

## Modelling of electron heating in a Penning-Malmberg trap by means of a chaotic map

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We have previously shown that an electron plasma can be produced within the ELTRAP (ELECTRON TRAP) Penning-Malmberg device applying a radio-frequency (RF) excitation on one of the trap electrodes [1]. While RF electric fields are routinely used to create plasmas via generation of discharges in neutral gases, the common features in plasma sources are the need for a significant input power and relatively high pressures ( $10^{-3} - 10$  mbar). On the contrary, a RF excitation with an amplitude of few volts and frequencies in the  $1 - 30$  MHz range is enough to observe the formation of an electron plasma with densities of  $10^6 - 10^8$  cm $^{-3}$ , i.e. comparable to the typical values obtained for Penning traps with more conventional sources. The method takes advantage of the peculiar characteristics of Penning devices: In trapping configuration, i.e. with proper voltages on the outermost electrodes (e.g.,  $\approx -100$  V) and high axial magnetic field, and thanks to the ultra-high vacuum (UHV) conditions, the mean free path of any electron present in the room-temperature residual gas can be as large as hundreds to thousands of meters. A RF excitation applied on any of the trap electrodes can thus effectively heat the free electrons beyond the first ionization energy of light gases, and an appreciable density of confined electrons is achieved within the time span of few hundreds of milliseconds.

Systematic studies of the total confined charge as a function of RF frequency, amplitude and axial position have shown a complicated, non-resonant behaviour characterized by a geometry-sensitive lower threshold in the MHz range and wide frequency bands beyond the threshold, where appreciable amounts of plasma are obtained (see Fig. 7 in Ref. [1]). In order to get a better understanding of the phenomenon, we have modelled the heating of a single electron by means of a two-dimensional (time-energy), area-preserving map where a particle bounces within a square potential well and an intermediate, oscillating square barrier of amplitude  $V_1$  represents the RF drive. This is a good approximation of the trap potential profile close to the electrodes' inner surface (see the sketch in the left panel of Fig. 1). The particle is given an initial longitudinal energy  $E_k$  and when it reaches the barrier can either be elastically reflected ( $E_k < eV_1$ ) or cross it ( $E_k > eV_1$ ) and continue with a new energy  $E_k - eV_1$ . Such one-dimensional model falls into the class of Fermi-like maps [2], and similar mappings have been used to describe a variety of physical situations. Taking as a reference the works of Mateos [3] and Leonel [4], who add a static barrier offset  $V_0 > V_1$ , we performed a characterization of our

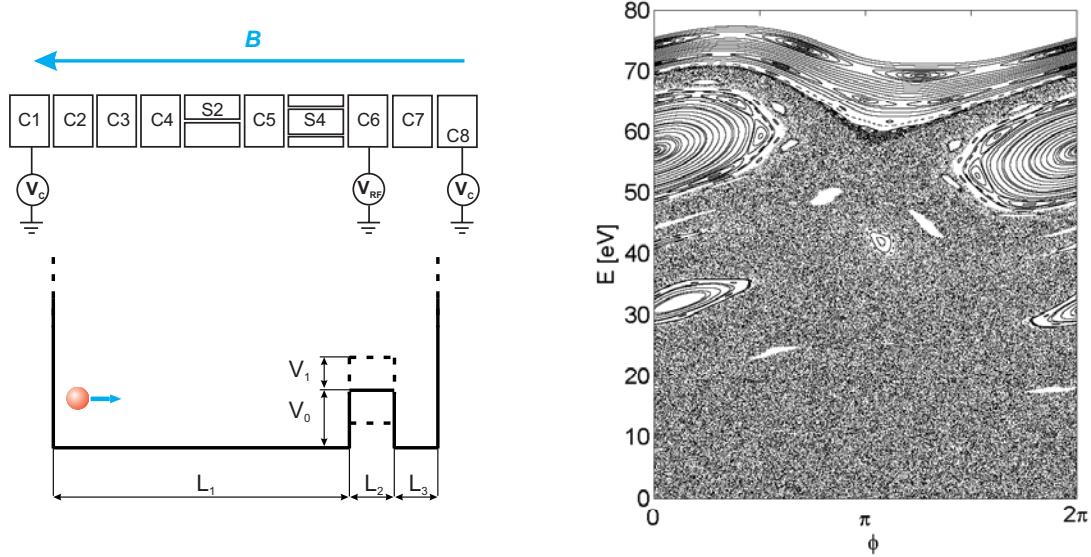


Figure 1: *Top left:* Sketch of ELTRAP electrode stack. C1-C8 electrodes are 90 mm long, S2, S4 are 150 mm long. Confining voltages  $V_c$  are applied on electrodes C1, C8. RF drive  $V_{RF} = V_1 \sin(2\pi vt)$  is applied on any inner electrode. *Bottom left:* Sketch of the map model, with  $V_0$  a generic static barrier offset. *Right:* Poincaré plot of the map shown on the left. The particle energy at the leftmost boundary is shown versus time normalized to the RF period for an electron bouncing in a well reproducing the ELTRAP configuration with RF on the C6 electrode, i.e.  $L_1 = 90$  mm,  $L_2 = 90$  mm,  $L_3 = 660$  mm,  $V_0 = 0$ , RF amplitude  $V_1 = 3.8$  V and frequency  $v = 8$  MHz.

map including the offset but eliminating the constraint  $V_0 > V_1$ . Therefore, depending on the geometry and  $V_0$ ,  $V_1$  amplitudes, one to three particle trapping regions are defined.

A representation of the particle evolution described by the map is given by the Poincaré plot, where the energy at each iteration (i.e. any time the particle reaches the leftmost boundary) is plotted versus the corresponding time instant expressed as the instantaneous phase  $\phi$  of the RF oscillation. An example is shown in the right panel of Fig. 1, where the energy states of several particles with different initial energy have been plotted over some  $10^6$  iterations using typical ELTRAP geometry and RF parameters. In the lower part, a chaotic region is present, which the particle can fully explore starting from any of its points. Notice that energies up to 70 eV can be reached, allowing for ionization of the residual gas.

The characterization of the chaoticity and efficiency of the heating process has been performed calculating the so-called roughness  $\omega$  for any given choice of the set of parameters. Roughness indicates the normalized standard deviation of energy states and is thus defined as

$$\omega = \frac{1}{P} \sum_{i=1}^P \sqrt{\overline{e_i^2}(n) - \overline{e_i}^2(n)} \quad (1)$$

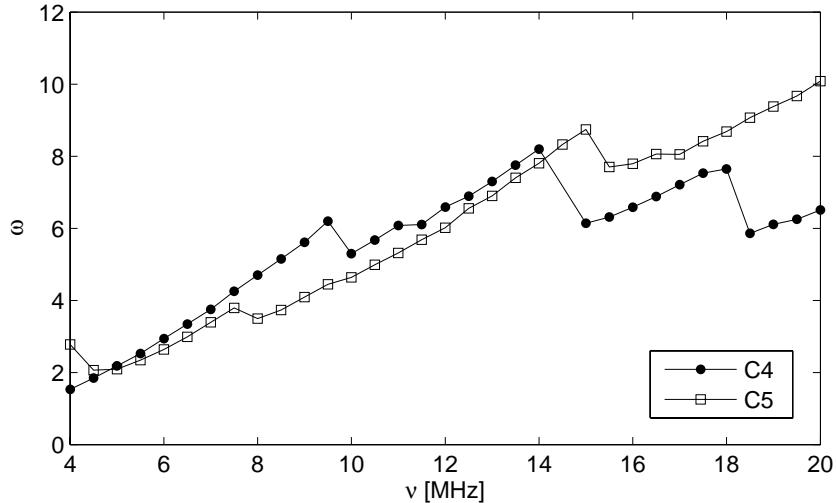


Figure 2: Roughness trend vs excitation frequency for the three-region map with ELTRAP geometry and different excitation electrode. Roughness values are averaged over 500 initial conditions in the lowest chaotic region of the map, iterated for  $2 \cdot 10^7$  cycles. Parameters:  $V_0 = 0$ ,  $V_1 = 3.8$  V.

where  $\bar{e}_i(n)$  and  $\bar{e}_i^2(n)$  are the mean over the  $n$  iterations of energy and squared energy states normalized to the barrier energy  $eV_1$  of the  $i$ -th particle, and an average is performed over  $P$  particles with initial phase uniformly distributed in the  $[0 - 2\pi)$  interval. Roughness initially increases with  $n$  and reaches a saturation value after a large number of iterations. From this point on we will always refer to its saturation value. Figure 2 depicts the behaviour of the roughness versus the RF excitation frequency for two possible ELTRAP geometries, in the range 4 – 20 MHz. The roughness shows piecewise power-law growth regions interrupted by abrupt transitions. Similar transitions have been observed for the Lyapunov exponent of similar maps by Leonel [4], and attributed to the collapse of invariant curves and merging of previously distinct chaotic regions.

An extensive characterization was performed for a simplified map with two regions only ( $L_3 = 0$  in the sketch of Fig. 1-left), which reduces the number of free parameters of the general case while retaining many of its properties. The left panel in Fig. 3 shows the roughness for maps with constant total length  $L_{tot} = 1$  m and varying the barrier length  $L_2$ , with the RF frequency as a parameter. The roughness exhibits a complicated structuring, with no clear trend with respect to the geometry. Nevertheless, for increasing RF frequency one can notice that the shape of  $\omega$  versus  $L_2$  shows an increase in the mean level together with a compression towards the left, suggesting the presence of a universal scaling law depending on the combination of barrier length and frequency. The right panel of Fig. 3 highlights how roughness is a piecewise smooth

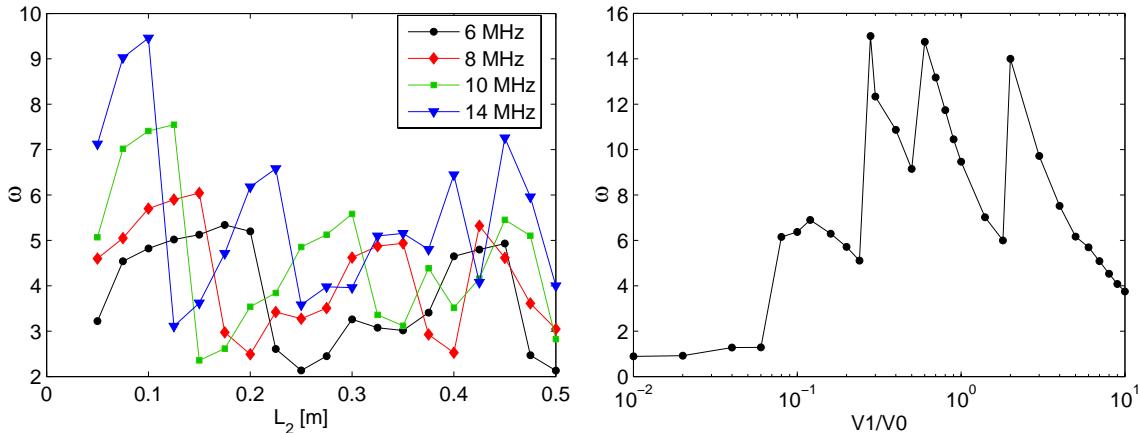


Figure 3: Roughness systematic study for the two-region map. Roughness values are averaged over 500 initial conditions in the lowest chaotic region of the map, iterated for  $2 \cdot 10^7$  cycles. Left: Roughness  $\omega$  as a function of the barrier width  $L_2$  for different excitation frequencies. Parameters:  $V_0 = 0$ ,  $V_1 = 3$  V,  $L_{tot} = 1$  m. Right: Roughness trend vs barrier amplitude. Parameters:  $V_0 = 0.5$  V,  $L_{tot} = 1$  m,  $L_2 = 0.1$  m.

function of the barrier amplitude interrupted by sharp transitions, in a similar fashion to the dependence on the frequency (Fig. 2).

Ionization of a light gas background was implemented via a Monte Carlo scheme. The first ionization cross section of  $H_2$  was used [5]. At a pressure of  $10^{-8}$  mbar, ionization introduces just a very weak dissipation in the map, and as a consequence we found out that the ion count follows well the roughness trend versus barrier length. While the presence of an offset was found to be of scarce influence on the roughness, the effect on ionization is significant as for  $V_0 > V_1$  a particle may lose enough energy by ionization to be indefinitely trapped within one of the map regions, without possibility of successive re-heating.

## References

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