Effect of strong poloidal modulation of anisotropic plasma pressure on the Shafranov shift in tokamaks

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1. Introduction. Selection of parameters and modes of operation of the designed systems with magnetic confinement is always based on the results of calculations of the plasma equilibrium. Usually they are made in assumption of isotropic plasma. However, in some ITER scenarios the plasma can be significantly anisotropic [1]. Strong anisotropy have been detected under intensive heating of the plasma in tokamaks JET (0.24 MW/m^3) and Tore Supra (0.28 MW/m^3) [2]. Also, plasma anisotropy have been observed in spherical tokamak MAST and in stellarators CHS and LHD [3-5]. For comparison, in the new project based on T-15 tokamak the expectations are 0.42 MW/m^3 with total power up to 13 MW [6].

Here we focus on the changes in equilibrium configurations due to the plasma anisotropy in a tokamak with parameters of the projected modernized T-15 tokamak [6].

2. Formulation of the problem. The plasma equilibrium is modelled with the code SPIDER [7] modified for anisotropic pressure. The force balance is described by the equation

$$0 = -\nabla \cdot \overleftrightarrow{p} + \mathbf{j} \times \mathbf{B}, \tag{1}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{j} = \nabla \times \mathbf{B}$ is the current density and $\overleftrightarrow{p}$ is the pressure tensor,

$$\overleftrightarrow{p} = p_{\|}(\mathbf{BB}/\mathbf{B}^2) + p_{\perp}(\overleftrightarrow{\mathrm{I}} - \mathbf{BB}/\mathbf{B}^2) \tag{2}$$

with the unit dyadic $\overleftrightarrow{\mathbf{I}}$, $\mathbf{B} = \nabla\psi \times \nabla\zeta + F\nabla\zeta$, $2\pi\psi$ is the poloidal magnetic flux, $2\pi F$ is the similar flux of $\mathbf{j}$, $p_{\|}$ and $p_{\perp}$ are the plasma pressures along and perpendicular to $\mathbf{B}$ taken as

$$p_{\|}(\psi, B) = p_{\perp 0}(\psi) + [p_{\| 0}(\psi) - p_{\perp 0}(\psi)]b + 0.5 p_{\perp 1}(\psi)(1-b)^2 / b, \tag{3}$$

$$p_{\perp}(\psi, B) = p_{\perp 0}(\psi) + p_{\perp 1}(\psi)(1-b)/b. \tag{4}$$

Here $b = B/B_m$ with $B_m = const$. This form of $p_{\|}$ and $p_{\perp}$ guaranties the parallel force balance [8, 9]. At the beginning, we prescribe the pressure function by similar profiles, but with different amplitudes: $p_{\perp 0} = k_{\perp 0} p$, $p_{\| 0} = k_{\| 0} p$ and $p_{\perp 1} = k_{\perp 1} p$ with constant $k_{\perp 0}$, $k_{\| 0}$, and $k_{\perp 1}$, where $p(\psi)$ corresponds to the isotropic pressure in one of the basic scenarios proposed for T-15 in [10]. The SPIDER code numerically solves the equation

$$\mathrm{div}\frac{\sigma\nabla\psi}{r^2} = -\frac{\partial p_\parallel(\psi,B)}{\partial\psi} - \frac{F_K F_K'}{\sigma r^2} \tag{5}$$

with $\sigma = 1-(p_\parallel - p_\perp)/\mathbf{B}^2$ that comes from (1), (2) and the Maxwell equations at axial symmetry [11, 12]. We consider the same fixed plasma boundary shape as in [10].

3. Effect of plasma anisotropy on the equilibrium configuration. We start from studying the plasma equilibrium with $p_\parallel > p_\perp$ that can be expected in tokamaks with tangential NBI.

In anisotropic plasma the pressures vary on the surfaces $\psi = const$ (in contrast to the isotropic plasma with $p_\parallel = p_\perp = p(\psi)$). This affects the toroidal current density j_ζ and the total magnetic field B. Configuration with $p_\perp = p_\perp(\psi)$ for $\beta_\parallel/\beta_\perp \approx 3$ and $(\beta_\parallel + \beta_\perp)/2 \approx \beta_{iso}$ is shown in Fig. 1, where $\beta_{iso} \approx 0.91\%$ corresponds to the selected basic scenario with isotropic pressure [10]. In this case, the maximum of the parallel pressure is shifted to the HFS relative to the magnetic axis by ≈–4.4 cm (about 0.066 of the minor plasma radius a), compare Fig. 1(a) with Fig. 1(b). The distributions of $j_\zeta^{aniso} - j_\zeta^{iso}$ and $B^{aniso} - B^{iso}$ are presented in Figs. 1(c) and (d), respectively. At $p_\parallel > p_\perp$ the current density j_ζ is reduced at the centre of the plasma column and has significant poloidal modulation near the edge. Magnetic field B increases in the central region and decreases at the periphery. At fixed boundary the most significant differences (about 20%) in the toroidal current density are observed in the thin layer near the plasma edge. Within the error of 0.45%, the poloidal magnetic field B_{pol} at the plasma boundary remains the same as in isotropic plasma.

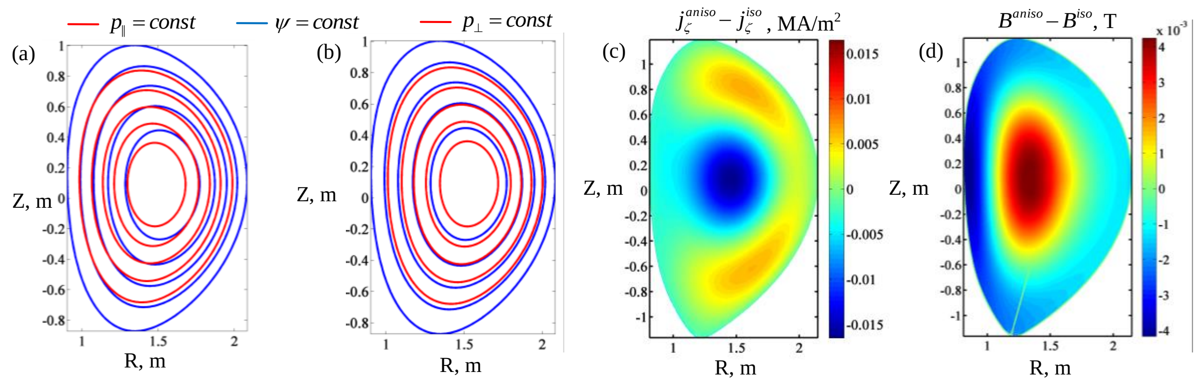

Fig. 1. Equilibrium configuration at $p_\perp = p_\perp(\psi)$ with $\beta_\parallel/\beta_\perp \approx 3$ and $(\beta_\parallel + \beta_\perp)/2 \approx \beta_{iso} \approx 0.91\%$.

At intensive off-axis heating, strong poloidal modulation of $p_\perp$ can occur. We describe it by $p_{\perp 1} \neq 0$ in (4). It also causes the poloidal variations of j_ζ, B and $p_\parallel$. The latter is always weak in agreement with theoretical prediction of [13], so that the maximum of $p_\parallel$ is not shifted. The results for the plasma with $p_{\perp 0} = p_{\parallel 0} = p$ and $p_{\perp 1} = 2p$ ($\beta_\parallel + \beta_\perp \approx 2\beta_{iso}$) are shown in Fig. 2. In this case, the maximum of $p_\perp$ is shifted to LFS by ≈ 11 cm relative to the magnetic axis (see Fig. 2(b)). The toroidal current changes significantly (up to 20%) only near the plasma boundary, while the relative change in B_{pol} is up to 0.4% only.

When $p_{\perp 1} = -2p$, in contrast to the previous configuration, we obtain a similar shift of the maximum of $p_\perp$ by ≈ 11 cm, but into the opposite (HFS) direction. Variation in j_ζ, B are similar to those shown in Figs. 2(c) and (d), but have the opposite sign.

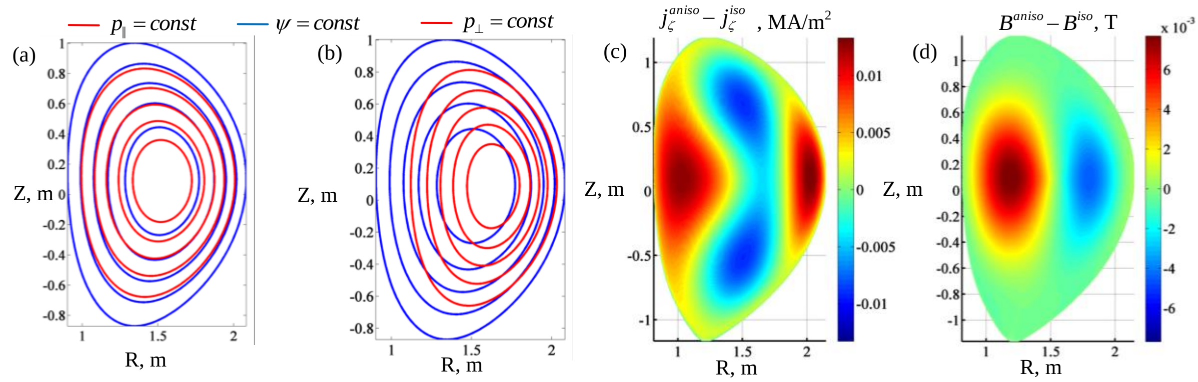

Fig. 2. Equilibrium configuration with $\beta_\parallel \approx \beta_\perp \approx \beta_{iso}$ and strong poloidal modulation of $p_\perp$ ($p_{\perp 1} = 2p$).

4. Effect of plasma anisotropy on the Shafranov shift. For tokamaks it was shown analytically [14] that Δ is mainly determined by $(\beta_\parallel + \beta_\perp)$. Our numerical calculations confirm this, but we found that Δ also slightly depends on the poloidal modulation of $p_\perp$ or differences in $p_{\parallel 0}$ and $p_{\perp 0}$ profiles. At similar profiles of $p_{\parallel 0}$ and $p_{\perp 0}$ and fixed $(\beta_\parallel + \beta_\perp)$, Δ is larger when $p_\perp$ is maximal at HFS and smaller in the opposite case, see Fig. 3(b). We found that the maximal possible changes in Δ due to poloidal modulation of $p_\perp$ at $\beta_\parallel + \beta_\perp = 2\beta_{iso}$ with $p_{\perp 1} = k_{\perp 1} p$ (at $k_{\perp 1}$ constrained by the natural requirements that $p_\parallel > 0$ and $p_\perp > 0$) is small and equal to -0.15 cm at $p_{\perp 1} > 0$ ($k_{\perp 1} \approx 2.22$) and to 0.16 cm at $p_{\perp 1} < 0$ ($k_{\perp 1} \approx -2.5$), while the Shafranov shift at isotropic pressure $\Delta_{iso} = 2.31$ cm. These differences in Δ are much smaller than the shifts (≈ 11 cm) of maximums of $p_\parallel$ and $p_\perp$ relative to the magnetic axis at $\beta_\parallel / \beta_\perp \approx 1$ and $k_{\perp 1} = \pm 2$.

To investigate the effect of pressure profiles on Δ we use $p'_{\perp 0} = k_{\perp 0} p'(0)(1-\psi_n^{\alpha_1})^{\alpha_2}$, where the prime denotes $\partial / \partial\psi$, $\psi_n = (\psi - \psi_a)/(\psi_b - \psi_a)$, ψ_a and ψ_b are the values of ψ at magnetic axis and at plasma boundary, respectively, and α_1, α_2 are constants, while still

$p_{\parallel 0} = k_{\parallel 0} p$. In Fig. 3(c), the results for $\alpha_1 = 2$ and $\alpha_2 = 4$ are presented. The lines of markers of the same colour ($k_{\perp 0} = const$) are parallel to the blue one, which means equilibriums with $p'_{\parallel 0} \propto p'_{\perp 0} \propto p'$. The lines with same form of markers ($k_{\parallel 0} = const$) are parallel to the red one corresponding to configurations with $p'_{\parallel 0} \propto p'_{\perp 0} \propto p'(0)(1-\psi_n^{\alpha_1})^{\alpha_2}$. At fixed $(\beta_\parallel + \beta_\perp)$ configurations with significantly different Δ are possible. For example, at $k_{\parallel 0} = 1$, $k_{\perp 0} = 4.75$ with $\alpha_1 = 2$ and $\alpha_2 = 10$ ($\beta_\parallel + \beta_\perp \approx 2\beta_{iso}$) the Shafranov shift is larger by 1.66 cm ($\approx 72\%$ Δ_{iso}) that Δ_{iso}. At the same time, B_{pol} on the plasma boundary changes only up to 0.6%.

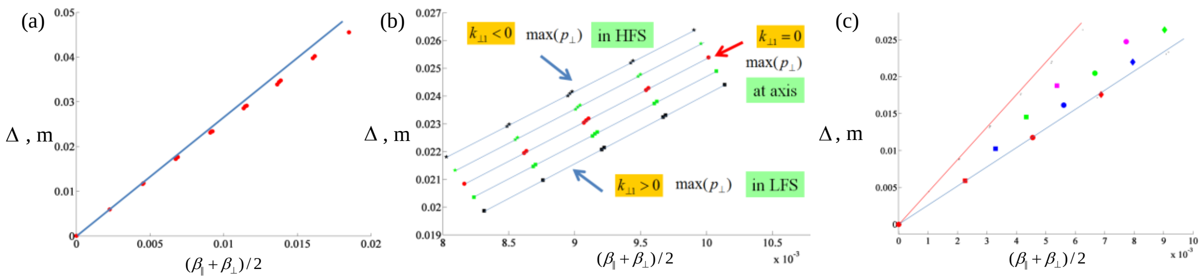

Fig. 3. The Shafranov shift Δ as function of $(\beta_\parallel + \beta_\perp)/2$ without (a) and with (b) poloidal modulation of $p_\perp$ at similar and different (c) profiles of $p_\parallel$ and $p_\perp$.

5. Conclusion. Our numerical calculations for anisotropic plasma with poloidal modulation of $p_\perp$ demonstrate that the Shafranov shift is mainly determined by $\beta_\parallel + \beta_\perp$ and weakly depends on poloidal variation of $p_\perp$. The magnetic axis position is almost unaltered even at significant shift of the maximum of $p_\perp$ (up to ± 11 cm relative to its position at isotropic pressure). At the same time, we found that equilibrium distributions and Δ at fixed $\beta_\parallel + \beta_\perp$ are sensitive to differences in the profiles of $p_{\parallel 0}$ and $p_{\perp 0}$. Calculations are made for T-15 tokamak with total plasma current $I = 2$ MA and toroidal magnetic field at axis $B_t = 2$ T.

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