Modeling adiabatic relativistic plasma expansion

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Abstract

Adiabatic relativistic plasma expansion is investigated by using two fluids model for electrons and ions. The fully ionized plasma is unmagnetized, collisionless and the dynamics governed by the combination of thermal pressure gradient and ambipolar electrostatic potential. Numerical investigations for a plasma created by high power laser is conducted based on quasi-neutrality assumption. The study focus on the effect of relativistic factor which is firstly considered only as a velocity factor and then, included in all plasma parameters. Important physical effects are inherent to fully relativistic model.

I. Introduction

Relativistic plasma has become an important subject inherent to the expansion in laboratory and space physics. Recent observation of the interaction between an intense laser pulse with a near critical gas jet provides clues of accelerated electrons at subrelativistic velocities[1]. Increasing the force exerted by light pulses having extreme intensity, electrons and even protons are accelerated to relativistic velocities[2]. Such particles accelerators suggest great utility in the fields of research in medicine, physics and engineering[3]. A major effort has been spent to study the behavior of electric field, energies and densities of particles created and accelerated by plasma expansion into another medium[4].

The expansion dynamics of relativistic plasma is studied by using two kinds of fluid models. In most of studies, authors used mathematical models in which relativistic effects are considered by using the equation of motion which can be conjectured from the classical Euler equation by simple replacement of the momentum mv with the relativistic factor momentum $m\gamma v$, where $\gamma = 1/\sqrt{1-v^2/c^2}$ is the relativistic factor. However, this approximation would sometimes lead to incorrect results even in the "weak" relativistic limit[5]. The aim of this work is to investigate the differences of such analysis in dealing with relativistic factor with an approach of fully relativistic plasma, based on a mathematical model derived from the covariant formulation of conservation of energy-momentum tensor [5]. A set of non linear differential equation is written and self-similar solution is numerically investigated.

II. Modeling

The mathematical model for a fully relativistic approach corresponds to :

$$\frac{\partial \gamma_j n_j}{\partial t} + \frac{\partial \gamma_j n_j v_j}{\partial x} = 0 \tag{1}$$

$$\frac{1}{c^2}\left(\frac{\partial h_j n_j v_j}{\partial t} + v_j \frac{\partial h_j n_j v_j}{\partial x}\right) + \frac{1}{\gamma_j n_j}\frac{\partial P_j}{\partial x} = \pm e \frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial \frac{P_j}{n_j^\alpha}}{\partial t} + v_j \frac{\partial \frac{P_j}{n_j^\alpha}}{\partial x} = 0 \tag{3}$$

Where, n_j and v_j are density and velocity respectively. P_j is the pressure, γ_j is the relativistic factor for plasma species. j=e(i) for electron (ions).
h is the enthalpy per fluid particle which is given by:

$$h_j = m_j c^2 + \frac{\alpha}{\alpha - 1}\frac{P_j}{n_j}$$

α is the polytropic index which is equal to $4/3$ for ultrarelativistic limit and $5/3$ in the non-relativistic fluid, q is the charge, m_j is the rest mass, and ϕ is the electrostatic potential.
Fluid equations $(1)-(3)$ are closed by invoking the quasi-neutrality assumption :

$$n_i = \frac{\gamma_e}{\gamma_i} n_e \tag{4}$$

The second relativistic fluid model assumes relativistic effect as a correction of th momentum $m_j v_j$ by adding the relativistic factor γ_j:

$$\frac{\partial n_j}{\partial t} + \frac{\partial n_j v_j}{\partial x} = 0 \tag{5}$$

$$\left(\frac{\partial \gamma_j v_j}{\partial t} + v_j \frac{\partial \gamma_j v_j}{\partial x}\right) + \frac{1}{n_j}\frac{\partial P_j}{\partial x} = \pm e \frac{\partial \phi}{\partial x} \tag{6}$$

$$P_j = P_{j0} N^\alpha \tag{7}$$

The quasi-neutrality assumption in this case is :

$$n_e = n_i \tag{8}$$

Using the self-similar variable in the system dynamic equations $\xi = x/(ct)$, where c is the speed of light, and the normalization of the plasma parameters as follows:

$N = \frac{n_j}{n_0}, V = \frac{v_j}{c}, \psi = \frac{eE}{\omega_{pi} m_i c}$. We derive a set of non linear-differential equations that can numerically handled with the initial condition iterative scheme.

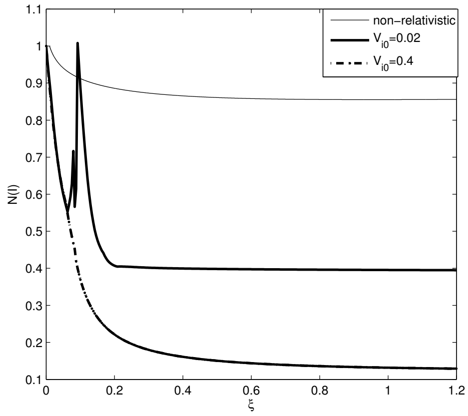

Figure 1: *Normalized density of the model I versus the self-similar variable for different regimes.*

III. Numerical results

We solve Eqs. (1-3; 5-7) for different initials values of ionic velocity. For that purpose a plasma of $n_i < 10^{22}\ cm^{-3}$ with, initial electron velocity $V_{e0} = 0.55$ and adiabatic process $(\alpha = 3)$ is used.

In fig.1, the normalized density is plotted for different situations to compare between, nonrelativistic, weakly relativistic and relativistic cases versus self-similar variable. We note that ionic density vanishes at lower values of self-similar variable when the initial electronic velocity increases. For non-relativistic case we recover the common result that is density depletion versus the self-similar variable. In weakly relativistic regime a spike-like structure appears which is associated with quasi-neutrality break down. This last result was not noted in the second model in fig.2. Dimensionless normalized ionic velocity is plotted versus self-similar variable, where the model I in black curves and model II in red ones (Fig.3).

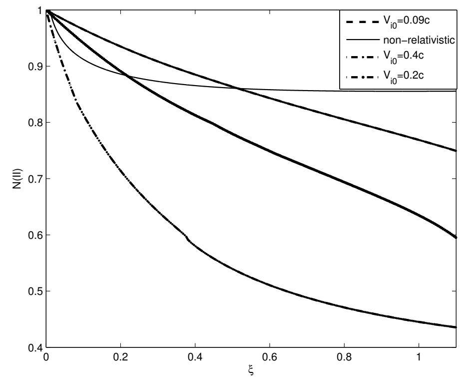

Figure 2: *Normalized density of the model II versus the self-similar variable for different regimes.*

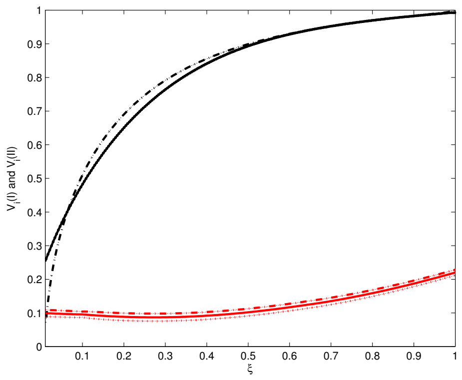

Figure 3: *Normalized velocity for the model I (Black) and model II (red) versus the self-similar variable for different initial values of ionic velocity.*

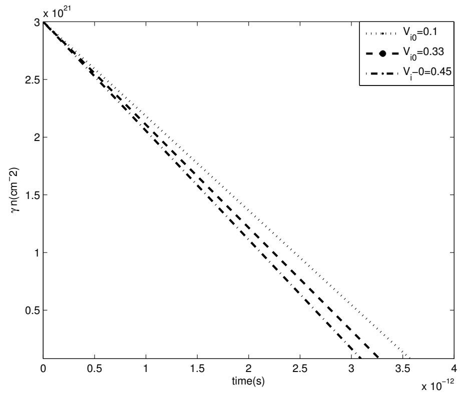

Figure 4: *Density in the co-moving frame for model I versus the self-similar variable for different initial values of ionic velocity.*

A higher acceleration of ions in the first model is observed. Our plasma model is inspired from an experiment of a jet gas target interacting with laser to create plasma with electron density 3.10^{21} cm^{-3}, within spatial size of less than 1 mm. In this experience a collisionless plasma expansion over 150 μm at relativistic velocity $C/3$. By including this experimental values in our numerical code, we get the profile of plasma density given by fig.4, which shows that the increase of the initial ionic velocity lead to increase the rate of density depletion for higher ionic velocities.

IV. conclusion

Relativistic adiabatic plasma expansion is investigated by using two set of equations. Numerical results show that the expansion is slower when the relativistic factor is included only in momentum equation. A fully relativistic model that assumes a relativistic factor contribution in all plasma parameters, leads to particles acceleration. This results is coherent with data provided par experiments on high intensity laser interacting with gas jet target. The first model did not sustain the quasi-neutrality assumption far away from the plasma source region.

References

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