Study of the reserved shear Alfven eigenmodes with down sweeping frequency in HL-2A experiment at $q_{\mathbf{min}} \approx 1$

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Recently, the RSAEs have been observed in two typical shots of HL-2A experiments during the tangial NBI with $E = 40keV$ + ECRH and current ramp-up[1]. It was found that the mode with down-sweeping frequency and the mode with up-sweeping frequency appeared in the spectrogram of Mirnov signal.

Figure 1(a) shows the toroidal mode number $n = 2-5$ modes appear simultaneously with downward frequency sweeps lasting $30 \sim 40$ ms for shot I. The begining downward frequencies increase with the toroidal mode numbers from above 90 kHz to 150 kHz. These frequencies sweep down approximately to the same freqency finally, i.e. ≥ 50 kHz. For shot II being supplied a different ECRH power, the downsweeping RSAEs with the toroidal mode number $n = 2-3$ are shown in Figure 1(b). Therefore, the modes are also called Alfven cascades. The same features of the modes as in shot I are found with starting with different frequencies increasing with the toroidal mode number and approaching the same final frequency.

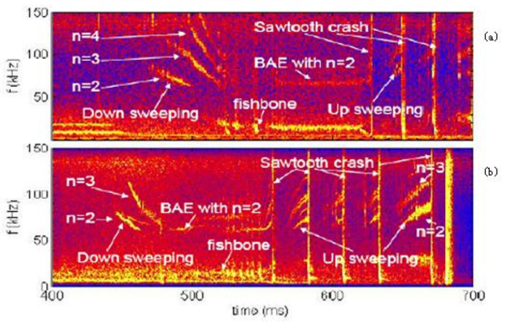

Figure 1: *Spectrogram of Mirnov signal for shot I (up) and shot II (down)*

To investigate the modes of the down sweeping frequency, we adopt the eigenmode equation for shear Alfven waves with effects of thermal ion FLR and parallel electric field as following[2],

$$\nabla \cdot \left(\frac{\omega^2}{v_A^2} \nabla_\perp \delta\phi \right) + \mathbf{B} \cdot \nabla \left(\frac{1}{B^2} \nabla \cdot B^2 \nabla_\perp Q \right) - \nabla \left(\frac{J_\parallel}{B} \right) \cdot \nabla Q \times \mathbf{B}$$

$$+2 \frac{\vec{\kappa} \cdot \mathbf{B} \times \nabla \delta P}{B^2} + \nabla_\perp^2 g_{km} \frac{1}{\rho} \nabla_\perp \rho \cdot \nabla_\perp \delta\phi = 0, \qquad (1)$$

with $Q = \frac{1}{B} \mathbf{b} \cdot \nabla \delta\phi$, $g_{km} = \frac{3}{8} \rho_i^2 \frac{\omega^2}{v_A^2} + \frac{1}{2} \rho_s^2 (1 - i\delta) k_{\parallel m}^2$. Here, $\vec{\kappa} = \mathbf{b} \cdot \nabla \mathbf{b}$ is the magnetic field curvature. v_A is Alfven velocity. ω is the eigenvalue frequency. $\rho_i = (2 m_i T_i)^{1/2} / eB$ is thermal ion gyroradius, and $\rho_s = (m_i T_e)^{1/2}/eB$ is ion acoustic gyroradius, $k_{\parallel m} = (m - n/q)/R$ is parallel wave number, δ is the wave dissipation rate, the disspation is relate to the collisional and Landau damping of electron acted by parallel electric field. $J_\parallel$ is the parallel current density. ρ is mass density. In Eq. (1) , the physics meanings with respect to the first four terms are well known and are explored extensively, which give so called Alfven law. The last term of Eq. (1) results from kinetic effects of thermal ion FLR and parallel electric field, which plays an important role in resolving the physics in inertial layer, and should be treated non-perturbatively.

For studying the behaviour of the downsweeping RSAEs frequencies and mode structures, we use the non-perturbative kinetic Alfven eigenmode code(KAEC)[3] to solve Eq. (1) on the basis of experimental measurements and TSC results. The numerical results are given in the followings. The reversed q profile is given by

$$q(x) = q_0 + x^2 \left[q_a - q_0 + (dq_a - q_a + q_0)(1 - y_s)\left(x^2 - 1\right) / \left(x^2 - y_s\right)\right], \quad (2)$$

with $y_s = (dq_a - q_a + q_0) / [dq_0 + dq_a - 2(q_a - q_0)]$, $q_a = 4.2$, $dq_0 = -9$, $dq_a = 4$. q_0 is the safety factor at the magnetic axis. The density profile is given by $n(x) = n_0 \left(1.00001 - \alpha x^\beta\right)^\gamma$ with $\alpha = 0.8$, $\beta = 1.6$, $\gamma = 1.8$. n_0 is the density at the magnetic axis, which is equal to $1.2 \times 10^{13} cm^{-3}$. $\beta \equiv 2\mu_0 P/B^2$ is the ratio of plasma pressure to magnetic pressure. The value of β_0 at the magnetic axis is approximated as 8.4×10^{-3} for HL-2A plasma with $B = 1.33T$, $T_{e0} = 2.4keV$ and $T_{i0} = 0.64keV$. The profile of β is shown in Figure 2.

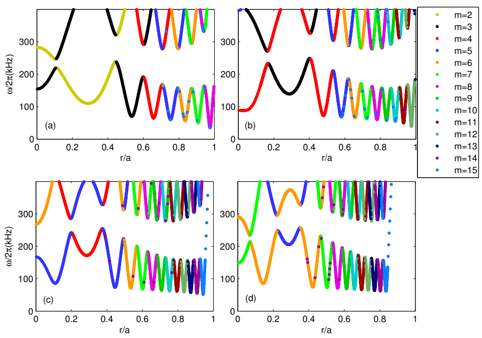

Figure 3: *Alfven continuum spectrum for* $q_0 = 1.34$. *(a)n=2, (b)n=3, (c)n=4, (d)n=5.*

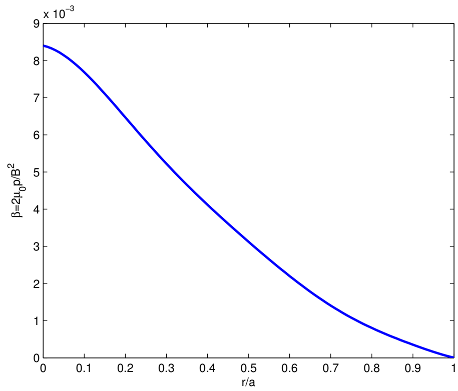

Figure 2: $\beta\left(\equiv 2\mu_0 P/B^2\right)$ *vs* r/a.

In Fig. 3, the Alfven continua spectrums are plotted for $q_0 = 1.34$. As is known, the frequency of RSAEs with toroidal mode number n and polodial mode number m can be approximated as $\omega = k_{\parallel m} v_A = (n - m/q_{\min}) v_A/R$ and it is corresponding to the local minimum of the Alfven continuum that is called the accumulation point(AP). The discrete global modes can easily exist at APs due to the less continua damping. Fig. 3 indicate that the RSAEs can be found near APs, whose position is at about $r/a = 0.25$. Due to finite β plasma, the geodesic compressibility is accounted for, which gives the geodesic acoustic frequency ω_{geo} and results in the beta induced Alfven eigenmode gap.

The global eigenmodes can easily exist at APs due to the less continua damping. However, in the case of the downsweeping mode with $q_{\min} > 1$, an effective potential hill of Eq. (1) with the kink term proportional to the gradient of $J_{\|}$ is produced in ideal MHD limit instead of a well[4], which can not localize the RSAEs. Including the non-ideal effects of Eq. (1), i.e. FLR and finite $E_{\|}$ field, the kinetic eigenmodes, named kinetic reversed shear Alfven eigenmodes(KRSAEs), can be obtained just above the AP[5]. The non-ideal effects discrete the continuum, thus, ignore the continua damping, in order to estabalish the eigenmodes. Figure 4 shows the caculated mode structures of KRSAEs of normalized thermal ion gyroradius $\rho_i/a = 0.005$, corresponding to Figure 3. They are the local modes near the position of APs, which are consistent with the observation.

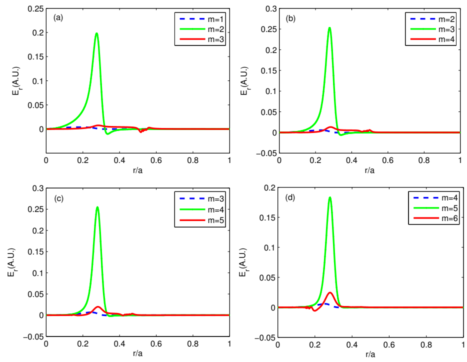

Figure 4: *The mode structure of KRSAEs for* $q_0 = 1.34$, $\rho_i/a = 0.005$

Figure 5 shows the frequencies of RSAEs of different torodial mode number n versus $q_{\min}$. The solid curve respesents the eigen frequencies of KRSAEs with $\rho_i/a = 0.005$, while the dash curve respesents the frequencies at the APs of the Alfven continuum. The mode frequencies sweep from $120-200kHz$ down to $80kHz$ as in Figure 1. For the same mode, the frequency of KRSAEs is just above the APs and the mismatch of the two kinds frequency increases with toroidal mode number n.

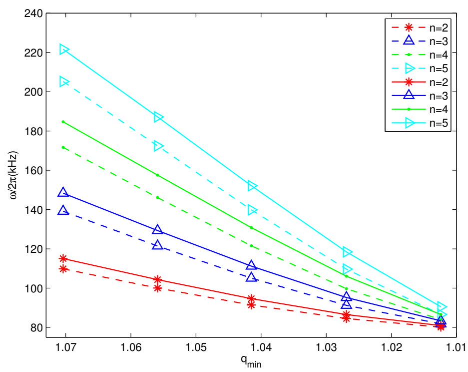

Figure 5: *The RSAE frequencies versus* q_{min}

Theoretically, the compact form of the vorticity equation for reversed shear Alfven eigenmodes, i.e. Eq. (1) , is expressed by[5]

$$\frac{d}{dx}\left[\left(x^2+S\right)\frac{d\delta\phi}{dx}\right] + \left(Q-S-x^2\right)\delta\phi - (1-i\mu)\Lambda\frac{d^4\delta\phi}{dx^4} = 0, \quad (3)$$

where $\Lambda \propto \rho_i^2$. It is convenient to treat Eq. (3) of forth order differetial equation in the Fouier space. Applying the Fouier transformtion to the above equation, .e.g, $\frac{1}{\sqrt{2\pi}}\int\cdots e^{ipx}dx$, we obtain the mode equation in p space,

$$\frac{d}{dp}\left[\left(p^2+1\right)\frac{d\delta\hat{\phi}\left(p\right)}{dp}\right]+\left[Q-\left(1+p^2\right)S-\left(1-i\mu\right)\Lambda p^4\right]\delta\hat{\phi}\left(p\right)=0, \tag{4}$$

with $\delta\hat{\phi}\left(p\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\delta\phi\exp\left(ipx\right)dx$. To explore the basic features of the mode Eq. (4), we can use the variational principle since this equation is a second order ordinary differetial equation. The functional L $\left[\langle\rangle\equiv\int_{-\infty}^{+\infty}dx\right]$is given by

$$L=-\langle\left(1+p^2\right)\delta\hat{\phi}'^2\rangle+\langle\left[Q-\left(1+p^2\right)S-\left(1-i\mu\right)\Lambda p^4\right]\delta\hat{\phi}^2\rangle. \tag{5}$$

We focus on the mode of the lowest eigenfrequency, and choose a trial function with no zeros for finite p, $\delta\hat{\phi}\left(p\right)=e^{-\alpha p^2/2}$, $\Re\alpha>0$. Therefore, we can obtain

$$L\propto\left[3/4+\alpha/2-\left(Q-S\right)+S/2\alpha+3\left(1-i\mu\right)\Lambda/2\alpha^2\right]. \tag{6}$$

Using the equations of $L=0$ and $\partial L/\partial\alpha=0=1-S/\alpha^2-3\Lambda/\alpha^3$, we can solve α for S[6]. For KRSAEs, the modes have broad structures in p ($\alpha\ll 1$) due to the Λ-dependent term that provide the confining potential. we obtain the mode width in p ,$\Delta p=\sqrt{\frac{1}{\left(1-\frac{4}{3}Q\right)^{1/2}\Lambda^{1/2}}}$. As is known, p and x are conjugate Fourier variables, i.e., $\Delta p\Delta x\sim 1$. Thus, the radial width of this mode is $\Delta x=\sqrt{\left(1-\frac{4}{3}Q\right)^{1/2}\Lambda^{1/2}}$, which is shown in Figure 6.

In conclusion, the reserved shear Alfven eigenmodes (RSAEs) whose frequencies sweep down have been studied in HL-2A plasmas at $q_{\min}\approx 1$ using KAEC. It is shown that the kinetic effect due to FLR and finite $E_{\|}$ field and the kink term proportional to the gradient of $J_{\|}$ play cruial roles in estabilishing the modes. Specially, including the kinetic effect and keeping the kink term in the vorticity equation, the modes called kinetic reversed Alfven eigenmodes (KRSAEs) are obtained for the down sweeping case of $q_{\mathbf{min}}>1$; if only including the kink term, the modes with down sweeping frequency cannot exist. It is shown that the modes have localized mode structure, which is consistent with the observations. Furthermore, the mode width of KRSAEs is proportional to $(\rho_i)^{1/2}$ is investigated numerically and theoretically.

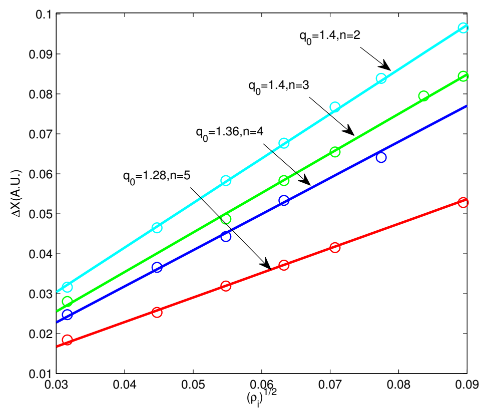

Figure 6: *The width of KRSAEs* Δx *versus* $(\rho_i/a)^{1/2}$*. Here,* Δx *is fitted linearly with* $(\rho_i/a)^{1/2}$.