Revisited thin-wall model of wall forces produced by kink modes in tokamaks

D.V. Mironov and V.D. Pustovitov

National Research Centre "Kurchatov Institute", Moscow 123182, Russia,

Moscow Institute of Physics and Technology, Dolgoprudny 141700, Moscow Region, Russia

1. Introduction. This task is related to evaluation of the forces on the conducting structures in ITER during plasma disruptions [1, 2]. In [1], an analytic approach has been proposed to calculate the wall forces produced by kink modes in tokamaks. The expressions have been derived within the thin-wall cylindrical model that is widely used in the resistive wall mode (RWM) studies, see explanations and references in [3–5]. In addition to the standard thin-wall constraints, in [1] the toroidal current density j_z in the plasma was assumed constant. Also, the calculations there have been performed by using a truncated formula for the wall force. Later the result has been confirmed in [2].

Here we include the term intentionally disregarded in the starting formula in [1] and prove that it should be retained in the case considered. Besides, we do not impose any restriction on the plasma parameters. Though in other aspects we follow the same route as in [1], the first improvement essentially changes the final result compared to those in [1, 2]. The second one extends the applicability range of the predictions by allowing realistic current profiles. Besides, our model allows for the mode rotation as another new element.

2. Formulation of the problem. As in [1], we consider a cylindrical plasma with nearby resistive wall of radius r_w, thickness d_w and uniform resistivity η. The plasma-wall gap and space behind the wall are treated as vacuum. The magnetic field is described as $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, where $\mathbf{B}_0$ is the axisymmetric equilibrium field ($\partial\mathbf{B}_0 / \partial t = 0$) and $\mathbf{b}$ is the perturbation. The latter induces the eddy current $\mathbf{j} = \sigma\mathbf{E}$ in the wall, where $\mathbf{E}$ is the electric field governed by $\nabla\times\mathbf{E} = -\partial\mathbf{b}/\partial t$. In the wall, the volume density of the force is

$$\mathbf{f} = \mathbf{j}\times\mathbf{B} = \mathbf{j}\times\mathbf{B}_0 + \mathbf{j}\times\mathbf{b}\,. \qquad (1)$$

Following [1], we take a radial component of the linear term and introduce

$$f_{r1} \equiv \frac{1}{d_w}\int_{in}^{out}(\mathbf{j}\times\mathbf{B}_0)\cdot\mathbf{e}_r dr \qquad (2)$$

with integration across the wall. If the wall is geometrically thin ($d_w << r_w$), we have

$$\int_{in}^{out} \mathbf{j} dr = \mathbf{e}_r \times \mathbf{b}\Big|_{in}^{out} \qquad \text{so that} \qquad f_{r1} d_w = \mathbf{B}_0 \cdot \mathbf{b}\Big|_{out}^{in} . \tag{3}$$

This corresponds to Eq. (36) in [1]. We use it as a starting point.

3. Calculation of the force. Here $\mathbf{b} = \nabla\varphi$ is the vacuum field at the wall surfaces. Therefore,

$$mRb_\zeta = -nrb_\theta \tag{4}$$

for a single-mode perturbation depending on the poloidal and toroidal angles θ and ζ as

$$u = \mathrm{Re}[u_c(r,t)\exp(im\theta - in\zeta)] \tag{5}$$

with different (complex) u_c at the opposite sides of the wall. Trivially obtained from $\mathbf{b} = \nabla\varphi$ with φ in the form (5), relation (4) does not need any comment except that it differs from (incorrect) equation (38) for b_ζ in [1]. The consequences will follow.

With (4), Eq. (3) yields

$$f_{r1} = \frac{B_{0\theta}}{md_w}(m - nq)\cdot b_\theta\Big|_{out}^{in} , \tag{6}$$

where $B_{0\theta}$ and $q \equiv rB_{0\zeta}/(RB_{0\theta})$ are taken at the wall ($r = r_w$), R is the major radius.

Next we have to calculate the right-hand side of (6) and compare f_{r1} with the result of [1]. For perturbations prescribed by (5), at $nr << mR$ equation $\nabla^2\varphi = 0$ gives us

$$\varphi_c = -\frac{r_w}{m} B_m \left[x^{-m} + \frac{\Gamma_m}{2m}(x^{-m} + x^m) \right] \tag{7}$$

in the plasma-wall vacuum gap with $x \equiv r/r_w$ and amplitudes B_m and Γ_m. Then

$$b_{\theta c}\Big|_{in} = -i(1 + \Gamma_m/m) b_{rc}(r_w) , \tag{8}$$

where $b_{rc} \equiv \partial\varphi_c/\partial r$ is the complex amplitude of $\mathbf{b}\cdot\nabla r$. Similarly we have $b_{\theta c}\big|_{out} = -ib_{rc}(r_{w+})$ just behind the wall, where $r_{w+} \equiv r_w + d_w$ is the outer minor radius of the wall, and

$$K_b \equiv -b_{\theta c}\Big|_{out}^{in} = ib_{rc}(r_w)\left[\frac{\Gamma_m}{m} + 1 - \frac{b_{rc}(r_{w+})}{b_{rc}(r_w)} \right]. \tag{9}$$

Using this definition and complex notation introduced by (5) we obtain from (6)

$$f_{r1c} = -\frac{B_{0\theta}}{d_w}(1 - g_0)K_b \qquad \text{with} \qquad g_0 = \frac{n}{m} q_{pl} \frac{r_w^2}{r_{pl}^2} , \tag{10}$$

where q_{pl} is q at the plasma boundary ($q \propto r^2$ in the vacuum) and r_{pl} the plasma minor radius. Eq. (10) has the same structure as Eq. (40) in [1], but differs from it by the factor of

$$\frac{1 - g_0}{1 + g_1} , \qquad \text{where} \qquad g_1 = g_0 \frac{m^2}{n} \frac{1}{\gamma\tau_w + mr_w^2/R^2} \tag{11}$$

for perturbations with $u_c \propto \exp(\gamma t)$ and $\tau_w \equiv r_w d_w / \eta$. Expression for g_1 here corresponds to Eq. (41) in [1] with definition of τ_w next to Eq. (31) there. Eq. (8) in [2] gives a different g_1.

The analytical model in [1] was developed for evaluation of the wall force dependence on current and wall resistivity. Equations (10) and (11) show that our model will give an essentially different dependence on $\gamma\tau_w$ at the same K_b as in [1].

In [1], the force is expressed finally through the radial component ξ_r of the plasma displacement $\boldsymbol{\xi}$. To move in this direction we use the consequence of (7) (see also [6])

$$\frac{b_{rc}(r)}{b_{rc}(r_w)} = \left[1 + \frac{\Gamma_m}{2m}(1 - x^{2m})\right] \cdot x^{-m-1} \tag{12}$$

valid in the plasma-wall gap. This allows to transform (9) into

$$K_b = -\Gamma_m (1+\alpha_1)\frac{\psi_c(r_w)}{r_w} = -2\frac{m}{r_w}\frac{\kappa(1+\alpha_1)}{2m/\Gamma_m + 1 - \kappa^2}\psi_c(r_{pl}), \tag{13}$$

where $\kappa \equiv (r_{pl}/r_w)^m$, $\psi_c \equiv -irb_{rc}(r)/m$ and

$$\alpha_1 \equiv \frac{m}{\Gamma_m}\left[1 - \frac{b_{rc}(r_{w+})}{b_{rc}(r_w)}\right]. \tag{14}$$

We introduced ψ_c to facilitate comparisons with the models representing the perpendicular component of $\mathbf{b}$ (fully described here by $\nabla\varphi$ in vacuum) as $\nabla\psi \times \nabla z$, Eq. (20) in [1].

Equation (13) can be used for any plasma. If it is assumed ideal as in [1], we have $\mathbf{b} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B}_0)$, $b_r = \mathbf{B}_0 \cdot \nabla \xi_r$ and

$$m\psi_c = B_{0\theta}(m - nq)\xi_{rc} \tag{15}$$

at the plasma boundary. Note that a large multiplier $B_{0\zeta}$ is missing in similar Eq. (42) in [1].

4. Comparisons and discussion. Up to now the ratio $b_r(r_{w+})/b_r(r_w)$ and the quantity Γ_m are free parameters in our formulas. In [1, 2], a thin shell approximation has been used that requires continuity of the normal component of $\mathbf{b}$ (here, b_r) at the wall. Then $\alpha_1 = 0$ and Γ_m is related to the growth rate γ by (see [3–6] and the references therein)

$$\Gamma_m = \gamma\tau_w \qquad \text{with} \qquad \tau_w \equiv r_w d_w / \eta . \tag{16}$$

Combining this with (13) and the ideal-plasma relation (15), we obtain from (10)

$$f_{r1c} = 2\frac{1-g_0}{1-\kappa^2 + 2m/\gamma\tau_w}\frac{B_{0\theta}^2}{r_w d_w}(m - nq_{pl})\kappa\xi_{rc}(r_{pl}). \tag{17}$$

This can be compared with Eq. (43) in [1] and Eqs. (7)–(9) in [2] for the same force.

One can easily note the absence of r_w in the denominator of Eq. (43) in [1]. Another more serious disagreement of our expression (17) from the corresponding results in [1, 2] has deeper roots: it comes from the use of Eq. (4) here instead of Eq. (38) in [1] for $b_\zeta(r_{w+})$ combined with disregard of $b_\zeta(r_w)$ in Eq. (36) there. Finally, this difference is quantified by the factor (11) with $1+g_1$ varying with γ, while $1-g_0$ remains constant.

It follows from (17) that f_{r1c} monotonically increases with γ from $f_{r1c}=0$ at $\gamma=0$:

$$\frac{f_{r1c}}{f_{r1c}^{\max}}=\frac{y}{1+y} \qquad \text{with} \qquad y\equiv\frac{\gamma\tau_w}{2m}(1-\kappa^2)\,, \tag{18}$$

where $f_{r1c}^{\max}$ is the maximal value of f_{r1c} (the saturation level achieved at $\gamma\tau_w\to\infty$). On the contrary, Eq. (43) in [1] shows non-monotonic dependence of the force on γ with a maximum at $\gamma\tau_w\approx 1$. This was emphasized [1, 2] as a merit of the model, but actually is a mistake. That equation and our solution (18) are compared in Fig. 1. In [1] the dependence of sideways force on $\gamma\tau_w$ (dashed curve) was found to agree moderately well with the simulations. Fig. 1 shows that it should be considered as purely accidental because of strong disagreement of the correct result (solid curve) with the result from [1] obtained under unjustified simplifications. Let us note that Eqs. (7)–(9) in [2] give a finite force at $\gamma=0$.

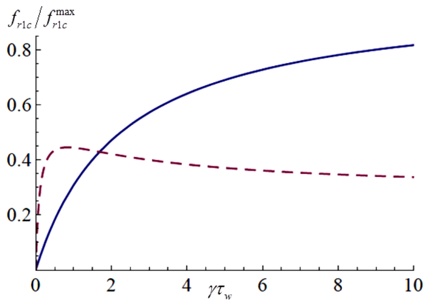

Fig. 1. The force calculated by (18) (solid) and by Eq. (43) in [1] (dashed curve) with the same normalization at $n=m=1$, $q_{pl}=0.5$, $r_w=3r_{pl}$, $R=3r_w$.

5. Conclusion. In the model [1], the plasma pressure is zero, $j_z=const$ in the plasma, and γ is real. Here the plasma parameters are not restricted in any way and γ can be complex (the latter allows the mode rotation). Also, we do not employ the thin-wall approximation in the main part of our calculations. With applicability range much wider than in [1] and without artificial constraints on b_ζ used there, our model is more realistic and precise. We recommend it for numerical and experimental testing.

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