

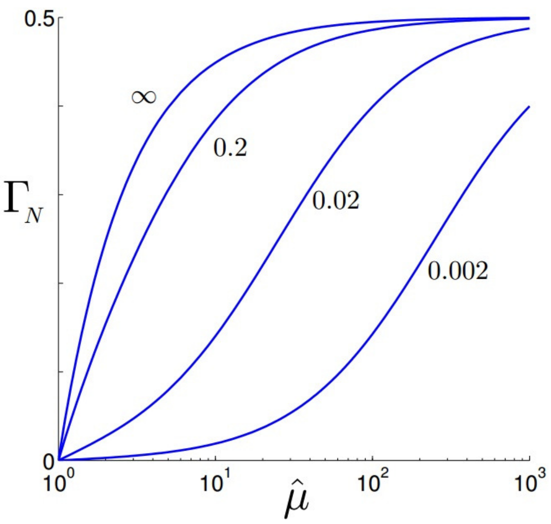

Fig. 1. Normalized shift Γ_N of the stability boundary vs. $\hat{\mu}$ for the $m = 2$ mode at d_w/r_w =0.002, 0.02, 0.2 and ∞, as indicated from bottom to top.

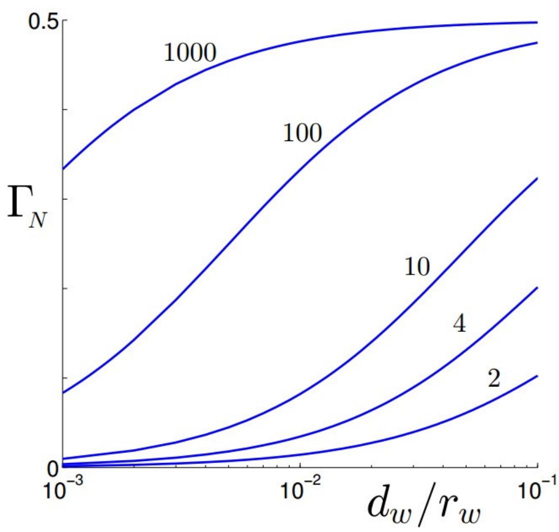

Fig. 2. Normalized shift Γ_N of the stability boundary vs. d_w/r_w for the $m = 2$ mode and $\hat{\mu}$=2, 4, 10, 100 and 1000, as indicated from bottom to top.

$d_w/r_w = 0.002$ (lower) and $d_w/r_w = 0.02$ (upper set of curves) is plotted in Fig. 3, while Fig. 4 shows Γ_N for the same values of m, but versus d_w/r_w at $\hat{\mu}=2$ (lower), 10 (intermediate) and 1000 (upper set of curves). At $d_w/r_w = 0.02$ and $\hat{\mu}=4$ Eq. (4) gives $\Gamma_N = 0.11$ at $m=4$ and 0.18 at $m=8$ ($\Gamma_N = 0.06$ at $m=2$), while with parameters relevant to the line-tied pinch experiments [9] ($d_w/r_w = 0.002$ and $\hat{\mu}=1000$) we have $\Gamma_N = 0.33$, 0.40, 0.44 and 0.47 at $m=1$, 2, 4 and 8, respectively.

4. Discussion. For arbitrary plasma with a linear response to external perturbations the downward shift of the stability boundary caused by the presence of a ferritic wall is described by Γ_N depending on $\hat{\mu}$, d_w/r_w and m only and equal zero at $\hat{\mu}=1$. Larger Γ_N means stronger effect and vice versa. In a linear model, Γ_N must be proportional to β for the pressure-destabilized modes. The stability deterioration in β up to 20% for $d_w/r_w \approx 0.06$ and $\hat{\mu}=4$ was stated in [5]. At these parameters Eq. (4) gives $\Gamma_N = 0.15$ for the $m=2$ mode and $\Gamma_N = 0.22$ for the $m=4$. In [5], the calculations have been performed for a plasma with parabolic current and pressure profiles. Being free from such constraints, our approach is more general. Let us add that our results do not confirm the (unexplained) conclusion in [5] that, with the permeability effect, the critical beta saturates or even decreases with the thickness of the wall above a threshold value of the thickness. On the contrary, we have always $\partial\Gamma_N/\partial\varepsilon_w < 0$ at $\hat{\mu}>1$ and any d_w/r_w.

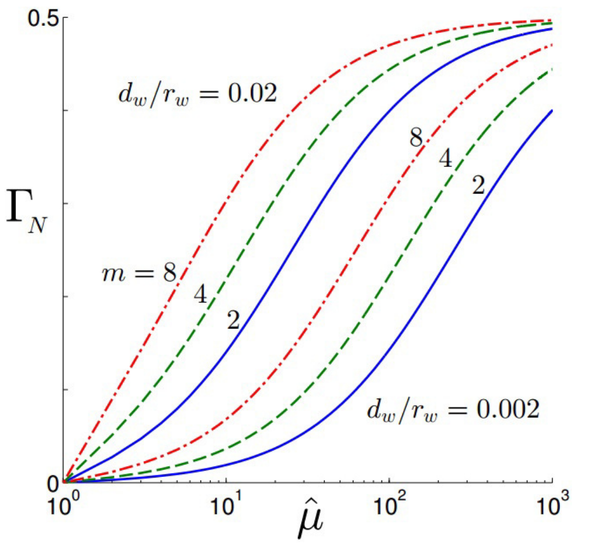

Fig. 3. Normalized shift Γ_N of the stability boundary vs. $\hat{\mu}$ for the modes with $m=2$ (solid), 4 (dashed) and 8 (dashed-dotted lines) at $d_w/r_w = 0.002$ (lower set) and $d_w/r_w = 0.02$ (upper set of curves).

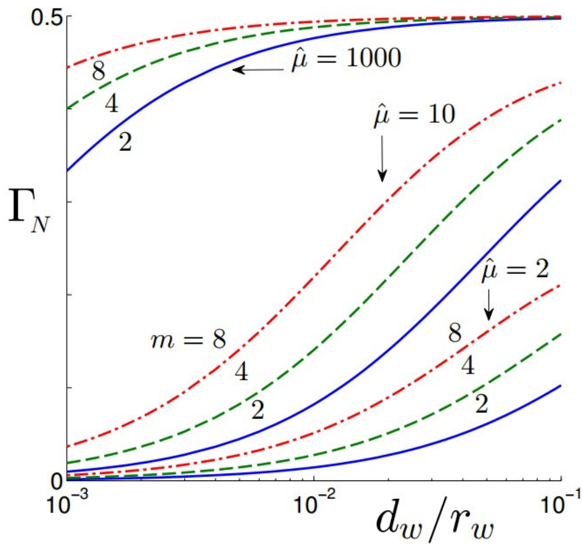

Fig. 4. Normalized shift Γ_N of the stability boundary vs. d_w/r_w for the modes with $m = 2$ (solid), 4 (dashed) and 8 (dashed-dotted lines) at $\hat{\mu}$=2, 10, 1000.

Small Γ_N at $\hat{\mu} = 2 - 4$ are consistent with results on JFT-2M with no adverse effect on plasma stability [10–12]. However, quite strong destabilization was observed in experiments on the line-tied pinch at $d_w / r_w = 0.002$, $m = 1$ and $\hat{\mu} = 1200$ [9]. This is also reproduced in our model: in this case we obtain $\Gamma_N = 0.35$ (70% of the maximal shift).

The maximal possible $\Gamma_N = 0.5$ gives $\gamma\tau_w = m$ at $\Gamma_m = 0$ (the stability boundary when $\hat{\mu} = 1$). The modes with such γ can be easily counteracted by feedback systems of modern tokamaks [2, 13]. More dangerous is the increase of γ by factor of $\hat{\mu}$ at $\Gamma_m > r_w / d_w$ [8].

5. Conclusion. The model predicts lowering of the stability boundary of RWMs with larger wall permeability, especially combined with larger d_w / r_w. It also shows that the stability deterioration must be more pronounced for RWMs with higher m. This can be of interest because the edge modes play an important role in some tokamak regimes. In the existing and future tokamaks (say, at $\hat{\mu} = 2 - 4$) the destabilizing effect cannot be strong, but closer look on the problem may be needed in designing the high-β experiments on JT-60SA tokamak [3, 4]. On the Wisconsin rotating wall machine [9] it can be useful to compare the stability deterioration with additional layers of the ferromagnetic foils until its saturation at $\Gamma_N = 0.5$. If necessary, our analysis can be extended to the cases with two walls as considered in [7].

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