Heat equilibrium and stability of L-mode in impurity seeded tokamak plasmas

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1. Introduction

One of the most important problems of tokamak-reactor is the problem of the critical density. Particularly, the thermal quench may initiate the discharge disruption. The periphery temperature falls down, thermal front achieves the magnetic surface $q = 2$, and initiate the fast MHD mode. The thermal equilibrium is analyzed in the present paper. The equilibrium description including SOL is performed. The stability of the mode $m = 0$, $n = 0$ is discussed. As it will be shown below the mode is stable for any possible equilibrium.

2. Boundary condition

Boundary condition at the Last Closed Magnetic Surface (LCMS) is significantly two dimensional. Nevertheless, one dimensional approximation is used in many papers, and one dimensional boundary condition models two dimensional one. Two types of conditions at the LCMS are accepted in different papers. The simplest one $T(r = a) = 0$ is used [1-3]. More complex condition is proposed in papers [4-6],

$$-\kappa_{\perp}\frac{\mathrm{d}T}{\mathrm{d}x}=\frac{T_{\mathrm{LCMS}}}{\delta_T}. \qquad (1)$$

Here $\kappa_{\perp}$ is the perpendicular heat conductivity, $x = r / a$, a is the radius of the Last Closed Magnetic Surface (LCMS), $T_a = T_{\mathrm{LCMS}}$ is the temperature at radius of the LCMS.

As it will be shown below one can receive the simplified boundary condition $T(r = a) = 0$ as a limit case of the small depth of SOL.

One has to match the solution of the heat equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\kappa_{\perp}\frac{\mathrm{d}T}{\mathrm{d}x}\right)=a^2 n n_{\mathrm{I}} L \qquad (2)$$

inside LCMS and the solution in SOL. Here n and n_{I} are the plasma and impurity densities respectively, and L is the radiation function. The equation describing the heat flow in SOL takes the form

$$\kappa_{\perp}\frac{\partial^2 T}{\partial x^2}+\kappa_{\parallel}\frac{\partial^2 T}{\partial l^2}=a^2 n n_I L(T)\,. \qquad (3)$$

Inside SOL one can write $\frac{\partial}{\partial l}=\frac{H_r}{H_T}\frac{\partial}{\partial x}=\frac{\varepsilon}{q}\frac{\partial}{\partial x}$. The equality $H_r \approx H_\theta$ is supposed. The equation (3) may be rewritten as

$$\frac{\partial^2 T}{\partial x^2}\left(\kappa_{\mathrm{SOL}}\right)=n n_{\mathrm{I}} L(T) \qquad (4)$$

with the boundary condition $T(x = x_{\mathrm{w}}) = 0$, $x_{\mathrm{w}} = r_{\mathrm{w}} / a$. The functions $\kappa_{\perp}$ and κ_{SOL} are assumed to be constant. The standard model for radiation losses is used,

$$L=0,\ \text{if}\ T_1,\ \text{and}\ L_0=\text{const},\ \text{if}\ T<T_1\,. \qquad (5)$$

Matching of the solutions at the points $x = x(T_1)$, $x = 1$, and $x = x_{\mathrm{w}}$ one can find

$$T=\frac{A_2}{2}y^2+B_2 y,\ y<y_a;\qquad T=\frac{A_1}{2}y^2+B_1 y+\frac{A_1-A_2}{2}y_a^2,\ y_a<y<y_1\,. \qquad (6)$$

Here

$$A_1=a^2 n n_{\mathrm{I}} L_0/\kappa_{\perp},\ A_2=a^2 n n_{\mathrm{I}} L_0/\kappa_{\mathrm{SOL}},\quad B_1=\pm\sqrt{a^2M^2+A_1\left(A_1-A_2\right)y_a^2-2A_1T_1};$$

$$B_2=B_1-\left(A_1-A_2\right),\quad \mathrm{y}_1=x_{\mathrm{w}}-x_1,\ y_a=x_{\mathrm{w}}-1,\ M=-\kappa_{\perp}a\left.\frac{dT}{dy}\right|_{y=y_1}.$$

The temperature must increase with y. Hence, $B_1 > 0$, and only one sign in the expression for B_1, and only one solution are acceptable. Also, one can write

$$\left(x_{\mathrm{w}}-x_1\right)\alpha L_0=M\left(1-\sqrt{1-\frac{\alpha L_0}{M^2}\left(2-\alpha L_0\left(x_{\mathrm{w}}-1\right)\right)}\right). \qquad (7)$$

The left hand side of (7) is equal to the re-radiated power. The critical regime is related to the total re-radiated power equal to the total power input. Usually $x_{\mathrm{w}} - 1 << 1$, and total radiation losses inside SOL are small. Hence, one can put $T(1) = 0$.

3. Critical equilibrium

As it is noted above $T(x = a) = T\,'(x = a) = 0$ for the critical density. Equilibrium has been analyzed in the earliest papers (in particular, in [4-6]) solving the equation (2). For simplicity the heating source has been localized at the plasma center.

Multiplying equation (2) by $\mathrm{d}T/\mathrm{d}y$ and integrating over y one can get

$$\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)^2=C+Qnn_{\mathrm{I}}\int_0^T L(T)dT\,. \qquad (8)$$

The function d^2T/dy^2 is positive, and one can find.

$$\frac{dT}{dy} = \sqrt{M^2 - Qnn_I \int_T^{T(1)} L(T)dT} \ . \qquad (9)$$

The function $L(T)$ decreases with the temperature rapidly [7] for light impurities. The upper limit in the integral may be replaced by the value T^*. The critical density is defined by the condition

$$M^2 = Qnn_I \int_T^{T(1)} L(T)dT \text{, or } nn_I = M^2 / Qg,\ \ g = \int_0^{T^*} L(T)dT \qquad (10)$$

For example, for Carbon $g \approx 6.7 \cdot 10^{-18}$ erg^{2}cm^{2}s^{-1}. Here the bremsstrahlung is ignored.

One has to compare the condition (10) and Greenwald's limit $n_c \sim I$ [8]. The Ohmic heating power M is defined by the expression $M = (I^2\rho)/(\pi a^2)$, where I is the total toroidal current, and ρ is the plasma column resistance. The plasma resistance depends on the temperature, $\rho = A_1 T^{-3/2}$. Total energy balance yields $3nT/\tau_E = M$. The energy confinement time is defined by the empiric scaling [9] $\tau_E \sim A_2 I^{\alpha} n^{\beta} M^{-\gamma}$. One can find $T = \frac{A_2}{3} I^{\alpha} n^{\beta-1} M^{1-\gamma}$, and $M \sim I^{\frac{4-3\alpha}{3\gamma-1}} n^{\frac{3(\beta-1)}{3\gamma-1}}$. For the critical density $nn_I \sim M^2$. Ohmic scaling [10] $\tau_{OH} \sim q^{0.5} \cdot n^1 \sim I_p^{-0.5} \cdot n^1,\ \gamma = 0$ yields the critical density to be approximately proportional to the total current and looks like the Greenwald criterion, $n_c \approx I^{1.04}$.

4. Stability problem

The stability of the discharge based on the radiation model (5) has been investigated using boundary conditions $T(1) = 0$ ([1]), and (1) ([4,5,6,8]) respectively. Drake [1] has been shown the mode $m = 0$, $n = 0$ to be stable under the boundary condition $T(1) = 0$ using the simplest model for radiation losses. S. Deshpande [3] also has found the stability with the exponential function $L(T) \sim \exp(-T/T_0)$. It is shown below that the plasma is stable for more general radiation model.

As it well known the development of axially symmetric mode corresponds to the thermal disruption. The temperature perturbation $\tilde{T}$ is described by the equation

$$\frac{\partial^2 \tilde{T}}{\partial y^2} - 3\frac{\gamma a^2}{\kappa_\perp}\tilde{T} - \alpha\frac{\partial L}{\partial T}\tilde{T} = 0 \, . \qquad (11)$$

Here γ is the growth rate, $\alpha = a^2 nn_I/\kappa_\perp$. The equation (11) coincides with the Schrödinger equation. The thermal equilibrium is stable if there is no energy level in the potential well. In order to get the sufficient condition of the positive γ existence one can expand the potential well and to transform it into the rectangular potential box,

$$L = L_0 \frac{T}{T_1} + L_1, \text{ if } T < T_1; \quad L_0 - \alpha\left(T - T_1\right), \text{ if } T_1 < T < T_1 + \frac{L_0}{\alpha}; \quad 0, \text{ if } T > T_1 + \frac{L_0}{\alpha}. \qquad (12)$$

Usually the value T_1 is small, $T_1 << T_2$ and one can put $T_1 = 0$. The equilibrium is stable if $nn_{\mathrm{I}} < \frac{\pi^2}{4} \frac{M^2 \kappa_\perp}{L_0^2 T_2 a^2}$. The instability cannot develop if the equilibrium exists.

5. Conclusion

Two thermal equilibria in tokamaks are discussed. Thermal balance in SOL is included into consideration in the present paper. It is shown that only one thermal equilibrium exists. The critical plasma density is shown to be defined by the balance of the radiation losses and the power input. The condition of the thermal quench for Ohmic heating $n_c \approx I^{1.1}$ is close to Greenwald condition, $n_c \approx I$. The thermal stability is investigated. It is shown that the equilibrium is violated earlier than the thermal instability appears.

Acknowledgements

This work was partially supported by the grant No.3328.2014.2 of President of Russian Federation for Leading Scientific Schools.

References

[1] Drake J F 1987 *Phys. Fluids* **30** 2429

[2] Rebut P H and Green B J 1977 *Proc. 6th IAEA Conf. on Plasma Phys. and Controll. Nucl. Fusion Res.* vol 2 p IAEA-CN-35/B6

[3] Deshpande S 1993 *Phys. Plasmas* **1** 127

[4] Ohyabu N 1979 *Nucl. Fusion* **9** 1491

[5] Tokar M Z 2000 *Phys. Plasmas* **7** 2432

[6] Morozov D Kh and Mavrin A A 2014 *Contrib. Plasma Phys* **54** (to be published)

[7] Post D E *et al* 1977 *Atomic data and Nuclear Data Tables* **20** 397

[8] Greenwald M *et al* 1988 *Nucl. Fusion* **28** 2199

[9] ITER Physics Basis 1999 *Nucl. Fusion* **39** 2208

[10] Bracco G and Thomsen K 1997 *Nucl. Fusion* **37** 759