Development of nonlinear collision operator for the Monte Carlo code in toroidal plasmas

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The nonlinear collision operator is formulated in order to study the collision effect between high energy particles. The Monte Carlo collision operator is derived to be easily implemented to the orbit following type code such as GNET. The nonlinear collision effect on the alpha particle confinement is investigated for heliotron reactor. It is found that the pitch angle scattering is enhanced and the energy loss fraction increases about 1% due to the nonlinear collision effect.

Introduction

In a D-T fusion plasma α particles (E =3.5MeV) are generated by the fusion reaction and those α particles mainly collide only with electrons because of very high velocity compared to the thermal ion ones. Thus it is considered that the pitch angle scattering is very small for the α particle during the energy slow down. On the other hand, if we consider the relative velocity between the α particles this relative velocity sometimes becomes very small. In this case they would experience an additional pitch angle scattering. Although the density of high-energy particle is much less than that of thermal other ions, the collisions between α-particles would have some effect on pitch angle scattering. It was reported that the nonlinear collisions by each fast α particle has sometimes larger effect than that by other background ions [1].

This nonlinear collision effect may lead to deteriorate the α particle confinement, because of increase of pitch angle scatterings. Especially, in helical systems, high-energy particle trajectories are complicated due to the three dimensional magnetic configuration and the small increase of pitch angle scattering might affect the high-energy particle confinement significantly. Therefore, the analysis including the both complicated orbit and the nonlinear collisions are necessary to make clear the α-particle confinement in toroidal plasmas. However, the nonlinear collision operator has not yet been formulated for the orbit following type of Monte Carlo code.

In this paper, we formulate the nonlinear collision operator and the Monte Carlo collision operator which can be easily implemented to the orbit following type codes is derived. We study the nonlinear collision effect on the alpha particle confinement in the heliotron reactor by GNET[2] applying the developed collision model.

Nonlinear Collision Model

We first consider the Fokker-Plank equation, which is useful for the study of plasma kinetic behaviors. Because Coulomb collisions in plasma are typically dominated by small pitch angle scattering, the collision term can be written using the flux as

$$C(f_a, f_b) = -\nabla \cdot \overleftrightarrow{S}_c^{a/b}, \tag{1}$$

where $\overleftrightarrow{S}_c^{a/b}$ is the collisional flux in velocity space and is expressed by Landau collision integral as

$$\overleftrightarrow{S}_c^{a/b} = \frac{q_a^2 q_b^2}{8\pi\varepsilon_0^2 m_a} \ln\Lambda^{a/b} \int \overleftrightarrow{U}(u) \cdot \left(\frac{f_a(v)}{m_b}\frac{\partial f_b(v')}{\partial v'} - \frac{f_b(v')}{m_a}\frac{\partial f_a(v)}{\partial v} \right) d^3v' \tag{2}$$

ε_0 is the dielectric constant of in vaccume, $\ln\Lambda^{a/b}$ is the Coulomb logarithm, and

$$\overleftrightarrow{U}(u) = \frac{u^2\overleftrightarrow{I} - uu}{u^3}, \quad u = v - v'. \tag{3}$$

Using an equivalent representation in terms of Rosenbluth potentials, we can rewrite the Eq. (2). Also applying the convenient notation of Trubnikov, we define two Rosenbluth potentials as

$$\phi_b(v') = -\frac{1}{4\pi}\int \frac{f_b(v')}{|v'-v|} d^3v', \quad \psi_b(v') = -\frac{1}{8\pi}\int |v-v'| f_b(v') d^3v'. \tag{4}$$

Thus, $\overleftrightarrow{S}_c^{a/b}$ can be expressed in terms of these potentials as

$$\begin{aligned} \overleftrightarrow{S}_c^{a/b} &= -\overleftrightarrow{D}_c^{a/b} \nabla f_a(v) + F_c^{a/b} f_a(v), \\ \overleftrightarrow{D}_c^{a/b} &= -\frac{4\pi\Gamma^{a/b}}{n_b}\nabla\nabla\psi_b(v), \quad F_c^{a/b} = -\frac{4\pi\Gamma^{a/b}}{n_b}\frac{m_a}{m_b}\nabla\phi_b(v), \end{aligned} \tag{5}$$

where $\overleftrightarrow{D}_c^{a/b}$ and $F_c^{a/b}$ are *a diffusion tensor* and *a friction force*, respectively, and

$$\Gamma^{a/b} = \frac{n_b q_a^2 q_b^2 \ln\Lambda^{a/b}}{4\pi\varepsilon_0^2 m_a^2}. \tag{6}$$

Monte Carlo operators which represent a random walk in the velocity space (v, λ) with a time step Δt are now given by changing the values of the invariants I^i at the n th step to those at the next step, as follows

$$I^i_{n+1} = I^i_n + \Delta I^i, \tag{7}$$

where the quantitie ΔI^i it the stochastic value. The expectation value (the friction force) and covariance (the diffusion) of them are given by

$$E(\Delta I^i) = \left[F_c^{a/b,i} + \frac{1}{g}\frac{\partial}{\partial I_j}\left(g \overleftrightarrow{D}_c^{a/b,ij} \right) \right] \Delta t, \quad C(\Delta I^i, \Delta I^j) = 2\overleftrightarrow{D}_c^{a/b,ij} \Delta t, \tag{8}$$

where $g = v^2$.

In general, the covariance matrix $C(\Delta I^i, \Delta I^j)$ is symmetric and positive definite, and can be diagonalized. We introduce the matrix $L^{kl} = \mathrm{diag}(\sqrt{\Lambda_1}, \sqrt{\Lambda_2})$, and write the covariance matrix as

$$C(\Delta I^i, \Delta I^j) = M^{ik} L^{km} L^{lm} M^{jl}. \tag{9}$$

From this relation, as well as Eqs. (8), ΔI^i is expressed as

$$\Delta I^i = E(\Delta I^i) + M^{ik} L^{kl} \xi^l, \tag{10}$$

where ξ^l is $\xi = \pm 1$ and this sign is chosen at random.

In order to evaluate the steady state distribution with the nonlinear collision model, we need the distribution of the energetic particle before the calculation. So we apply the iteration method to obtain the steady state distribution with the nonlinear collision model as shown in Fig. 1.

In this model, charge neutrality between background plasma and high-energy ions becomes important. In order to maintain the charge neutrality, $n_\mathrm{e} = n_\mathrm{i} + n_\mathrm{s}$,where n_e is the density of background electron, n_i is of background ion, and n_s is of high-energy ions spices s.

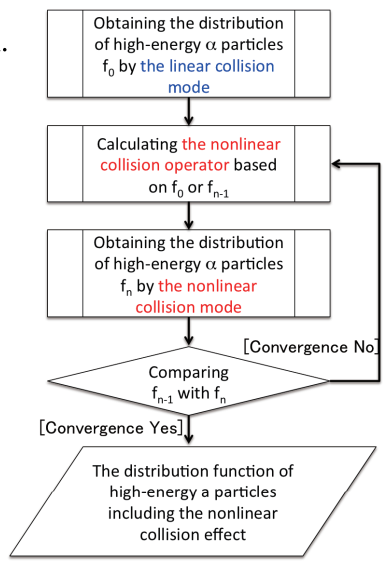

Figure 1: The flow chart of the nonlinear collision simulation model

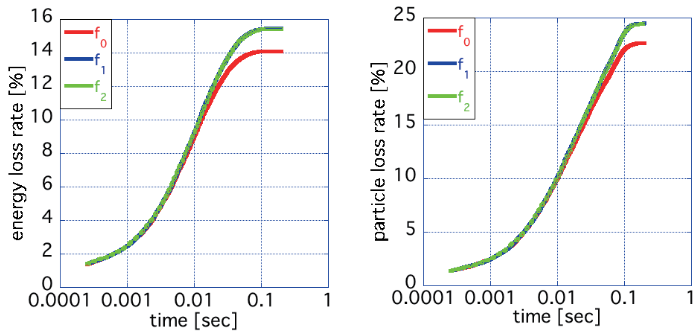

Figure 2: Effects of nonlinear collisions on the energy (left) and particle (right) loss fractions of alpha particles in the heliotron type reactor.

Simulation Results

We implement the nonlinear collision operator to GNET[2], which can solve the drift kinetic equation in the 5D phase space. We study the nonlinear collision effect on the alpha particle confinement in the heliotron reactor assuming FFHR-d1[3] configuration. The similar plasma parameters are assumed as in the previous papers[4, 5].

First we have evaluate the steady state distribution of alpha particle using the linear collision operator and, then, perform the iterative calculations using the nonlinear collision operator. The energetic ion distribution almost saturates in the two iterations, $f \sim f_2$.

It is found that the pitch angle scattering is enhanced by the nonlinear collisions and energetic trapped particle distribution is increased. As a result the loss of the energetic particle is increased. Figure 2 shows the energy and particle loss fractions of energetic alpha particle in the FFHR-d1. We compare the results of f_0 with the linear collision operator and the first and second iteration results with the nonlinear collision operator. We can see that the first and second iteration results show almost same results and that the loss fractions are increased a few percent by the nonlinear collisions.

References

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