# Extended neoclassical rotation theory for tokamaks and neoclassical predictions of transport and poloidal asymmetries

C. Bae[1], W.M. Stacey[2], S.G. Lee[1], L. Terzolo[1]

[1] *National Fusion Research Institute, Daejeon, South Korea*

[2] *Fusion Research Center, Georgia Institute of Technology, Atlanta, U.S.A.*

The extended neoclassical rotation theory[1,2] incorporating poloidal asymmetries in density, rotation velocities, and electrostatic potential was recently extended[3,4] to an elongated poloidally asymmetric and Shafranov-shifted flux surface geometry[5,6]. The theory[1-4] is based on the Braginskii's closure[7], the Stacey-Sigmar(S-S) representation of the poloidal asymmetries[8,9] and its viscosity model based on the Braginskii's flow rate of strain[7] is extended to the Miller flux surface model by Stacey and Bae[10] and to arbitrary collisionality by the use of the Shaing-Sigmar neoclassical viscosity model[9]. Along with the reported measurements of the poloidal variations in density and velocity in many tokamaks[11-15], a neoclassical investigation of their effect on plasma rotation and transport is reported in a recent paper[16], which is summarized in this report.

The momentum balance equation in Eq. (1),

$$n_j m_j \left(\vec{V}_j \cdot \nabla\right)\vec{V}_j + \nabla P_j + \nabla \cdot \overleftrightarrow{\boldsymbol{\pi}}_j = n_j e_j (\vec{E} + \vec{V}_j \times \vec{B}) + \vec{F}_j^{\,1} + \vec{S}_j^{\,1} - m_j \vec{V}_j S_j^{\,o} \tag{1}$$

with the first term coming from the Reynolds stress, $n_j m_j \nabla \cdot \left(\vec{V}_j \vec{V}_j\right)$, thus called "inertial" term in the extended theory[1-4] is decomposed into three components( $r, \theta, \phi$ ) and converted into the curvilinear geometry by Stacey and Bae[11]. The final curvilinear form of the poloidal momentum balance equation is shown in Eq. (2) with the sources( $S_j^{\,o}$ and $\vec{S}_j^{\,1}$ ) and friction( $\vec{F}_j^{\,1}$ ) replaced with the actual calculation models.

$$\begin{aligned} &n_j m_j \left[\left(\vec{V}_\mathrm{j} \cdot \nabla\right)\vec{V}_\mathrm{j}\right]_\theta + \left(\nabla \cdot \overleftrightarrow{\boldsymbol{\pi}}_\mathrm{j}\right)_\theta + \frac{1}{h_\theta}\frac{\partial p_j}{\partial \theta} - M_{\theta j} + n_j m_j \nu_{jk}\left(V_{\theta j} - V_{\theta k}\right) \\ &+ n_j e_j \left(V_{rj} B_\phi - E_\theta\right) + n_j m_j \nu_{ionj} V_{\theta j} + n_j m_j \nu_{elcxj} V_{\theta j} = 0 \end{aligned} \tag{2}$$

where $h_\theta$ is the poloidal metric coefficient and $M_j$ is the external momentum. The lowest order Fourier expansion of density, velocity, and electrostatic potential is of the form,

$$X_j(r,\theta) \approx \overline{X}_j(r)\left[1 + X_j^c(r)\cos\theta + X_j^s(r)\sin\theta\right] \tag{3}$$

for a given plasma parameter $X$ of the species $j$ with the overbar indicating the average over the flux surface. The superscript "c" and "s" indicate poloidally "in-out" and "up-down" asymmetries respectively.

The cross-field toroidal angular momentum transports are represented by the inertial and gyroviscous transport frequencies ($\nu_{nj}$ and $\nu_{dj}$ respectively) shown below.

$$\nu_{nj} \approx \frac{\overline{V}_{rj}}{R_0}\left[\frac{\partial R_0}{\partial r}\left\langle\frac{1}{h_r}\right\rangle + \left\langle\frac{\cos\xi}{h_r}\right\rangle - R_0 L_{V_\phi}^{-1}\right] + \overline{V}_{\theta j}\varepsilon\tilde{V}_{\phi j}^{s}\left(\left\langle\cos\theta\frac{1}{h_\theta}\right\rangle + \frac{\left\langle\frac{1}{R}\frac{\partial R}{\partial\theta}\sin\theta\frac{1}{h_\theta}\right\rangle}{\left\langle\sin^2\theta\frac{1}{h_\theta}\right\rangle}\left\langle\cos^2\theta\frac{1}{h_\theta}\right\rangle + \varepsilon\left\langle\cos\theta\frac{\cos\xi}{h_\theta}\right\rangle + \frac{1}{R_0}\left\langle\frac{\partial R}{\partial\theta}\sin\theta\frac{1}{h_\theta}\right\rangle\right) \tag{4}$$

$$\nu_{dj} \approx \nu_{dj}^{1} + \nu_{dj}^{2} \tag{5}$$

where

$$\nu_{dj}^{1} = -\frac{T_j}{R_0^2 e_j \overline{B}_\phi}\varepsilon\left[\begin{array}{l}\tilde{V}_{\phi j}^{s}\left(2R_0\left\langle\cos\theta\frac{\cos\xi}{h_\theta h_r}\right\rangle + \left\langle R\cos\theta\frac{\cos\xi}{h_\theta h_r}\right\rangle + \left\langle R\sin\theta\frac{\sin\xi}{h_\theta h_r}\right\rangle + x\left\langle R\sin\theta\cos\theta\frac{\sin\xi}{h_\theta h_r}\right\rangle\right) \\ +\tilde{n}_j^{s}\left(\left\langle R\sin\theta\frac{\sin\xi}{h_\theta h_r}\right\rangle + x\left\langle R\sin\theta\cos\theta\frac{\sin\xi}{h_\theta h_r}\right\rangle\right)\end{array}\right], \tag{6}$$

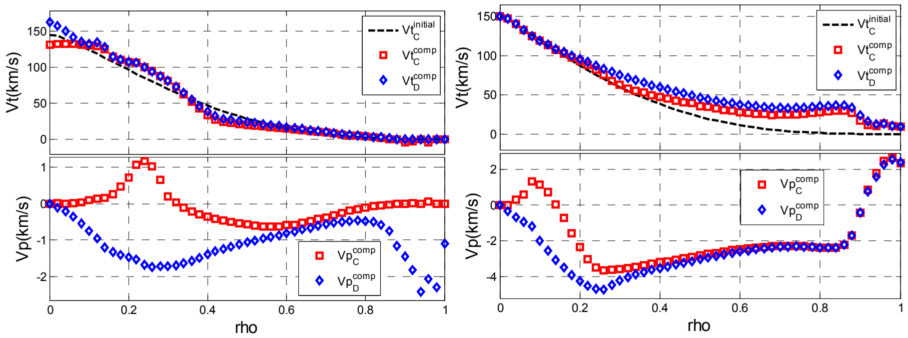
$$\nu_{dj}^{2} \equiv \frac{1}{2}\frac{\tilde{\theta}_j G_j T_j}{R_0^2 e_j \overline{B}_\phi} \tag{7}$$

where $G_j$ and $\tilde{\theta}_j$ can be found in Ref. [16], $\partial R_0/\partial r$ represents the Shafranov shift[5], $L_X^{-1} = -1/X\,(\partial X/\partial r)$ is the gradient length scales, $h_r$ is the radial metric coefficient, $\varepsilon \equiv r/R_0$, $\tilde{V}_{\phi j}^{c,s} \equiv V_{\phi j}^{c,s}/\varepsilon$, and $\tilde{n}_j^{c,s} \equiv n_j^{c,s}/\varepsilon$. The extended rotation theory[1-4] neglects the perpendicular viscosity due to Braginskii's $\eta_{gv} >> \eta_\perp$ ordering and the gyroviscosity, calculated as a strong function of poloidal asymmetries, replaces the role of predicting the neoclassical viscous damping. The theory[3,4] takes the cosine and sine moments of Eq. (2), which reduce to Eqs. (8) and (9) respectively (expressed in generic forms with $j$ being either $i = deuterium$ or $I = carbon$ and $k$ being the other), to solve for $\tilde{n}_j^{c,s}(r)$ and the other asymmetries are coupled with $\tilde{n}_j^{c,s}(r)$.

$$A_{C1}\tilde{n}_j^{c} + A_{C2}\tilde{n}_j^{s} + A_{C3}\tilde{n}_k^{c} = B_C, \tag{8}$$

$$A_{S1}\tilde{n}_j^{c} + A_{S2}\tilde{n}_j^{s} + A_{S3}\tilde{n}_k^{s} = B_S \tag{9}$$

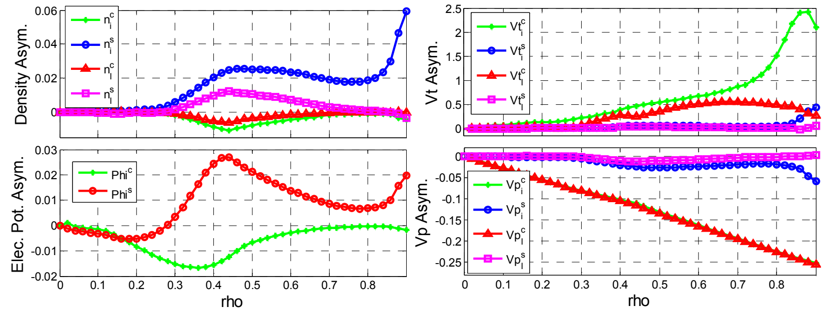
Two similar KSTAR NBI H-mode discharges were analyzed recently[16]. Fig. 1 shows the toroidal (Vt) and poloidal (Vp) velocities predicted by GTROTA[17], the rotation and transport calculation code written for the extended rotation theory[3,4]. The predicted Vt ( $Vt_C^{comp}$ ) results are shown to stay within approximately <10 % to the experimentally measured profiles ( $Vt_C^{initial}$ ) in the core ranges.



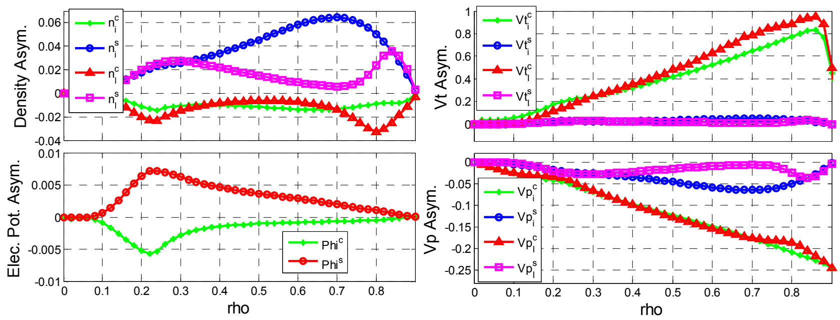

**a) #5505(2500ms)** **b) #5953(2500ms)**

**FIG. 1. Vt and Vp in two KSTAR discharges (positive CW for Vt / upward at outboard midplane for Vp)**

Figs. 2 show all the calculated poloidal asymmetries for the reliable range ( $\rho < 0.9$ ) with the current theory[3,4]. Fig. 3 presents the calculated inertial ( $\nu_{nj}$ ) and gyroviscous ( $\nu_{dj}$ ) transport frequencies, and their additions ( $\nu_{nj} + \nu_{dj}$ ), showing no indication of gyroviscous cancellation in realistic tokamak geometry.




**a) #5505(2500ms)**




**b) #5953(2500ms)**

**FIG. 2. Poloidal asymmetries of KSTAR #5505(2500ms) and #5953(2500ms) discharges.**

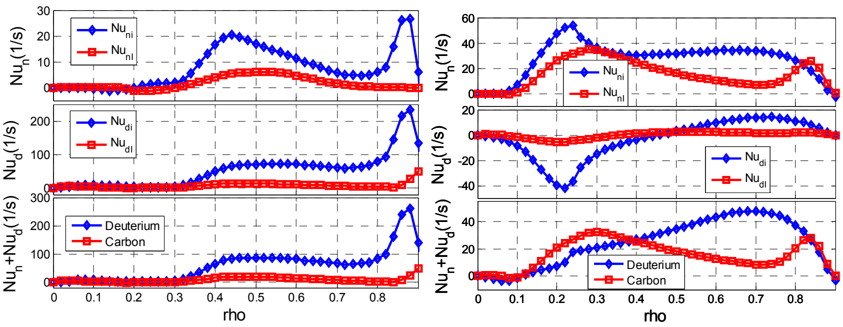



**FIG. 3. Transport frequencies of two KSTAR discharges (left: #5505-2500 ms/right: #5953-2500 ms)**

In conclusion, the extended rotation theory[3,4] allows unprecedented neoclassical calculations of the poloidal asymmetries not only in density but also in velocities and electrostatic potential, thus neoclassically predicting both the inertial and gyroviscous transport contributions as functions of these asymmetries. The numerical calculations for the two KSTAR discharges show the following ordering relation between the asymmetries.

$$\left\{\Phi^{c/s} \le O\left(10^{-2}\right)\right\} < \left\{n^{c/s} \sim V^{s}_{\phi,\theta} \sim O\left(10^{-2}\right)\right\} < \left\{V^{c}_{\phi,\theta} \sim O\left(10^{-1}\right)\right\} \tag{10}$$

The extended rotation theory[3,4] will be further developed in the future to increase its accuracy in the plasma edge.

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