

Turbulent Equipartition Pinch of Toroidal Momentum in Spherical Torus

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The TEP momentum pinch originates from the fact that magnetic curvature can modify acceleration of ions along the magnetic field, as can be appreciated from the modern nonlinear gyrokinetic equation [1]. When the magnetic curvature, $(\mathbf{b} \cdot \nabla)\mathbf{b}$, changes its direction as one moves from the low B field (bad curvature) side to the high B field (good curvature) side, the variation of fluctuation amplitude along the magnetic field (a property of ballooning fluctuations in toroidal geometry) can yield a net acceleration. This symmetry breaking mechanism due to magnetic curvature [2], alongside the k_{\parallel} -symmetry breaking due to the $\mathbf{E} \times \mathbf{B}$ shear responsible for the residual stress [3], constitutes the unified “ \mathbf{B}^* - symmetry breaking” as discussed in [2]. We show that a careful treatment of geometric effects due to nonuniform \mathbf{B} with nonvanishing curvature yields a novel pinch mechanism for parallel angular momentum density.

The nonlinear electrostatic gyrokinetic equation with proper conservation laws in general geometry is given by Eqs. (19), (21) and (22) of [1] :

$$\frac{\partial F}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \nabla F + \frac{dv_{\parallel}}{dt} \frac{\partial F}{\partial v_{\parallel}} = 0 \quad (1)$$

with

$$\frac{d\mathbf{R}}{dt} = v_{\parallel} \frac{\mathbf{B}^*}{B^*} + \frac{c\mathbf{b}}{e_i B^*} \times [e_i \nabla \langle \delta \phi \rangle] + m_i \mu \nabla B, \quad (2)$$

and

$$\frac{dv_{\parallel}}{dt} = -\frac{\mathbf{B}^*}{m_i B^*} \cdot [e_i \nabla \langle \delta \phi \rangle + m_i \mu \nabla B]. \quad (3)$$

Here, the gyrokinetic Vlasov equation, Eq. (1) is written in terms of the gyro-center distribution function $F(\mathbf{R}, \mu, v_{\parallel}, t)$, with $\mu \equiv v_{\perp}^2/2B$, and $\langle \dots \rangle$ denotes an average over the gyrophase. \mathbf{B}^* is defined by

$$\mathbf{B}^* \equiv \mathbf{B} + \frac{m_i c}{e_i} v_{\parallel} \nabla \times \mathbf{b}$$

We can derive the nonlinear evolution of the parallel momentum density per ion mass, $nU_{\parallel} \equiv 2\pi \int d\mu dv_{\parallel} B^* F v_{\parallel}$, by taking a moment of the nonlinear gyrokinetic equation, Eq. (1), or equivalently of a conservative form of the nonlinear gyrokinetic equation (Eq. (24) of [1]):

$$\frac{\partial (FB^*)}{\partial t} + \nabla \cdot \left(FB^* \frac{d\mathbf{R}}{dt} \right) + \frac{\partial}{\partial v_{\parallel}} \left(FB^* \frac{dv_{\parallel}}{dt} \right) = 0. \quad (4)$$

With the Mach number using the sound speed $M_s \equiv \frac{U_0}{C_s}$, we adopt an ordering $k_\theta \rho_s > \frac{a}{qR} M_s$, and assume $M_s < 1$ so that we can ignore $\mathbf{B} \cdot \nabla n U_\parallel^2$ in comparison to $\mathbf{B} \cdot \nabla P_\parallel$. The pressure moments are defined as usual. With these considerations, we can write a nonlinear evolution equation for the parallel momentum, by multiplying Eq. (4) by v_\parallel and integrating over the velocity space, to obtain

$$\begin{aligned} \frac{\partial}{\partial t}(m_i n U_\parallel) = & -c \mathbf{b} \times \nabla \delta \phi \cdot \nabla \left(\frac{m_i n U_\parallel}{B} \right) - \frac{2c m_i n U_\parallel}{B} \mathbf{b} \times (\mathbf{b} \cdot \nabla) \mathbf{b} \cdot \nabla \delta \phi \\ & - \frac{m_i c}{e_i} \mathbf{b} \times \nabla B \cdot \nabla \left(\frac{P_\perp U_\parallel}{B^2} \right) - 3 \frac{m_i c}{e_i} \mathbf{b} \times (\mathbf{b} \cdot \nabla) \mathbf{b} \cdot \nabla \left(\frac{P_\parallel U_\parallel}{B} \right) \\ & - n_i e_i \mathbf{b} \cdot \nabla \delta \phi - \mathbf{b} \cdot \nabla P_\parallel. \end{aligned} \quad (5)$$

The 2nd term on the RHS of Eq. (5) originates from the magnetic curvature modification of the parallel acceleration in Eq. (3). The last two terms are the origin of the $\mathbf{E} \times \mathbf{B}$ shear. This has been known to produce a nondiffusive radial flux of the parallel flow and reviewed in [4]. The physics of residual stress has been extensively discussed in [5]. Therefore, from this point, we don't keep these terms in this paper, which focuses only on the inward pinch driven by toroidal effects.

In addition, as identified in [2], the 3rd and 4th terms on the RHS of Eq. (5) are responsible for the geodesic curvature driven momentum flux which is subdominant to the standard $\mathbf{E} \times \mathbf{B}$ fluctuation induced momentum flux. It is also obvious that these terms vanish in the cold ion limit of $T_i/T_e \rightarrow 0$. An expression related to the magnetic curvature in the second term on the RHS of Eq. (5) can be expressed as

$$\mathbf{b} \times (\mathbf{b} \cdot \nabla) \mathbf{b} = (\nabla \times \mathbf{b})_\perp = \frac{\mathbf{b}}{B^2} \times \nabla \left(\frac{B^2}{2} + 4\pi P \right) \quad (6)$$

using the MHD equilibrium condition $\frac{1}{c} \mathbf{J} \times \mathbf{B} = \nabla P$. Note that in our previous works on momentum pinch in conventional high aspect ratio tokamaks [2], a low- β approximation has been used dropping the last term proportional to ∇P . Then, Eq. (5) can be further reduced to

$$\frac{\partial}{\partial t} \delta(n U_\parallel) = -c \mathbf{b} \times \nabla \delta \phi \cdot \nabla \left(\frac{n U_\parallel}{B} \right) - 2 \frac{c n U_\parallel}{B^3} \mathbf{b} \times \nabla \left(\frac{B^2}{2} + 4\pi P \right) \cdot \nabla \delta \phi \quad (7)$$

The radial flux of angular momentum carried by fluctuating $\mathbf{E} \times \mathbf{B}$ flow is

$$\begin{aligned} \Pi_{Ang} &= \left\langle \sum_{\mathbf{k}} \delta(M_i n U_\parallel R)_{\mathbf{k}} \delta u_{\mathbf{E}\mathbf{k}}^* \right\rangle \\ &= M_i R^2 \left\langle \sum_{\mathbf{k}} \delta(n \omega_\phi)_{\mathbf{k}} \delta u_{\mathbf{E}\mathbf{k}}^* \right\rangle \end{aligned} \quad (8)$$

From now, we use the angular frequency of toroidal rotation, $\omega_\phi = u_\parallel/R$, which is usually a flux fluctuation as the main variable, With this in mind, Eq. (7) can be written in the following form,

$$\begin{aligned} \frac{\partial}{\partial t} \delta(n\omega_\phi) &= -\frac{c\mathbf{b} \times \nabla \delta\phi}{B} \cdot \left[\nabla(n\omega_\phi) - n\omega_\phi \left\{ -\frac{B}{R} \nabla \left(\frac{R}{B} \right) + \frac{2}{B^2} \nabla \left(\frac{B^2}{2} + 4\pi P \right) \right\} \right] \\ &= -\delta\mathbf{u}_E \cdot \left[\nabla(n\omega_\phi) - n\omega_\phi \left\{ \frac{R}{B^3} \nabla \left(\frac{B^3}{R} \right) + \frac{8\pi}{B^2} \nabla P \right\} \right] \end{aligned} \quad (9)$$

Eq. (9) shows that the fluctuations in $n\omega_\phi$ is not only driven by the radial gradient of $n\omega_\phi$, which eventually leads to a diffusive momentum flux, but also by the gradients of B^3/R and of P . We can further arrange it as follows:

$$\delta(n\omega_\phi) = -(-i\omega_{\mathbf{k}} + \triangle\omega_{\mathbf{k}})^{-1} \delta v_{\mathbf{r},\mathbf{k}} \left[\frac{\partial}{\partial r} (n\omega_\phi) - n\omega_\phi \left\{ \frac{R}{B^3} \frac{\partial}{\partial r} \left(\frac{B^3}{R} \right) + \frac{8\pi}{B^2} \frac{\partial P}{\partial r} \right\} \right] \quad (10)$$

From Eq. (9) and (10), we obtain the radial flux of angular momentum,

$$\Pi_{Ang,r}/M_l R^2 = \left\langle \sum_{\mathbf{k}} \delta(n\omega_\phi) \delta v_{\mathbf{r}}^* \right\rangle = -\chi_{Ang} \frac{\partial}{\partial r} (n\omega_\phi) + V_\phi^{TEP} n\omega_\phi \quad (11)$$

with the angular momentum diffusivity,

$$\chi_{Ang} = \left\langle \sum_{\mathbf{k}} (Re\tau_{\mathbf{k}}) |\delta v_{\mathbf{r},\mathbf{k}}|^2 \right\rangle, \quad (12)$$

and the TEP pinch

$$V_{Ang}^{TEP} = \left\langle \sum_{\mathbf{k}} (Re\tau_{\mathbf{k}}) |\delta v_{\mathbf{r},\mathbf{k}}|^2 \left[\frac{R}{B^3} \frac{\partial}{\partial r} \left(\frac{B^3}{R} \right) + \frac{8\pi}{B^2} \frac{\partial P}{\partial r} \right] \right\rangle \quad (13)$$

with $Re(\tau_{\mathbf{k}}) = Re\left(\frac{1}{-i\omega_{\mathbf{k}} + \triangle\omega_{\mathbf{k}}}\right)$.

An expression in Eq. (13) should be understood as a fluctuation intensity ($\propto |\delta \mathbf{v}_{\mathbf{r},\mathbf{k}}|^2$) weighted flux surface average. Therefore, assuming that the fluctuation amplitude peaks strongly at the low field side, the final expression for the TEP momentum pinch satisfies the relation

$$V_{Ang}^{TEP}/\chi_{Ang} = \left(\frac{R}{B^3} \frac{\partial}{\partial r} \left(\frac{B^3}{R} \right) + \frac{8\pi}{B^2} \frac{\partial P}{\partial r} \right), \quad (14)$$

where the expression can approximately be evaluated at the point of the low field side mid-plane for a given flux surface. Of course, the minus sign expected for typical equilibrium B and P indicates an inward pinch. Note that Eq. (14) reduces to our previous expression of $-4/R$ in the limit of high aspect ratio ($B \propto \frac{1}{R}$) and low- β ($p \ll \frac{B^2}{8\pi}$) [2]. Figure 1 illustrates the pinch to diffusion ratio expected from typical equilibria of NSTX and VEST [7] spherical tori. We

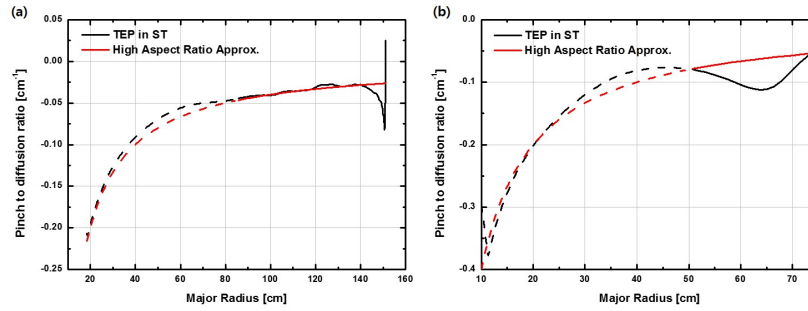


Figure 1: Pinch to diffusion ratio for typical equilibria of (a) NSTX ($\beta = 0.11$) and (b) VEST ($\beta = 0.04$). The black lines correspond to our new result in Eq. (14). The red lines are $-4/R$ from Ref. [2].

also remind readers that the $\partial P / \partial r$ term in Eq. (14) shows up when we convert the magnetic curvature term $(\nabla \times \mathbf{b})_{\perp}$ contained in the \mathbf{B}^* term in the conservative modern gyrokinetics [1], to $\frac{\mathbf{b}}{B^3} \times \nabla \cdot (\frac{B^2}{2} + 4\pi P)$. Therefore, we should still consider it as a part of TEP momentum pinch which originates from the magnetic field inhomogeneity. We also note that it's total P , not P_i which appears in an ITG-specific formula [6].

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