On O-X mode conversion in 1D inhomogeneous turbulent plasma

A. Popov

Ioffe Institute, St.Petersburg, Russia

Introduction. Heating plasmas by means of microwaves is a widespread tool nowadays. A possibility of energy transfer, which applies for most fusion devices, is the well-known concept of the electron cyclotron resonance heating (ECRH). The conventional ECRH in overdense plasmas appears to be inefficient due to high density at comparatively low magnetic field. To overcome this difficulty the electron Bernstein wave (EBW) heating was proposed. The EBW is excited in a vicinity of the upper hybrid resonance via the O-X-B mode conversion scheme in which an ordinary (O) wave, being launched obliquely from the low magnetic field side, transforms to an extraordinary (X) wave, which then totally converts to the EBW, propagating to the plasma core without further density cutoff and absorbed there at harmonics of the ECR. The O-X mode conversion being a key element of the scheme determines its efficiency entirely. The phenomenon of the linear O-X mode conversion was investigated last four decades [1]-[4] in detail. Nevertheless, the influence of low frequency drift turbulence on the efficiency of this linear mode conversion still remains unresolved problem and appeals for further theoretical analysis. In this paper we focus on the O-X mode conversion in 1D inhomogeneous plasma with gaussian random dense fluctuations those correlation length exceeds the length of the conversion region.

The physical model. We assume that plasma parameters depend on a single coordinate x of a Cartesian coordinate system the origin of which lies on the O-mode cutoff surface. The x-axis is directed towards the plasma core, and the z-axis is along the magnetic field line. The distribution of the electromagnetic field components in a cold magnetized plasma is governed by Maxwell's wave equations. In the Stix's notation the dielectric tensor contains the diagonal components solely $\varepsilon_{\pm} = 1 - \omega_{pe}^2/(\omega(\omega \pm \omega_{ce}))$, $\varepsilon_{\parallel} = 1 - \omega_{pe}^2/\omega^2$, and the electric field $\mathbf{E}$ has the components $\mathbf{E} = (E_+, E_-, E_z)$, $E_{\pm} = (E_x \pm iE_y)/\sqrt{2}$. In the 1D inhomogeneous plasma a beam of the electromagnetic waves with an arbitrary polarization can be represented as

$$\mathbf{E}(\mathbf{r}) = \int_{-\infty}^{\infty} \frac{dk_y dk_z}{(2\pi)^2} A(k_y, k_z)\, \mathbf{E}(x, k_y, k_z) \exp(ik_y y + ik_z z),$$

where $A(k_y, k_z)$ is a beam distribution over the wavevector components k_y and k_z, determined by the launching antenna (O-X mode conversion) or the EBW emission spectrum (X-O mode conversion), $\mathbf{E}(x, k_y, k_z)$ is the wave's amplitude. Linear mode conversion is a phenomenon which occurs in a region where the wavevectors and the polarizations of the coupled waves are close one another. For a particular angle of incidence [1], it is possible for the O electromagnetic wave to be totally converted into the X wave. This particular angle is determined by the components of the wavevector $k_y = 0$, $k_z = \omega/c\,(1+\omega/\omega_{ce})^{-1/2}$, conserved in a plasma with 1D inhomogeneity. In a immediate vicinity of the point $x = 0$ the dielectric tensor components can be represented as follows

$$\varepsilon_+ (x) \simeq \frac{\omega_{ce}(0)}{\omega+\omega_{ce}(0)} - \frac{\omega}{\omega+\omega_{ce}(0)}\frac{x}{L_n} - \frac{\delta n(x)}{n_c}, \varepsilon_\parallel (x) \simeq -\frac{x}{L_n} - \frac{\delta n(x)}{n_c},$$

where $L_n = d\ln n(x)/dx|_0^{-1}$, $n_c = m_e\omega^2/\left(4\pi e^2\right)$ and $\delta n(x)$ corresponds to the density fluctuation. Assuming $x/L_n \ll 1$, $\delta n/n_c \ll 1$, $k_y/k_z \ll 1$ and $|\partial/\partial x|/k_z \ll 1$ and keeping the first order terms in Maxwell's wave equations, we get a system describing the O-X waves coupling in a vicinity of the O mode cutoff surface

$$\left(i\frac{\partial}{\partial\xi} + \xi + \delta\tilde{n}\right)a + (iq_y + q_z)\,b = 0,\ \left(i\frac{\partial}{\partial\xi} - \xi - \delta\tilde{n}\right)b + (iq_y - q_z)\,a = 0, \tag{1}$$

where new notations have been introduced:

$$\xi = \frac{x}{\delta_x} + q_z,\ q_z = 2^{1/2}\left(\frac{\omega+\omega_{ce}(0)}{\omega}\right)^{1/2}\delta k_z\delta_x,\ q_y = k_y\delta_x,\ \delta_x = \left(\frac{L_n c}{\omega}\right)^{1/2}\left(\frac{\omega_{ce}(0)}{2\omega}\right)^{1/4},$$

$$\delta\tilde{n} = \frac{\delta n}{n_c}\frac{L_n}{\delta_x}, (a,b) = \left(\frac{\omega}{\omega+\omega_{ce}(0)}\right)^{1/4} E_+ \pm \left(\frac{\omega+\omega_{ce}(0)}{\omega}\right)^{1/4} E_z,\ E_- \ll E_+, E_z$$

Combining the lines in Eq. (1), we get the second order differential equation for b describing the X-O mode conversion

$$b'' + \left[(\xi+\delta\tilde{n}(\xi))^2 - Q^2 - i\left(1+\delta\tilde{n}'(\xi)\right)\right]b = 0, \tag{2}$$

where $Q^2 = q_y^2 + q_z^2$ and the superscript $'$ means the derivative over ξ. We seek the required solution b of Eq. (2) in a form

$$b(\xi) = \int_{-\infty}^{\infty} G(\xi,\zeta;\delta\tilde{n})\,b_0(\zeta)\,d\zeta \tag{3}$$

with b_0 being chosen in such a way that it describes the incident WKB X wave propagating against the coordinate axis $b_0 = \exp\left[i\zeta^2/2 - (\zeta-\zeta_0)^2/L_0^2\right]$ with finite but still wide envelope $1 \ll L_0 \ll \zeta_0$. The Green's function $G(\xi,\zeta;\delta\tilde{n})$ can be represented in terms of the path integral

$$G(\xi,\zeta;\delta\tilde{n}) = \int_0^\infty dt\int_\zeta^\xi Du(\cdot)\exp\left[-\frac{i}{2}\int_0^t\left[\dot{u}^2 + (u+\delta\tilde{n}(u))^2 - Q^2 - i\left(1+\delta\tilde{n}'(s)\right)\right]ds\right], \tag{4}$$

where $\dot{u} = \partial u(s)/\partial s$, $\zeta = u(0)$, $\xi = u(t)$, that is best suited for investigation of the statistical properties of the O-X mode conversion in the presence of the drift-wave density fluctuations

$\delta\tilde{n}$. Further, we assume statistically homogeneous turbulence, which however possesses an arbitrary wave number spectrum $<\delta n(\kappa)^2>$ related to the correlation function of the density fluctuations with the density perturbation level $\sqrt{\delta n_0^2}$ and the correlation length l_{cx}. The probability distribution function $P[\delta n(\kappa), \delta n(\kappa')]$ which we need for statistical averaging over an ensemble of the density fluctuations takes a form

$$d[\rho(\delta n(\kappa))] = D\delta n[\cdot]\exp\left[-\frac{1}{2}\int_{-\infty}^{\infty} d\kappa d\kappa' \alpha(\kappa,\kappa')\delta n(\kappa)\delta n(\kappa')\right] \quad (5)$$

where $\alpha(\kappa,\kappa')^{-1} = <\delta n(\kappa)\delta n(\kappa')> = 2\pi\delta(\kappa+\kappa') < \delta n(\kappa)^2 >$ and $\delta(...)$ is the Delta function. This is a generalization of the Gaussian distribution to a continuous variable $\delta n(\kappa)$ [5]. Utilizing the averaging procedure Eq. (5) and omitting tedious calculations, we arrive at the average value of the electric field amplitude Eq. (3) representation in terms of convolution of the averaged Green's function Eq. (4) and the initial distribution [7]

$$< b(\xi) > = \int d[\rho(\delta n(\kappa))] b(\xi;\delta n(\kappa)) = \frac{\exp(-\pi Q^2/4)}{\sinh(Q^2\pi/2)}\int_{-\infty}^{\infty}\frac{d\mu}{\sqrt{2\pi\delta\tilde{n}_0^2}}\exp\left[-\frac{\mu^2}{2\delta\tilde{n}_0^2}\right]\times \quad (6)$$

$$\left\{D_{-iQ^2/2}\left(h\tilde{\xi}\right)\int_{-\infty}^{\xi} d\zeta D_{iQ^2/2-1}\left(\bar{h}\tilde{\zeta}\right) + D_{-iQ^2/2}\left(-h\tilde{\xi}\right)\int_{\infty}^{\xi} d\zeta D_{iQ^2/2-1}\left(-\bar{h}\tilde{\zeta}\right)\right\} b_0(\zeta),$$

$h = \exp(-i\pi/4)$, $\bar{h} = h^*$. Substituting b_0 in Eq. (6) and evaluating integration over ζ yields

$$< b(\xi) > = \int_{-\infty}^{\infty}\frac{d\mu}{\sqrt{2\pi\delta\tilde{n}_0^2}}\exp\left[-\frac{\mu^2}{2\delta\tilde{n}_0^2}\right] C(\mu) D_{-iQ^2/2}(-h(\xi+\mu)) \quad (7)$$

where $C(\mu) = \int_{-\infty}^{\infty} d\zeta D_{iQ^2/2-1}\left(-\bar{h}(\zeta+\mu)\right) b_0(\zeta)$. The asymptotic expression for the parabolic cylinder function for large value of the argument $|\xi| \gg 1$ and a fixed value of Q^2 are given by [7]

$$D_{-iQ^2/2}\left(-h\tilde{\xi}\right) \approx \exp\left(i\frac{\xi^2}{2} - i\frac{Q^2}{2}\ln\left(\sqrt{2}\xi\right) + \frac{3\pi}{8}Q^2\right) - O(1/|\xi|)$$

at $\xi \to \infty$, and

$$D_{-iQ^2/2}\left(-h\tilde{\xi}\right) \approx \exp\left(i\frac{\xi^2}{2} - i\frac{Q^2}{2}\ln\left(\sqrt{2}\xi\right) - \frac{\pi}{8}Q^2\right)$$

at $\xi \to -\infty$. Thus, in the WKB region the solution Eq. (7) describes the wave propagating against axis: at $\tilde{\xi} > 0$ it corresponds to the incident X wave, and $\tilde{\xi} < 0$ it relates to the outgoing O wave. The conversion coefficient can be introduced in a following way

$$T_{XO} = < b(\xi) >_{\xi\to-\infty} / < b(\xi) >_{\xi\to\infty} \quad (8)$$

In the case $\delta n_0^2 \to 0$, Eq. (8) reduces to the familiar expression $T_{XO} = \exp\left(-\pi Q^2/2\right)$ [1]. In Fig. 1 we illustrate the conversion coefficient Eq. (8) dependence on the dimensionless parameter $\delta\tilde{n}_0^2$ at $Q = 0$. As we can see, the density fluctuations, affecting the coordinate dependence of the solution of the wave equation Eq. (2), lead to reduction of the efficiency of the X-O mode conversion and anomalous reflection of the incident wave.

Conclusion. In the paper the linear X-O (O-X) mode conversion at electron cyclotron frequencies in the turbulent 1D inhomogeneous plasma has been investigated. We expand Maxwell's equations in the region near the intersection of the two cutoff surfaces and find the approximate full wave equations appropriate for this region and describing the modes coupling in the turbulent plasma. The solutions are found by means of convolution of Green's function of the approximate equation with the incident WKB X wave. We use an assumption of the long wavelength density fluctuations, the typical wavelength of those is much greater than the length of the evanescent layer. In this case the density fluctuations are strongly correlated throughout the region of interest. Averaging the solutions, we get the average value of the electric field components. Identifying the incoming and outgoing waves, we obtain the conversion coefficient. The density fluctuations are shown to affect the coordinate dependence of the solution, that leads to reducing of the efficiency of the X-O (O-X) mode conversion and anomalous reflection of the incident wave.

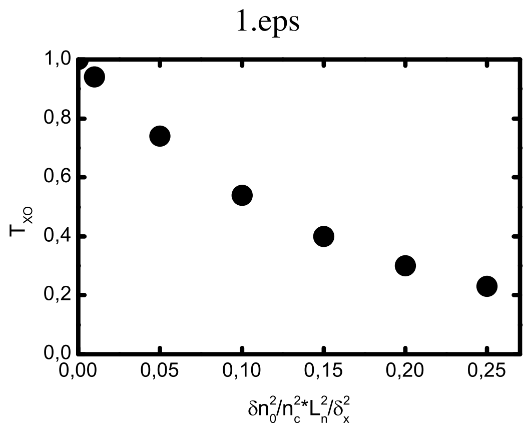

Figure 1: T_{XO} *versus the dimensionless parameter* $\delta\tilde{n}_0^2 = \delta n_0^2/n_c^2 \cdot L_n^2/\delta_x^2$, $Q = 0$.

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