

Plasma shape control with a linear model for Globus-M tokamak

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The paper presents the first preliminary results of the plasma equilibrium reconstruction, generation of linear models of a tokamak-plasma system from the obtained plasma current density distribution and further designing controllers for plasma position, current, and shape in Globus-M tokamak (Ioffe Inst., St. Petersburg, Russia) [1]. In this study the plasma shape is controlled by means of stabilization of poloidal fluxes on the plasma separatrix of the discharge divertor phase (isoflux control) [2] and 6 gaps between separatrix and the first wall [3].

To reconstruct plasma poloidal flux and current density distribution satisfying the Grad-Shafranov equation the Picard's iteration scheme is used [4] in the numerical code FCDI developed in the MATLAB environment. The plasma current density is modelled as a ψ -polynomial in which coefficients are estimated by minimization of

$$\sum_{i=1}^{n_M} k_i (\psi_i - \psi_i^M)^2 + k_I (I - I^M)^2 + k_F F_z^2 \text{ where } \psi_i^M \text{ and } I^M \text{ are measured values of poloidal flux}$$

and plasma current respectively, F_z is the vertical force produced by vacuum magnetic field acting on the plasma, k_i , k_I and k_F are scaling coefficients. Fig. 1a shows one of reconstructed equilibrium flux distributions which is used for linear model deriving.

The linear model is obtained as $\dot{x} = Ax + Bu$, $y = Cx$ where x is the state vector containing variations of the control currents in the poloidal field coils and currents on vacuum vessel (VV), y is the output vector of plasma geometric parameters, poloidal flux variations on the plasma separatrix, and the plasma current variation. It is assumed that the plasma moves rigidly and changes its total current holding the shape and current profiles. The VV currents are represented with the help of a set of filaments [5]. The dynamics of tokamak-plasma system is described by the linearized Kirchhoff's voltage law:

$$M\dot{I} + M_p\dot{I}_p + RI + \frac{\partial\Psi}{\partial\vec{r}_p}\dot{\vec{r}}_p = U, \quad M_p^T\dot{I} + M_{pp}\dot{I}_p + \frac{\partial\Psi_{pc}}{\partial\vec{r}_p}\dot{\vec{r}}_p = 0, \quad \vec{r}_p = \begin{bmatrix} r_p \\ z_p \end{bmatrix}, \quad M_p = \begin{bmatrix} M_{pc} \\ M_{pv} \end{bmatrix}, \quad (1)$$

$$M = \begin{bmatrix} M_{cc} & M_{cv} \\ M_{vc} & M_{vv} \end{bmatrix}, \quad R = \begin{bmatrix} R_c & 0 \\ 0 & R_v \end{bmatrix}, \quad U = \begin{bmatrix} U_c \\ 0_{N_v \times 1} \end{bmatrix}, \quad \frac{\partial\Psi}{\partial\vec{r}_p} = \begin{bmatrix} \partial\Psi_{cp}/\partial r_p & \partial\Psi_{cp}/\partial z_p \\ \partial\Psi_{vp}/\partial r_p & \partial\Psi_{vp}/\partial z_p \end{bmatrix}, \quad \frac{\partial\Psi_{pc}}{\partial\vec{r}_p} = \begin{bmatrix} \partial\Psi_{pc}/\partial r_p \\ \partial\Psi_{pc}/\partial z_p \end{bmatrix}^T.$$

Here subscripted c , v and p correspond to poloidal coils, VV and plasma volume respectively, M_{ab} is inductance matrix between conductors a and conductors b , Ψ_{ab} is the matrix of fluxes at conductors a due to conductors b , r_p and z_p denote plasma horizon and vertical magnetic axis displacements, R_c and R_v are resistance diagonal matrices of active coils and passive structures. The plasma resistivity and mass are neglected.

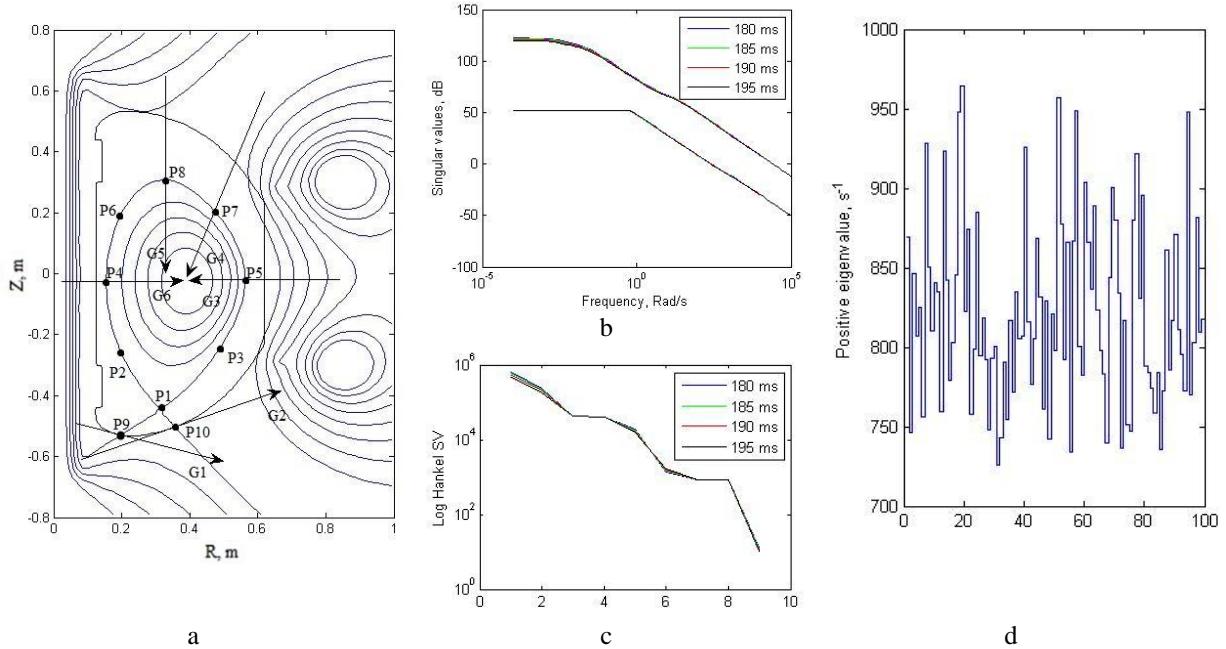


Fig. 1. (a) Reconstructed plasma equilibrium with points P1-P10 chosen on the plasma separatrix at which the poloidal flux has to be controlled and directions of 6 gaps between separatrix and the first wall. (b) Maximal and minimal singular values and (c) Hankel singular values for stable parts of obtained models for different time moments of the discharge. (d) Unstable pole changes in disturbed models.

A force balance equation comes from the fact that plasma resultant force equals zero:

$$F_p \vec{r}_p + F_I I = 0, \quad F_p = \begin{bmatrix} \partial F_r / \partial r_p & \partial F_r / \partial z_p \\ \partial F_z / \partial r_p & \partial F_z / \partial z_p \end{bmatrix}, \quad F_I = \begin{bmatrix} \partial F_r / \partial I_c & \partial F_r / \partial I_v \\ \partial F_z / \partial I_c & \partial F_z / \partial I_v \end{bmatrix}. \quad (2)$$

Combining (1) and (2) one can obtain the linear model in the form $\tilde{M} \dot{I} + RI = U$ where

$$\tilde{M} = M - \frac{\partial \Psi}{\partial \vec{r}_p} F_r^{-1} F_I - M_p M_{pp}^{-1} \left(M_p^T - \frac{\partial \Psi_{pc}}{\partial \vec{r}_p} F_r^{-1} F_I \right), \quad A = -\tilde{M}^{-1} R, \quad B = \tilde{M}^{-1} \begin{bmatrix} 1_{N_c \times N_c} & 0_{N_c \times N_v} \end{bmatrix}^T,$$

matrix C is composed from rows corresponding to plasma displacement perturbations of poloidal flux at the set Γ of fixed points, and total plasma current respectively $\delta \vec{r}_p = -F_r^{-1} F_I I$,

$$\delta \Psi_{\Gamma p} = \left(M_{\Gamma I} - \frac{\partial \Psi_{\Gamma p}}{\partial \vec{r}_p} F_r^{-1} F_I - M_{\Gamma p} M_{pp}^{-1} \left(M_p^T - \frac{\partial \Psi_{pc}}{\partial \vec{r}_p} F_r^{-1} F_I \right) \right) I, \quad \delta I_p = -M_{pp}^{-1} \left(M_p^T - \frac{\partial \Psi_{pc}}{\partial \vec{r}_p} F_r^{-1} F_I \right) I.$$

Obtained linear model for Globus-M tokamak has 8 inputs (voltages on PF coils), 75 states (8 currents in PF coils and 67 currents in filaments of the VV), 27 outputs (8 PF currents, vertical and radial displacements of plasma magnetic axis, the plasma current, the flux at 10 points of the separatrix, and 6 gaps) (Fig. 1a). Maximal and minimal singular values of the model transfer function are exhibited in Fig. 1b, and Fig. 1c displays Hankel singular values [6] of A, B, C, D -realization of the model for different time moments of the discharge which do not significantly change during the shot of No 31648. Each obtained linear model has single positive pole (right eigenvalue of A matrix). For the model used to design controller the eigenvalue is equal to 795 s^{-1} . To test robustness of the derived model the random set of disturbed plasma equilibria was generated. The plasma current was varied within 3% and vertical position of plasma was shifted within 2.5 cm in each disturbed equilibrium. For that

case the positive pole of corresponding linear model is demonstrated in Fig. 1d.

In Globus-M the plasma shape control is done by poloidal magnetic fluxes and gaps stabilization approaches. The control system structure is presented in Fig. 2. Here signals δZ and δR are the plasma vertical and horizon position variations respectively, $I_{PF} \in \mathbb{R}^6$ are poloidal

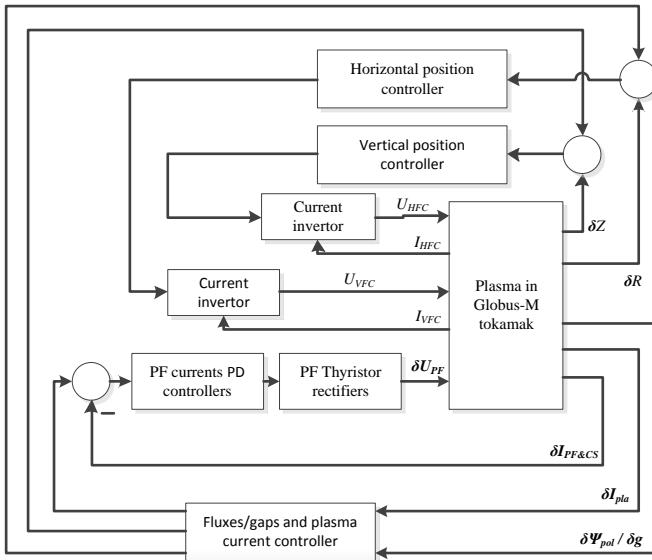


Fig. 2. Multivariable control system structure.

field coil current variations, I_{pla} is the plasma current variation, $F_{pol} \in \mathbb{R}^{10}$ are the poloidal magnetic fluxes variations. The control signals U_{HFC} and U_{VFC} are voltages on horizontal and vertical field coils respectively, I_{HFC} and I_{VFC} are the currents in these coils. The control signal $U_{PF} \in \mathbb{R}^6$ is the vector of the poloidal field coils voltage variations. Plasma position stabilization is done with using impulse current invertors [7] as nonlinear actuators in a self-exciting mode of about 3 kHz. The PF multiphase thyristor rectifiers with transport delay of 3.3. ms are used to control the poloidal magnetic fluxes/gaps, and plasma current.

Robust H_∞ loop shaping design [3, 6] was applied to synthesis poloidal magnetic fluxes/gaps and plasma current controllers on the base of the linear models as well as the plasma position controllers. The robust stability margin equals more than 0.2 [6]. Z, R -references are adjusted by the gap controller. PF currents are controlled by tuned PID-controllers. Responses at plasma

position initial disturbance of 0.01 m of vertical and horizontal position simultaneously are shown in Fig. 3,a for isoflux control in closed-loop system. The largest displacements are 0.015 m for vertical and 0.005 m for horizontal position. The system settling time is equal to 15 ms. For gaps control at 0.01 m reference position step-action (Fig. 3,b) the settling time is 10 ms. The gap maximum deviation is 0.05 m at the steady-state accuracy of about 0.005 m. The vertical position maximum deviation is 0.024 m when the deviation itself converges to zero.

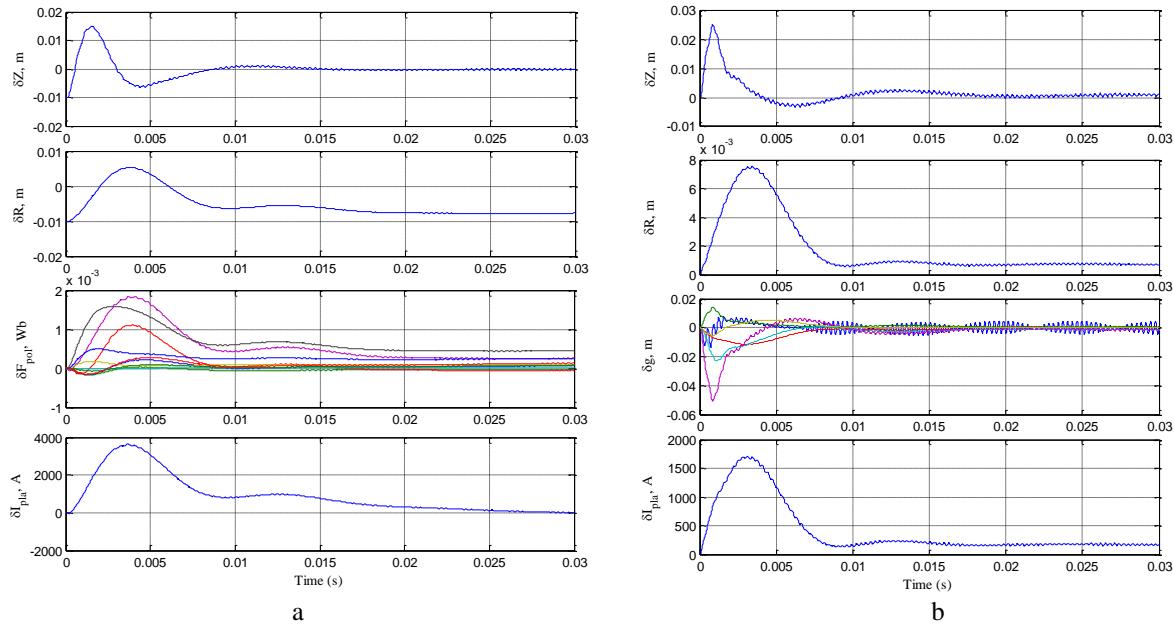


Fig. 3. Transient processes in control system: (a) isoflux and (b) gap control.

CONCLUSIONS

- Equilibrium reconstruction code of Flux and Current Distribution Identification (FCDI) was created and applied to the experimental data from Globus-M tokamak;
- Linear models of Globus-M experimental plasma were derived;
- Multivariable H_∞ robust systems for control of (i) poloidal flux on the separatrix and (ii) gaps between the separatrix and the first wall were designed and simulated;
- Obtained results are assumed to be applied on Globus-M tokamak in real time.

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