Low-threshold parametric excitation of UH wave trapped in a blob in the first harmonic O-mode ECRH experiment

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Introduction. High power ECRH is widely used nowadays and it is considered for application in ITER. In the community since 80th nonlinear effects such as parametric decay instabilities (PDIs) were believed to be deeply suppressed in the first harmonic ordinary mode and second harmonic extraordinary mode ECRH experiments in toroidal magnetic fusion devices [1]. Nevertheless, during the last decade a number of anomalous phenomena observations have been reported in second harmonic ECRH experiments [2-4] such as fast ion generation [2, 3] and anomalous backscattering [4]. An explanation of these observations proposed recently is based on possibility of low-threshold parametric excitation of decay waves trapped in plasma due to non-monotonic behaviour of plasma density in radial direction [5, 6]. This mechanism is not specific for the second harmonic ECRH and can occur in the case of the first harmonic O-mode heating in contemporary devices and in ITER. Here we analyse a possibility of low-threshold parametric excitation by the first harmonic O-mode pump of the upper hybrid (UH) and ion acoustic (IA) waves in axially symmetric which can be considered as a model of excitation in filament or blob elongated in the magnetic field direction but can be observed in the linear plasma device [7], as well.

The basic equations. We consider the decay of the ordinary pump wave propagating perpendicular to the magnetic field and possessing the electric field given by expression $E_{iz} = \sqrt{\frac{8P_i}{\pi XYc}} e^{-\frac{z^2}{2Z^2}-\frac{y^2}{2Y^2}} e^{-i\omega_i t}$, where x, y are directions transverse to the magnetic field and P_i is a pump wave power. The wave number of the pump wave is supposed to be negligible compared to wave numbers of the decay waves. We use cylindrical coordinate system below with the axial direction along the magnetic field. The basic equations describing generation of UH wave $\varphi_{uh} = \varphi_{uh}(r)e^{ik_z z+im\phi-i\omega_{uh}t}$ and IA wave $\varphi_s = \varphi_s(r)e^{-ik_z z-im\phi-i\Omega t}$ and their convective losses from the decay region are as follows:

$$div\, \hat{\varepsilon}_{uh}\, \vec{\nabla}\varphi_{uh} = 4\pi\rho_{uh} \qquad (1)$$

$$\left(\frac{\omega_{pi}^2}{c_s^2} + \frac{\omega_{pi}^2}{\Omega^2}\Delta\right)\varphi_s = 4\pi\rho_s \qquad (2)$$

Here $\rho_s = ia_B\varepsilon_{si}\varepsilon_{uhe}\dfrac{\Delta\varphi_{uh}^{(0)*}}{4\pi}$, $\rho_{uh} = ia_B\varepsilon_{si}\varepsilon_{uhe}\dfrac{\Delta\varphi_s^{*}}{4\pi}$, $\varepsilon_{uhe} = -\dfrac{\omega_{pe}^{2}}{\omega^2-\omega_{ce}^{2}}$, $\varepsilon_{si} = -\dfrac{\omega_{pi}^{2}}{\Omega^2}$,

$a_B = \dfrac{E_{iz}k_z}{\omega_{uh}^{2}}\dfrac{\bar{e}}{m_{\bar{e}}}$ and $\varphi_{uh}^{(0)}$ stands for the solution of Eq. (1) with the omitted wave interaction.

The transverse component of the permittivity tensor of the UH wave includes the thermal correction $\tilde{\varepsilon}_{uh} = 1+\varepsilon_{uhe}-\beta\Delta_\perp$, $\beta = \dfrac{\omega_{pe}^{2}v_{Te}^{2}\left(\omega^2+3\omega_{ce}^{2}\right)}{\left(\omega^2-\omega_{ce}^{2}\right)^3}$.

The simplified model. In the WKB approximation Eq. (1) without wave interaction takes the following form

$$D(k_r, r, \omega_{uh}) = \left(\varepsilon_{uh}(r)+\beta\left(k_r^{2}+\frac{m^2}{r^2}\right)\right)\left(k_r^{2}+\frac{m^2}{r^2}\right)+\eta_{uh}k_z^{2}=0 \qquad (3)$$

In the case of axially symmetric plasma the UH wave is trapped in the radial directionbetween the UHR and the internal cut off which appears due to refraction making the plasma axis evanescent. At high azimuthal numbers the finite transparency region degenerates into a point $r = r_c$ (see Fig. 2), where r_c , ω_c are defined by the equations

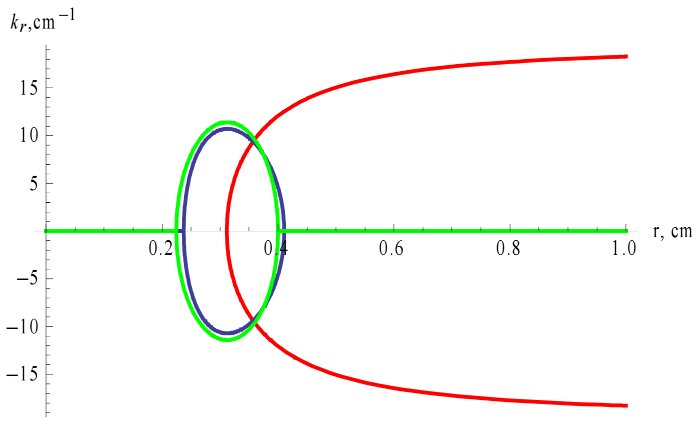

Fig. 2.Exact (blue line) and approximated (green line) dispersion curves of the UH wave and dispersion curve of the IS wave (red line)

$D(0, r_c, \omega_c) = 0$ and $D_r(0, r_c, \omega_c) = 0$. Since the wave is also trapped in the azimuthaldirection due to the plasma axialsymmetry there are only axial convective losses for this wave. In the case of narrow UH wave transparency regionthe differential equation for the UH wave can be simplified as

$$\left[\frac{1}{2}D_{rr}\left(r-r_c\right)^2-\frac{1}{2}D_{k_rk_r}\frac{\partial^2}{\partial r^2}+D_\omega\left(\omega-\omega_c\right)\right]\varphi_{uh}^{(0)}=0 \qquad (4)$$

Here subscripts r , k_r , ω indicate differentiation of $D(k_r, r, \omega)$ with respect to corresponding parameter, which is performed at $r = r_c$, $\omega = \omega_c$.

The corresponding expressions for the eigenmode trapped in the radial direction and eigenvalues are

$$\varphi_{uh}^{(0)}(n,r) = \exp\left[-(r-r_c)^2 \sqrt[4]{\frac{D_{rr}}{D_{k_r k_r}}}\right] H_n\left((r-r_c)^2 \sqrt[4]{\frac{D_{rr}}{D_{k_r k_r}}}\right), \quad \omega_n = \omega_c - (2n+1)\frac{\sqrt{D_{k_r k_r} D_{rr}}}{D_\omega}$$

where $H_n(\zeta)$ stands for Hermitian polynomial.

As it is known from the PDI theory, the three-wave interaction is effective only in the case the decay conditions for the interacting wave numbers is fulfilled. In our case it makes obligatory the intersection of UH and IA dispersion curves in Fig.2. In this case the region of interaction is asmall neighbourhood of IA wave cut-off point where a solution of the Eq. (3) can be found in the form

$$\varphi_s = c_1(r) Ai\left(\frac{R-r}{l_A}\right) + c_2(r) Bi\left(\frac{R-r}{l_A}\right) \qquad (5)$$

where R is radial coordinate of the IA wave cut-off point and $l_A = \left(dk_r^2/dr\right)^{-\frac{1}{3}}$ is Airy length. The natural boundary conditions for the parametrically driven IA wave are the asymptotic suppression of the wave in the evanescent regionand absence of the wave incident on the plasma from the outside.

The UH wave generation due to the nonlinear interaction and axial convective losses can be described with the help of the perturbation theory:

$$\left[\frac{1}{2} D_{rr}(r-r_c)^2 - \frac{1}{2} D_{k_r k_r}\frac{\partial^2}{\partial r^2} + D_\omega(\omega-\omega_c)\right]\varphi_{uh} = 4\pi\rho_s - D_{k_z}\delta k_z \varphi_{uh}^{(0)} \triangleq \hat{V}\varphi_{uh}^{(0)} \qquad (6)$$

Since the eigen frequencies should not change, the diagonal matrix element of the perturbation $\left\langle \phi_{uh}^{(0)}(n,r)\middle|\hat{V}\middle|\phi_{uh}^{(0)}(n,r)\right\rangle$ should be equal to zero, so the addition to the axial wave vector δk_z is defined by the expression

$$\delta k_z = \frac{4\pi \int \rho_{uh}(r)\varphi_{uh}^{(0)}(n,r)dr}{D_{k_z}\int \varphi_{uh}^{(0)}(n,r)^2 dr} \qquad (7)$$

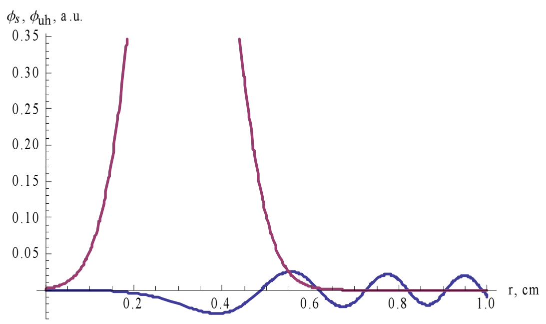

Fig. 3.UH wave (red line) and IS wave (blue line) potentials

The imaginary part of δk_z defines the UH wave convective amplification and the threshold of the PDI onset. It should be noted that the threshold is the lower then the axial wave vector k_z is the higher, because the generation matrix element $\int \rho_{uh}(r)\varphi_{uh}^{(0)}(n,r)dr \sim k_z^2$ and the term responsible for axial convective losses $D_{k_z} \int \varphi_{uh}^{(0)}(n,r)^2 dr \sim k_z$. The maximal value of k_z is determined by the existence of appropriate IA wave.

In the case of Granit device parameters [7] (argon plasma, $B = 0.05\,T$, $T = 1\,eV$, $n_0 = 4.6\ 10^{10}\ cm^{-3}$, $f_i = \omega_i/2\pi = 2.1\,GHz$) for $k_z = 10$ and $m = 12$ we obtain the threshold value $P \approx 100W$ which can be overcome in experiment.

Trapping of UH wave in blob. The possibility of the UH wave trapping in filaments or blobs possessing density maximum and aligned with magnetic field in the case of ITER ECRH experiment ($T_{e0} = 10\,keV$, $n_0 = 10^{20}\ m^{-3}$) was also investigated with the help of ray tracing procedure under the assumption that density variation in blob is $\delta n/n = 10\%$ and its radius $r = 1\,cm$.It is shown that the UH wave is trapped in blob in radial as well as in poloidal direction. Moreover it is localised in the toroidal direction as well (see Fig. 4, 5).Therefore it is expected that if an appropriate low frequency partner exists under this parameters (IBW for example) the threshold of PDI onset will be exceptionallylow.

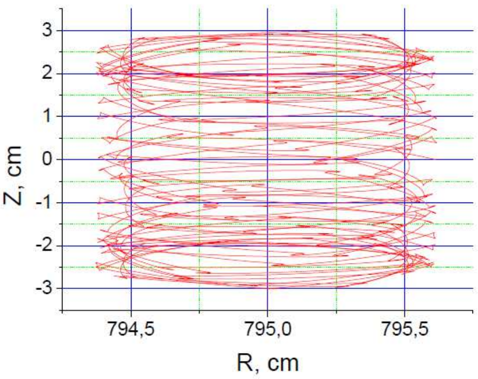

Fig. 4.UH wave ray trajectory in the poloidal section

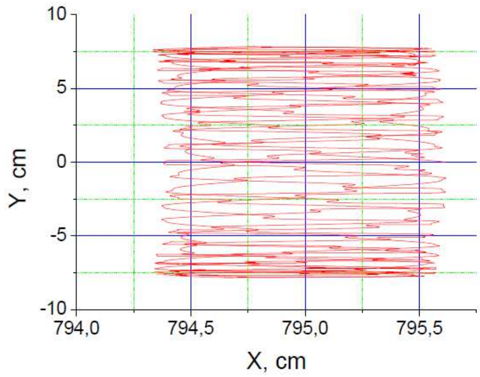

Fig. 5.UH wave ray trajectory in thetoroidal section

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