Long rows of the magnetic vortices in turbulent plasma

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I. The mechanisms for generation of fluid motions with additional symmetry, frequently refered to as flows, have been studied extensively in recent years both in plasmas and fluid dynamics. In a plasma allowing for inhomogeneities, the gradient-specific modes known as drift type modes are able to propagate in the direction of translation symmetry, i.e. perpendicular to the gradient. These modes can then spontaneously generate structures with higher saymmetry, the large scale flows, in a way similar to the established Reynolds stress in hydrodynamics, using the free energy stored in density and temperature gradients. The process of generation of such structures is energetically sustained through the well-known inverse cascade guaranteed in two-dimensional (and quasi-two-dimensional) fluids by the conservation of energy and enstrophy. Here we investigate the generation of large scale magnetic structures by underlying small scale magnetic electron drift (MED) wave turbulence. These turbulent magnetic fluctuations are drift-type modes excited in non-uniform initially non-magnetized plasma, characterized by a frequency range in-between the electron and ion plasma frequencies. These modes may be stable or unstable, depending on the combination of density and electron temperature gradients. Phenomena occurring in such time scales are important as a source of different magnetic structures encountered in space plasma, laser fusion plasma, as well as in a number of plasma devices. The particular interest in these studies is to formulate the conditions for the formation and existence of the infinitely long rows of vortices or vortex streets with finite vorticity associated with MED waves. To perform the analysis we use the two-field nonlinear model equations for MED modes. The source of the instability is the baroclinic vector in the electron fluid which gives rise to a finite vorticity. These equations support two spectrally conserved integrals of motion. The double cascade is a consequence of existence of these two integrals and it can be inferred that the mean square magnetic field cascades towards longer scales. Together with the continuous evolution, these properties are sufficient for existing the stationary localized 2D-solutions, the magnetic vortex structures. The stationary localized solutions of the vortex-street type are obtained. Finally, the evolution of nonlinearly interacting MED modes is illustrated by a simulation study of the model equations for different set of parameters.

II. The motion of the considered modes is assumed to take place in the plane perpendicular to the magnetic field and hence a quasi-two-dimensional analysis is applied, where only the perturbed magnetic field is directed along the third dimension, here chosen to be the z axis. These modes are placed in a non-uniform unmagnetized plasma with density and temperature gradients along the x axis. The temperature and density gradients of the fluctuations are in general not collinear, and this generates a vorticity in electron fluid. The consequent motion generates a perpendicular magnetic field (with vanishing equilibrium part), $B(x,y,t)\mathbf{z}$, which actually plays the role of a stream function. Due to a typical time scale of the MED modes, the ions play the role of a neutralizing background in the mode dynamics, whereas the electrons move fast enough to equalize any density perturbation in a relatively short time. Therefore, the electron density will be considered constant on time, $n = n_0$. The temperature can be written as the sum of an equilibrium value T_0 and a perturbation T. We assume that

the length scale of the fluctuations is much smaller than that of the equilibrium inhomogeneities, (this can be expressed by small parameters $\varepsilon_n \sim \left|\nabla \ln n_0\right|/k$ and $\varepsilon_T \sim \left|\nabla \ln T_0\right|/k$, where $k^{-1} \sim c/\omega_{pe}$ is the typical spatial scale of the fluctuations), and take $\varepsilon_n \sim \varepsilon_T \sim \varepsilon$. Starting then from the momentum equation and the energy equation, the model equations for MED mode turbulence can be derived up to the lowest non-vanishing order in ε and read in dimensionless form

$$\frac{\partial}{\partial t}\left(B - \nabla^2 B\right) - \left\{B, \nabla^2 B\right\} = -v_0 \frac{\partial T}{\partial y} \tag{1a}$$

$$\frac{\partial T}{\partial t} + \left\{B, T\right\} = -w_0 \frac{\partial B}{\partial y} \tag{1b}$$

Here, $v_0 = \left|\nabla \ln n_0(x)\right|$, $w_0 = T_0\left(2v_0/3 - \left|\nabla \ln T_0\right|\right)$ may be regarded as constant coefficients, the length unit is $\left(c/\omega_{pe}\right)$, the magnetic field and the temperature are normalized by $\left(e/m\right)B \to B$, and $\left(\omega_{pe}^2/c^2 m\right)T \to T$, the curl brackets denote the Poisson brackets and are defined as $\left\{a, b\right\} \equiv \left(\nabla a \times \nabla b\right)\cdot\mathbf{z}$. The dispersion relation of the linear version of Eqs.(1) is

$$\omega^2 = v_0 w_0\left[k_y^2/\left(1 + k^2\right)\right] \tag{2}$$

Note that a purely growing solution is possible for $\varepsilon_T > 2/3\varepsilon_n$, or $\left(v_0 w_0\right) < 0$, which can explain the measured strong magnetic fields in laser-produced laser experiments. Of course, due to this linear growth, the linear approximation breaks down and nonlinear effects have to be included. On the other hand, in a stable plasma, $\left(v_0 w_0\right) > 0$, the phase velocity of linear waves in the y direction has an upper limit, indeed, $-\left(v_0 w_0\right)^{1/2} < \omega/k_y < \left(v_0 w_0\right)^{1/2}$. Eqs.(1) have two invariants. Assuming an infinite (x, y) plane and sufficiently rapidly vanishing fields at the infinity, these integrals are found to be

$$E = \frac{1}{2}\int\left(B^2 + \left|\nabla B\right|^2 + \frac{v_0}{w_0}T^2\right)dxdy \quad \text{and} \quad X = \int\left(BT + \nabla B \cdot \nabla T\right)dxdy \tag{3}$$

Thus, whereas the T field is advected by the B field, it also drives it. In the full nonlinear system (1) these integrals are manifestation of this field coupling. In the weakly nonlinear approximation, it will turn out that these integrals play a role similar to that of energy and enstrophy in several 1-filed 2-D systems. In the frame of this approximation, the analysis of spectral properties of the MED modes within the model Eqs.(1) shows that the presence of these two invariants necessitates the double energy cascade as the key property of the MED mode turbulence. The nonlinear transfer of wave energy from small scales towards the long wave length region (the so-called "inverse cascade") is a cause of spontaneous generation and sustainment of large scale structures. *So, the MED mode turbulence is capable of generating the large scale wing of the wave spectrum.*

III. We now consider stationary solutions of Eqs.(1) which propagate with constant velocity $u\hat{\mathbf{y}}$. Setting $\partial/\partial t = -u\,\partial/\partial y$ and introducing the stream function $\psi = B - ux$ we find that the stationary solution will be given by

$$\nabla^2 \psi = r(\psi) + ux\left(1 - \frac{v_0}{u}s'(\psi)\right) \quad \text{and} \quad T = s(\psi) + w_0 x \qquad (4)$$

where r and s are arbitrary functions. We show now that in the set defined by (4) there exist stationary solutions which are localized in one direction and periodic in the other. To this end we choose $s(\psi) = \frac{u}{v_0}\psi$, so that the first expression in the set (4) is reduced to $\nabla^2\psi = r(\psi)$, which is the relation between the stream function (ψ) and the vorticity often used in the fluid dynamics. Consider two possible particular cases, namely,

$$r_1(\psi) = \xi\sinh\psi \quad \text{and} \quad r_2(\psi) = A\exp(-\psi/A)$$

which correspond to the "sinh-Poisson" equation and to the Liouville-equation. The solutions of these equations are well-defined in 2-D fluid dynamics and under some restrictions on free parameters they describe, physically, so-called "vortex streets". If $\nabla^2\psi = r_{1,2}(\psi)$, the solutions to these equations are known as the "breather", ψ_1, and Kelvin-Stuart cat's eyes, ψ_2, and are given by

$$\psi_1 = 4\arctan h\left(\sqrt{\frac{b}{a}}\frac{\sin\sqrt{a}\,y}{\cosh\sqrt{b}x}\right), \quad \text{and} \quad \psi_2 = 2A\ln\left[8b^2\left(2a\cosh bx + 2\sqrt{a^2-1}\cos by\right)\right] \quad (5)$$

where $\xi = b - a$, $a > 0$, $b > 0$ (ψ_1) and $a > 1$ (ψ_2), a and b are arbitrary constants. As can be seen from (5), these solutions describe vortex flow $\left(\nabla^2\psi \neq 0\right)$ which is localized in the x direction and periodic in y. In the Kelvin-Stuart cat's eyes solution, ψ_2, the parameter a describes the width of the cat's eyes. As a decreases to 1 the cat's eyes diminish and the limiting flow is purely zonal.

IY. The vortex solutions (5) are essentially nonlinear and therefore the velocity u cannot be arbitrary. Indeed, the localized solutions with finite energy must satisfy the boundary conditions $B \to 0$ and $T \to 0$ at infinity. This means that the streamlines $\psi = const$ are open and extend to infinity, except maybe in a finite region where the amplitude of B is large. In the region of open streamlines, the outer region, the functions $s(\psi)$ and $r(\psi)$ are determined uniquely by the boundary conditions at infinity and given by $s(\psi) = (w_0/u)\psi$, $r(\psi) = \rho^2\psi$, where $\rho^2 = 1 - v_0 w_0/u^2$. Taking into account these relations yields the vorticity equation for the outer region $\nabla^2 B = \rho^2 B$. Thus, $\rho^2 > 0$ is a necessary condition for the solution to be localized. The condition $u^2 > v_0 w_0$ can easily be understood from the linear dispersion relation. Any stationary structure propagating within the phase velocity interval is oscillating, while structures whose velocity is outside of this interval must have an exponentially decreasing profile. Since the amplitude of any localized structure must be small far away from the center, its velocity must therefore be outside the linear region. This is

the general property of many different nonlinear wave equations: localized, nonlinear structures and linear waves cover complementary regions in velocity space.

Y. We have performed a simulations study of the system (1) for different sets of parameters. The simulation code is based on a pseudospectral method to resolve derivatives in space with periodic boundary conditions with random fluctuations as initial conditions.
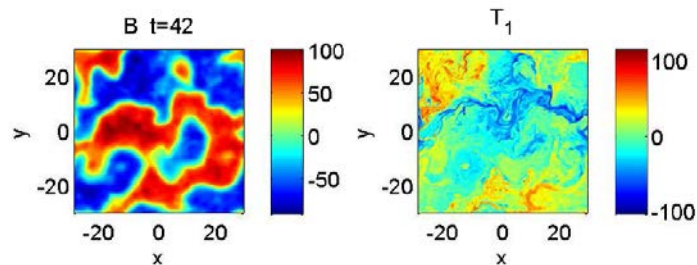

Figure 1: Linearly unstable regime. Magnetic field and temperature fluctuations

In the unstable regime, $v_0 w_0 < 0$, in Fig. 1, we could observe magnetic field generation and the formation of large scale magnetic structures, accompanied by small-scale turbulence visible in the temperature fluctuations.
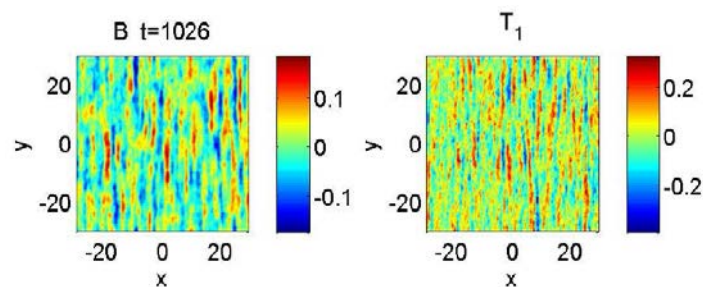

Figure 2: Linearly stable regime. Magnetic field and temperature fluctuations

In the linearly stable, small amplitude regime, $v_0 w_0 > 0$, in Fig. 2, we observe small-scale turbulence and the formation of long rows of vortex street flows (zonons).
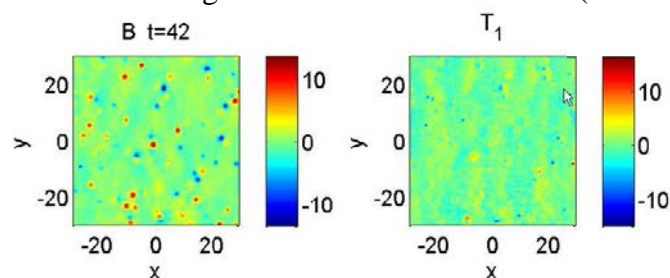

Figure 3: Large amplitude regime. Magnetic field and temperature fluctuations.

In the large amplitude regime, illustrated in Fig. 3, with initial amplitude 10 times larger than in Fig. 2, we observe the formation of vortices and vortex pairs.